EXIT Chart Based Design of Irregular LDPC Codes for Large-MIMO Systems

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***Abstract*—In this letter, we characterize the extrinsic information transