Lattice Reduction Aided Detection in Large-MIMO Systems

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***Abstract*—Lattice reduction (LR) aided detection algorithms are known to achieve the same diversity order as that of maximum-likelihood (ML) detection at low complexity. However, they suffer SNR loss compared to ML performance. The SNR loss is mainly due to imperfect orthogonalization and imperfect nearest neighbor quantization. In this paper, we propose an improved LR-aided (ILR) detection algorithm, where we specifically target to reduce the effects of both imperfect orthogonalization and imperfect nearest neighbor quantization. The proposed ILR detection algorithm is shown to achieve near-ML performance in large-MIMO systems and outperform other LR-aided detection algorithms in the literature. Specifically, the SNR loss incurred by the proposed ILR algorithm compared to ML performance is just 0.1 dB for 4-QAM and < 0.5 dB for 16-QAM in 16×16 V-BLAST MIMO system. This performance is superior compared to those of other LR-aided detection algorithms, whose SNR losses are in the 2 dB to 9 dB range.**

Keywords – *Lattice reduction, Seysen's algorithm, near-optimal detection, large-MIMO systems.*

I. Introduction

The capacity of multiple-input multiple-output (MIMO) wireless channels in rich scattering environments is known to increase linearly with the minimum of the number of transmit and receive antennas [1]. Multiple antennas at both the transmitter and the receiver can be exploited to achieve high spectral efficiency and/or diversity resulting in better error performance. Large-MIMO systems with tens to hundreds of antennas are getting increased research attention [2], [3]. Practical implementation of large-MIMO systems is challenging due to the exponential complexity of optimum MIMO signal detection. Sphere decoder is a well known maximum-likelihood (ML) detector. Its average case complexity increases exponentially with the size of the system, and hence becomes computationally infeasible for number of real dimensions more than 32 [5]. This motivates the need for alternate low complexity detection algorithms for large-MIMO systems with tens to hundreds of antennas.

Several low complexity algorithms based on local search [2], mixed Gibbs sampling [4], and message passing have been proposed in literature for large-MIMO detection. Lattice reduction (LR) aided detection is a promising approach [6], [7], [8], which needs investigation in the context of large-MIMO systems. LR-aided algorithms are a set of algorithms which take the closest lattice point search approach to the MIMO detection problem. This approach considers a lattice whose basis vectors are given by the columns of the MIMO channel matrix. For a given lattice, lattice reduction aims to find a set of small, nearly orthogonal vectors which forms the basis of the given lattice. Various algorithms such as Lagrange reduction algorithm, Gauss reduction, Hermite reduction algorithm, Korkine-Zolotareff reduction algorithm, and Minkowski reduction algorithm serve this purpose [6]. A major work in this regard came up in 1982, when A. K. Lenstra, H. W. Lenstra, and L. Lovász introduced a reduction algorithm known as LLL algorithm [9]. A few years later M. Seysen proposed another algorithm to find the reduced basis of a lattice and its dual simultaneously [10]. Recently, another LR algorithm known as D-ELR-SLB algorithm has been proposed in [11].

In [12], it has been proved that for a channel matrix with bounded condition number, the zero-forcing (ZF) detector achieves the same diversity order as that of the maximum-likelihood (ML) detector. Lattice reduction transforms the channel matrix into another matrix with near-orthogonal columns, and hence with a better condition number. Thus, linear detectors yield improved performance when operated in the transformed domain. Performance of the LR-aided ZF (LR-ZF) and LR-aided MMSE (LR-MMSE) detectors have been studied in [13]. In [13], it is shown that linear detectors using Seysen's reduction algorithm perform better than using LLL reduction algorithm, and is computationally less complex. LR-MMSE detector performs better than LR-ZF detector, but its performance is poorer compared to ML performance. In [14], an algorithm referred to as LR-AUG-ZF algorithm ('AUG' stands for 'augmented') was proposed to exploit the advantages of both lattice reduction and MMSE. Another improved LR decoder (referred to as 'LRdecimp') reported in [15] gives almost ML performance in small dimensional systems but suffers significant SNR loss compared to ML performance in large dimension systems.

The main reasons behind the SNR loss in LR-aided detectors compared to ML performance are imperfect orthogonalization and imperfect nearest neighbor quantization. In this paper, we propose an algorithm that addresses these issues. The proposed algorithm, which we refer to as improved LR-aided (ILR) algorithm, breaks the problem into smaller problems based on a reliability criteria. Solution of the smaller problem along with reliable solution gives an intermediate solution vector. This part of the algorithm targets to reduce the effect of imperfect orthogonalization. Further, a set of lattice points which are neighbors of the intermediate solution vector in the transformed domain is found. The best solution among the found neighbors based on the ML criterion is selected. This reduces the effect of imperfect quantization. We use Seysen's algorithm and D-ELR-SLB algorithm for lattice reduction. Simulation results show that the proposed ILR algorithm performs better than other LR-aided detection algorithms.

Notations: Bold lowercase and uppercase letters denote column vectors and matrices, respectively. r_j denotes the jth coordinate of a vector $\mathbf{r}$. $r_{i,j}$ denotes the entry in ith row and jth column of a matrix $\mathbf{R}$. $\mathbf{I}_n$ denotes the $n \times n$ identity

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matrix. $|\mathcal{A}|$ and $\mathcal{A}^c$ denote the cardinality and complement of set $\mathcal{A}$, respectively. $\mathcal{Q}(.)$ denotes the nearest neighbor quantization operation. $\Re(.)$ and $\Im(.)$ denote the real and complex parts of a complex argument, respectively. $(.)^T$ denotes the transpose operation.

II. System model

Consider a V-BLAST MIMO system with n_t transmit antennas and n_r receive antennas. The MIMO channel matrix is denoted by $\mathbf{H}_c$, whose (i,j)th entry $h_{c_{i,j}}$ denotes the complex channel gain between jth transmit antenna and ith receive antenna. $h_{c_{i,j}}, \forall i = 1,\cdots,n_r, j = 1,\cdots,n_t$ are circularly symmetric complex Gaussian distributed with mean zero and unit variance. Each transmit antenna transmits a complex symbol from a square constellation set $\mathcal{A} + j\mathcal{A}$, where $\mathcal{A}$ is a subset of $\mathbb{Z}$ (set of integers). For example, for 16-QAM constellation set, $\mathcal{A} = \{-3,-1,1,3\}$. The received signal vector at the receiver is given by

$$\mathbf{y}_c = \mathbf{H}_c\mathbf{x}_c + \mathbf{n}_c, \tag{1}$$

where $\mathbf{y}_c$ is the received vector of size $n_r \times 1$, $\mathbf{x}_c$ is the transmitted vector of size $n_t \times 1$, $\mathbf{n}_c$ is the complex additive white Gaussian noise vector of size $n_r \times 1$, and $\mathbf{n}_c \sim \mathcal{CN}(0, \sigma^2\mathbf{I}_{n_r})$. The average signal-to-noise ratio in dB is defined as $10\log_{10}\frac{n_tE_s}{\sigma^2}$, where E_s is the average signal energy. The complex-valued system model in (1) can be converted into a real-valued system model

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{2}$$

by the following transformations:

$$\mathbf{H} = \begin{bmatrix} \Re(\mathbf{H}_c) & -\Im(\mathbf{H}_c) \\ \Im(\mathbf{H}_c) & \Re(\mathbf{H}_c) \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} \Re(\mathbf{y}_c) \\ \Im(\mathbf{y}_c) \end{bmatrix},$$
$$\mathbf{x} = \begin{bmatrix} \Re(\mathbf{x}_c) \\ \Im(\mathbf{x}_c) \end{bmatrix}, \quad \mathbf{n} = \begin{bmatrix} \Re(\mathbf{n}_c) \\ \Im(\mathbf{n}_c) \end{bmatrix}. \tag{3}$$

Under the assumption that the channel matrix is known at the receiver, and that all the transmitted vectors are equally likely, the ML decision rule can be expressed as

$$\mathbf{x}_{ML} = \underset{\mathbf{x}\in\mathcal{A}^{2n_t}}{\operatorname{argmin}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2. \tag{4}$$

Brute force evaluation of the problem in (4) involves the computation of ML cost, i.e., $\|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2$, for all $|\mathcal{A}|^{2n_t}$ possible transmit vectors, which is computationally infeasible for large $|\mathcal{A}|$ and n_t. Hence, our interest is in low-complexity algorithms that can achieve close to ML performance in large dimensions.

III. LR-aided detectors

In this section, we present a brief summary of existing LR-aided detectors. Let us define lattice $\mathcal{S}$ as

$$\mathcal{S} \triangleq \{\mathbf{x} : x_i \in \mathcal{A}, 1 \le i \le 2n_t\}, \tag{5}$$

and

$$\mathcal{V} \triangleq \{\mathbf{v} : \mathbf{v} = \mathbf{H}\mathbf{x}, x_i \in \mathcal{A}, 1 \le i \le 2n_t\}. \tag{6}$$

Note that $\mathcal{S}$ denotes the transmit lattice and $\mathcal{V}$ is the lattice containing received points without noise. Reducing the channel matrix $\mathbf{H}$ using any LR algorithm, we get $\widetilde{\mathbf{H}} = \mathbf{H}\mathbf{T}$, where $\mathbf{T}$ is a unimodular matrix. Thus, the system model equation (2) changes to

$$\mathbf{y} = \widetilde{\mathbf{H}}\mathbf{z} + \mathbf{n}, \tag{7}$$

where $\mathbf{z} = \mathbf{T}^{-1}\mathbf{x}$. The detected solution vectors obtained using LR-ZF and LR-MMSE detectors, denoted by $\widehat{\mathbf{x}}_{\text{LR-ZF}}$ and $\widehat{\mathbf{x}}_{\text{LR-MMSE}}$ are computed as follows:

$$\widehat{\mathbf{x}}_{\text{LR-ZF}} = \mathcal{Q}\left(\mathbf{T}\,\mathcal{Q}\left([\widetilde{\mathbf{H}}^T\widetilde{\mathbf{H}}]^{-1}\widetilde{\mathbf{H}}^T\mathbf{y}\right)\right)$$

$$\widehat{\mathbf{x}}_{\text{LR-MMSE}} = \mathcal{Q}\left(\mathbf{T}\,\mathcal{Q}\left([\widetilde{\mathbf{H}}^T\widetilde{\mathbf{H}} + \frac{\sigma}{\sqrt{n_tE_s}}\mathbf{T}^T\mathbf{T}]^{-1}\widetilde{\mathbf{H}}^T\mathbf{y}\right)\right).$$

In [14], the authors have proposed an enhanced LR-aided linear detector by rewriting the system model equation as follows:

$$\begin{bmatrix} \mathbf{y} \\ \mathbf{0}_{2n_t} \end{bmatrix} = \begin{bmatrix} \mathbf{H} \\ \frac{\sigma}{\sqrt{n_tE_s}}\mathbf{I}_{2n_t} \end{bmatrix}\mathbf{x} + \begin{bmatrix} \mathbf{n} \\ -\frac{\sigma}{\sqrt{n_tE_s}}\mathbf{I}_{2n_t}\mathbf{x} \end{bmatrix}. \tag{8}$$

Performing ZF on (8) is equivalent to performing MMSE on (2). Let us define $\mathbf{y}_{\text{aug}} = \begin{bmatrix} \mathbf{y} \\ \mathbf{0}_{2n_t} \end{bmatrix}$ and $\mathbf{H}_{\text{aug}} = \begin{bmatrix} \mathbf{H} \\ \frac{\sigma}{\sqrt{n_tE_s}}\mathbf{I}_{2n_t} \end{bmatrix}$. Lattice reduction of matrix $\mathbf{H}_{\text{aug}}$ gives us $\widetilde{\mathbf{H}}_{\text{aug}} = \mathbf{H}_{\text{aug}}\mathbf{T}_{\text{aug}}$, where $\mathbf{T}_{\text{aug}}$ is a unimodular matrix. LR-aided ZF on augmented system (LR-AUG-ZF) detector performs LR-ZF on (8) to obtain $\widehat{\mathbf{x}}_{\text{LR-AUG-ZF}}$ as

$$\widehat{\mathbf{x}}_{\text{LR-AUG-ZF}} = \mathcal{Q}(\mathbf{T}_{\text{aug}}\mathcal{Q}((\widetilde{\mathbf{H}}^T_{\text{aug}}\widetilde{\mathbf{H}}_{\text{aug}})^{-1}\widetilde{\mathbf{H}}^T_{\text{aug}}\mathbf{y}_{\text{aug}})).$$

Though the LR-AUG-ZF detector in [14] improved the BER performance beyond the performance of LR-ZF and LR-MMSE detectors, it still performs significantly poorer compared to the ML detector.

In [15], another LR-aided detector known as 'LRdecimp' is proposed. In the LRdecimp algorithm, x_i is assumed to be known as $\mathcal{A}_j$ and its effect is removed from the system equation. LR-ZF or LR-MMSE or LR-AUG-ZF detection is performed on this altered system model to obtain a candidate solution. This operation is repeated for $1 \le i \le 2n_t$, and $1 \le j \le |\mathcal{A}|$, and the best among the candidate vectors is declared as the final solution. LRdecimp algorithm achieves very close to ML performance in small number of antennas, but its performance degrades with increasing system dimension.

The main reasons behind the poor performance in LR-aided detectors compared to ML performance are imperfect orthogonalization and imperfect nearest neighbor quantization. The reduced lattice is not perfectly orthogonal, and hence linear detectors perform sub-optimally. The SNR loss compared to the optimal performance increases with the number of dimensions. Also, the different coordinates of the coefficient vector in the transformed domain are not independent. This information is completely ignored during quantization, causing information loss which results in detecting a vector in the transformed domain $\mathbf{z}$ such that $\mathbf{z} \neq \mathbf{T}\mathbf{x}$ for any $\mathbf{x} \in \mathcal{S}$.

In the following section, we propose an improved LR-aided (ILR) detection algorithm which tries to alleviate both the above mentioned problems faced in LR-aided detection.

IV. Proposed Improved LR-aided (ILR) detection

In this section, we propose an improved LR-aided (ILR) detection algorithm which mitigates the effects of both imperfect orthogonalization and imperfect quantization. It can be observed that, if we perform lattice reduction on a subspace of a given space, the obtained basis vectors are more orthogonal than the basis vectors obtained by performing lattice reduction of the given space. Thus, we break the problem into problems of smaller dimensions where the task is to reduce the basis vectors of a subspace of the entire space spanned by the columns of the channel matrix $\mathbf{H}$. This gives us better orthogonality and the effect of imperfect orthogonalization can be reduced. We also alleviate the effect of imperfect nearest neighbor quantization by finding the best neighbor of the detected $\mathbf{z}$, which when transformed back into x-domain gives a valid lattice point in $\mathcal{S}$.

In the proposed ILR algorithm, for $1 \leq i \leq 2n_t$ and $1 \leq j \leq |\mathcal{A}|$, x_i is assumed to be known as $\mathcal{A}_j$ and its effect is removed from (2). Thus, the system model transforms to

$$\mathbf{y}' = \mathbf{H}'\mathbf{x}' + \mathbf{n}, \tag{9}$$

where $\mathbf{y}' = \mathbf{y} - \mathcal{A}_j\mathbf{h}_i$, $\mathbf{H}' = [\mathbf{h}_1\,\mathbf{h}_2 \cdots \mathbf{h}_{i-1}\,\mathbf{h}_{i+1} \cdots \mathbf{h}_{2n_t}]$, $\mathbf{x}' = [x_1\,x_2 \cdots x_{(i-1)}\,x_{(i+1)} \cdots x_{(2n_t)}]^T$. The solution of (9) along with $\mathcal{A}_j$ gives a candidate solution vector. For different (i,j) pairs, $2n_t|\mathcal{A}|$ candidate solutions are obtained. The final solution is chosen based on the ML cost among all the $2n_t|\mathcal{A}|$ candidate solutions. The candidate solution is found by the following procedure.

In order to get the advantages of both MMSE and LR together, we change (9) to

$$\mathbf{y}'_{\text{aug}} = \mathbf{H}'_{\text{aug}}\mathbf{x}' + \mathbf{n}'_{\text{aug}}, \tag{10}$$

where $\mathbf{y}'_{\text{aug}} = \begin{bmatrix}\mathbf{y}' \\ \mathbf{0}_{2n_t-1}\end{bmatrix}$, $\mathbf{H}'_{\text{aug}} = \begin{bmatrix}\mathbf{H}' \\ \frac{\sigma}{\sqrt{n_t E_s}}\mathbf{I}_{2n_t-1}\end{bmatrix}$, $\mathbf{n}'_{\text{aug}} = \begin{bmatrix}\mathbf{n} \\ -\frac{\sigma}{\sqrt{n_t E_s}}\mathbf{I}_{2n_t-1}\mathbf{x}'\end{bmatrix}$. Lattice reduction on $\mathbf{H}'_{\text{aug}}$ gives $\widetilde{\mathbf{H}}'_{\text{aug}}$ and $\mathbf{T}'_{\text{aug}}$ related as $\widetilde{\mathbf{H}}'_{\text{aug}} = \mathbf{H}'_{\text{aug}}\mathbf{T}'_{\text{aug}}$. The system equation (10) is solved to obtain $\mathbf{r}'$ and $\mathbf{z}'$ as

$$\mathbf{r}' = (\widetilde{\mathbf{H}}'^{T}_{\text{aug}}\widetilde{\mathbf{H}}'_{\text{aug}})^{-1}\widetilde{\mathbf{H}}'^{T}_{\text{aug}}\mathbf{y}'_{\text{aug}}, \tag{11}$$

$$\mathbf{z}' = \mathcal{Q}(\mathbf{r}'). \tag{12}$$

We decide $z'_i, i \in \{1, 2, \cdots, 2n_t\}$ to be reliable based on a reliability criteria. Reliable coordinates are assumed to be correctly detected and their effects are removed from $\mathbf{z}'$. Thus, we are left with an over determined system problem which is solved in the same way as the original problem, recursively. The more is the number of reliable coordinates, the better is the orthogonality in the next recursion. So, the reliable condition should be such that only the correctly detected coordinates are considered reliable, and, at the same time, correctly detected coordinates should not be considered unreliable. Thus, both too weak and too strong reliable condition will give us poor performance. We employ the following reliability criteria:

$$\begin{aligned} |z'_i - r'_i| &< \theta_1, \\ \sigma^2 g_{i,i} &< \theta_2, \quad \text{and} \\ (\sigma^2 g_{i,i})(|z'_i - r'_i|) &< \theta_3, \end{aligned} \tag{13}$$

where $\mathbf{G} = ((\widetilde{\mathbf{H}}'_{\text{aug}})^T(\widetilde{\mathbf{H}}'_{\text{aug}}))^{-1}$, and $\theta_1, \theta_2, \theta_3$ are heuristically chosen parameters. For a PAM constellation set with only odd symbols, e.g., $\{-3, -1, 1, 3\}$, $|z'_i - r'_i| < 1\ \forall i \in \{1, 2, \cdots, 2n_t\}$. So, the range of θ_1 is $0 < \theta_1 < 1$. Smaller the value of θ_1, stronger is the reliable condition. θ_2 is a measure of the covariance of noise. Large values of θ_2 means the coordinate with large variance of noise is considered reliable. θ_3 further strengthens the reliability condition and is always less than $\theta_1\theta_2$.

The above recursive algorithm is continued until no reliable coordinate is found or depth of recursion (d_p) is less than the maximum allowed number of recursions, denoted by max_depth. In this case, the quantized solution is assumed to be free from the effects of imperfect orthogonalization. We refer to above procedure as 'modification 1' (labeled as 'Mod.1'), which is listed in **Algorithm 1** below.

Algorithm 1 Mod.1_Procedure (Detection using reliability criteria)

Input: $\mathbf{y}$, $\mathbf{H}$, max_depth, d_p, θ_1, θ_2, θ_3;
Set n_t = number of columns in ($\mathbf{H}$);
Set $\mathcal{R}$ = NULL; /* $\mathcal{R}$ is a set of reliable coordinates */
$(\widetilde{\mathbf{H}}, \mathbf{T})$ = LR ($\mathbf{H}$); /* Reduce the channel matrix $\mathbf{H}$ */
$\mathbf{r} = (\widetilde{\mathbf{H}}^T\widetilde{\mathbf{H}})^{-1}\widetilde{\mathbf{H}}^T\mathbf{y}$;
$\bar{\mathbf{z}} = \mathcal{Q}(\mathbf{r})$;
$\mathbf{f} = \bar{\mathbf{z}}$;
for $i = 1 : n_t$ **do**
  if (13) is satisfied **then**
    Add i to $\mathcal{R}$.
  else
    Set $f_i = 0$.
  end if
end for
$\mathbf{y}_{\text{layer}} = \mathbf{y} - \widetilde{\mathbf{H}}\mathbf{f}$;
$\mathbf{H}_{\text{layer}} = \{\widetilde{\mathbf{h}}_i\}\ \forall i \in \mathcal{R}^c$;
if $|\mathcal{R}| == 0$ or $d_p > max\_depth$ **then**
  $\bar{\mathbf{z}}(\mathcal{R}^c) = \mathcal{Q}([(\mathbf{H}^T_{\text{layer}}\mathbf{H}_{\text{layer}})^{-1}\mathbf{H}^T_{\text{layer}}\mathbf{y}_{\text{layer}}])$;
else
  $[\bar{\mathbf{z}}(\mathcal{R}^c), \mathbf{B}]$ = Mod.1_Procedure($\mathbf{y}_{\text{layer}}$, $\mathbf{H}_{\text{layer}}$, max_depth, $d_p + 1$, θ_1, θ_2, θ_3);
end if
Output: $\mathbf{x} = \mathbf{T}\bar{\mathbf{z}}$, $\mathbf{T}$.

Let $\mathbf{x}'_{det}$ be the solution vector obtained from **Algorithm 1**. Let $\mathbf{z}'_{det} = \mathbf{T}'^{-1}_{\text{aug}}\mathbf{x}'_{\text{det}}$. Any lattice point $\mathbf{z}'_{lat} \in \mathcal{V}'$, where $\mathcal{V}' = \{\mathbf{v}' : \mathbf{v}' = \mathbf{H}'\mathbf{x}', \mathbf{x}' \in \mathcal{A}^{2n_t-1}\}$ is related to $\mathbf{z}'_{det}$ as

$$\mathbf{z}'_{lat} = \mathbf{z}'_{det} + \mathbf{w}, \tag{14}$$

where $\mathbf{w}$ is a quantized noise vector. The problem now reduces to finding a $\mathbf{z}'_{lat}$ close to $\mathbf{z}'_{det}$, such that $\|\mathbf{y}'_{\text{aug}} - \widetilde{\mathbf{H}}'_{\text{aug}}\mathbf{z}'_{lat}\|^2$ is minimum possible. Let $\mathbf{z}'^{*}_{lat}$ be the solution and the corresponding noise vector be $\mathbf{w}^*$. Let us consider $\mathcal{A} \in \{\pm 1, \pm 3, \cdots, \pm(2n-1)\}$ for some $n \in \mathbb{Z}$. Since $\mathbf{z}'^{*}_{lat} = \mathbf{T}'^{-1}_{\text{aug}}\mathbf{x}'$, $z'^{*}_{lat_i}$ can be written as

$$z'^{*}_{lat_i} = \sum_{j=1}^{2n_t-1} u_{i,j} x'_j, \tag{15}$$

where $\mathbf{U} = \mathbf{T}'^{-1}_{\text{aug}}$ is a unimodular matrix. Defining $l_{max} \triangleq (2n-1) \sum_{j=1}^{2n_t-1} u_{i,j}$, from (15), we have

$$z'^{*}_{lat_i} \in \{-l_{max}, (-l_{max}+2), \cdots, l_{max}\}. \tag{16}$$

Thus, in (14), we have $w_i^* \in \{0, \pm 2, \pm 4, \cdots\}$. As $\widetilde{\mathbf{H}}'_{\text{aug}}$ is well conditioned, the chances of $w_i^* \in \{\pm 4, \pm 6, \cdots\}$ for any $i \in \{1, 2, \cdots, (2n_t - 1)\}$ is very low. This motivates us to consider $\mathbf{w}^* \in \{0, \pm 2\}^{2n_t - 1}$ only. Again, as $\widetilde{\mathbf{H}}'_{\text{aug}}$ is well conditioned, the zeroth norm of $\mathbf{w}^*$ is directly related to its Euclidean norm. Thus, we find $\mathcal{Z} = \{\mathbf{z} : \mathbf{z}'_{det} + \mathbf{w}, \mathbf{w} \in \{0, \pm 2\}^{(2n_t - 1)}, \|\mathbf{w}\|_0 \leq 2\}$. The best vector in $\mathcal{Z}$ based on the ML cost is the solution vector of (9) in the transformed domain. The above procedure, termed as 'Modification 2' (labeled as 'Mod.2'), aims to reduce the effect of imperfect nearest neighbor quantization.

The overall ILR algorithm that uses both modifications Mod.1 and Mod.2 is listed in **Algorithm 2**.

Algorithm 2 Improved LR-aided (ILR) detection algorithm.

1: Set $count = 0$, $\mathbf{m} = [0_{2n_t|\mathcal{A}|}]$, $\mathbf{V} = [0_{2n_t \times 2n_t|\mathcal{A}|}]$;
2: **for** $i = 1 : 2n_t$ **do**
3:   **for** $j = 1 : |\mathcal{A}|$ **do**
4:     $\mathbf{y}' = \mathbf{y} - \mathcal{A}_j \mathbf{h}_i$;
5:     $\mathbf{H}' = [\mathbf{h}_1\, \mathbf{h}_2 \cdots \mathbf{h}_{i-1}\, \mathbf{h}_{i+1} \cdots \mathbf{h}_{2n_t}]$;
6:     $\mathbf{y}_{\text{aug}} = [(\mathbf{H}')^T\, \mathbf{0}^T_{(2n_t-1)}]^T$;
7:     $\mathbf{H}_{\text{aug}} = [(\mathbf{H}')^T\, \mathbf{I}_{(2n_t-1)}]^T$;
8:     $[\bar{\mathbf{x}}, \mathbf{T}] =$ Mod.1_Procedure ($\mathbf{y}_{\text{aug}}, \mathbf{H}_{\text{aug}}$, max_depth, 1, θ_1, θ_2, θ_3);
9:     $\mathcal{Z} = \{\mathbf{z} : \mathbf{z}'_{det} + \mathbf{w}, \mathbf{w} \in \{0, \pm 2\}^{(2n_t-1)}, \|\mathbf{w}\|_0 \leq 2\}$;
10:     $\mathbf{c} = [0_{|\mathcal{Z}|}]$;
11:     **for** $k = 1 : |\mathcal{Z}|$ **do**
12:       $c_k = \|\mathbf{y}_{\text{aug}} - \mathbf{H}_{\text{aug}} \mathcal{Z}_k\|^2$;
13:     **end for**
14:     $l = \underset{k}{\text{argmin}}\ c_k$;
15:     $\breve{\mathbf{z}} = \mathcal{Z}_l$;   /* Steps 9 to 15: Mod.2 Procedure */
16:     $\breve{\mathbf{x}} = \mathbf{T}\breve{\mathbf{z}}$;
17:     count=count+1;
18:     $\mathbf{v}_{count} = [\breve{x}_1 \cdots \breve{x}_{(i-1)}\, \mathcal{A}_j\, \breve{x}_{(2n_t-1)}]^T$;
19:     $m_{count} = \|\mathbf{y} - \mathbf{H}\mathbf{v}_{count}\|^2$;
20:   **end for**
21: **end for**
22: $p = \underset{l}{\text{argmin}}\, m_l$;
23: **Output** : $\mathbf{v}_p$;

V. Results and Discussions

In this section, we present simulation results on the BER performance and complexity of the proposed ILR algorithm. We compare them with those of other LR-aided algorithms. The following values of the parameters are used in the simulations: $\theta_1 = 0.5$, $\theta_2 = 0.3$, $\theta_3 = 0.05$ and $max\_depth = 3$.

First, in Figs. 1 and 2, we present the BER performance of the ILR detector in comparison with those of other LR-aided detectors using Seysen's reduction algorithm for 16×16 V-BLAST MIMO system with 4- and 16-QAM, respectively. From these figures, it is observed that the ILR detector performs close to within 0.1 dB and 0.5 dB of the ML performance in 4- and 16-QAM, respectively, which is quite attractive. At a BER of 10^{-3}, the ILR detector achieves an SNR gain of about 1.9 dB and 2.6 dB compared to the performance of LRdecimp (LR-AUG-ZF) detector for 4- and 16-QAM, respectively.

In order to gain a better insight on the overall performance of the proposed algorithm, the individual contributions of dealing with imperfect orthogonalization (using Mod.1 procedure) and imperfect nearest neighbor quantization (using

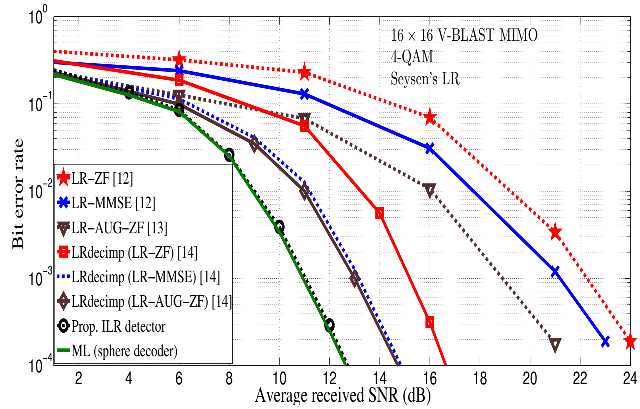

Fig. 1. BER performance of the proposed ILR detector in comparison with those of other LR-aided detectors in $n_t = n_r = 16$ V-BLAST MIMO system using 4-QAM.

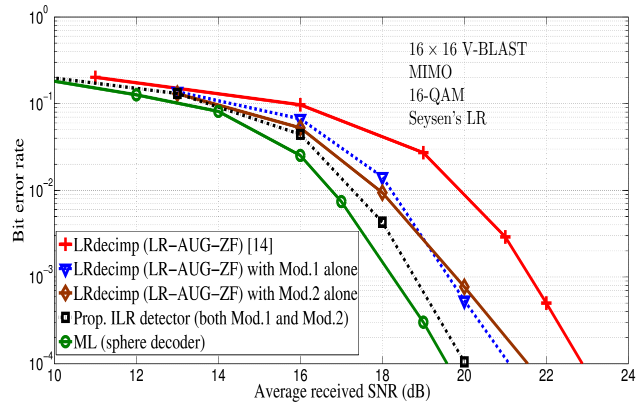

Fig. 2. BER performance of the proposed ILR detector in comparison with those of other LR-aided detectors in $n_t = n_r = 16$ V-BLAST MIMO system using 16-QAM.

Mod.2 procedure) are of interest. So, in Fig. 2, we have plotted separate BER curves for i) the basic LRdecimp (LR-AUG-ZF) algorithm, ii) LRdecimp with Mod.1 alone, iii) LRdecimp with Mod.2 alone, and iv) the overall ILR algorithm which uses both Mod.1 and Mod.2. It is seen that using either Mod.1 or Mod.2 alone results in some improvement, but not as good as using them simultaneously. For e.g., at a BER of 10^{-3}, individual use of Mod.1 and Mod.2 improves the performance of LRdecimp (LR-AUG-ZF) algorithm by 2 dB and 1.8 dB, respectively, whereas when both Mod.1 and Mod.2 parts are used simultaneously (i.e., ILR algorithm), performance close to ML is achieved.

The improved performance in ILR algorithm is achieved without much increase in complexity compared to LRdecimp algorithm complexity. The complexity comparison between the ILR detector and other LR-aided detectors is plotted in Fig. 3. It is observed that, to achieve a BER of 10^{-2}, the average number of real operations required by the ILR algorithm is roughly twice the average number of real operations required by the LRdecimp (LR-AUG-ZF) algorithm.

The SNR loss in various LR-aided detectors compared to the ML performance for 16×16 MIMO in both 4- and 16-QAM is presented in Table I. From Table I, it is seen that, at a BER of 10^{-4} in 16-QAM, the SNR loss incurred by the ILR detector compared to ML performance is just 0.4 dB,

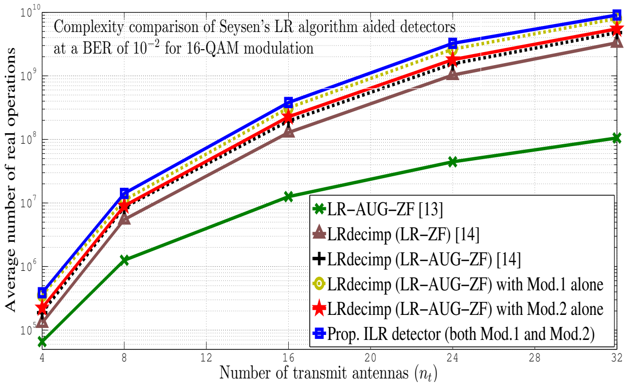

Fig. 3. Complexity comparison of LR-aided detectors at a BER of 10^{-2} for $n_t = n_r$ systems using 16-QAM.

which is 2.9 dB, 4 dB and 9.4 dB less than those incurred by LRdecimp (LR-AUG-ZF), LRdecimp (LR-ZF), and LR-AUG-ZF detectors, respectively.

Algorithm	SNR loss in dB compared to the ML performance at 10^{-2} BER and 10^{-4} BER for 16×16 MIMO			
	4-QAM		16-QAM	
	10^{-2}	10^{-4}	10^{-2}	10^{-4}
LR-AUG-ZF [14]	7.2 dB	8.2 dB	7.2 dB	9.8 dB
LRdecimp (LR-ZF) [15]	4.3 dB	4 dB	3.6 dB	4.4 dB
LRdecimp (LR-AUG-ZF) [15]	2.1 dB	2.2 dB	3.1 dB	3.3 dB
Proposed ILR detector	0.1 dB	0.1 dB	0.5 dB	0.4 dB

TABLE I
SNR LOSS IN LR-AIDED DETECTORS COMPARED TO ML PERFORMANCE FOR $n_t = n_r = 16$ V-BLAST MIMO SYSTEM USING 4- AND 16-QAM AT BER OF 10^{-2} AND 10^{-4}.

Next, Fig. 4 shows the BER performance of the ILR detector using Seysen's reduction algorithm and D-ELR-SLB reduction algorithm for $n_t = n_r = 24$ V-BLAST MIMO system using 16-QAM. Since the true ML detection for this system is computationally infeasible, we plot the unfaded SISO AWGN performance as a lower bound to the ML performance. From this figure, we observe that the performance of the ILR detector using Seysen's and D-ELR-SLB reduction algorithms are roughly the same at low to medium SNRs. At high SNRs, the D-ELR-SLB reduction algorithm offers slightly better performance than Seysen's reduction algorithm. For example, at a BER of 10^{-6}, an SNR gain of 0.5 dB is seen when D-ELR-SLB reduction algorithm is employed instead of the Seysen's reduction algorithm. In Fig. 4 also, we observe that simultaneous use of both Mod.1 and Mod.2 achieves significantly better performance compared to individual use of Mod.1 or Mod.2 alone.

VI. Conclusion

We proposed a lattice reduction (LR) aided MIMO detection algorithm which mitigates the imperfect orthogonalization and imperfect nearest neighbor quantization problems faced in LR-aided detectors. The proposed improved LR-aided (ILR) algorithm exhibited good performance and complexity attributes. It outperformed other LR-aided detection algorithms like LR-ZF, LR-MMSE, LR-AUG-ZF, and LRdecimp, and achieved near-ML performance in large-MIMO systems. While the SNR loss in other LR-aided algorithms compared to ML performance is in the range of 2 to 9 dB in 16×16 V-BLAST MIMO system with 4-QAM and 16-QAM, the SNR loss in the proposed ILR algorithm is just 0.1 dB for 4 QAM and < 0.5 dB for 16-QAM. Performance/complexity comparisons with LR based K-best algorithm [16] and variants of sphere decoding are possible future works.

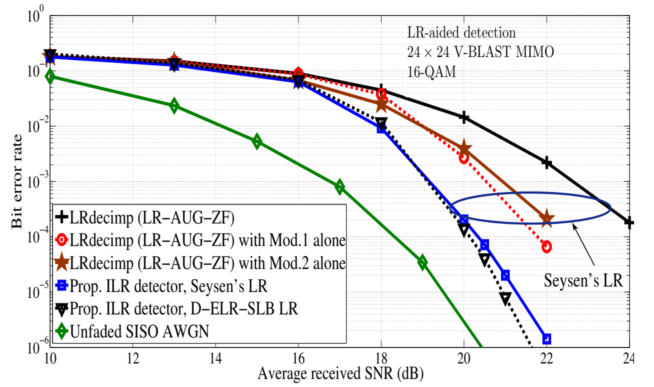

Fig. 4. BER performance of the ILR detector using Seysen's reduction algorithm in comparison with that of using D-ELR-SLB reduction algorithm for $n_t = n_r = 24$ V-BLAST MIMO system using 16-QAM.

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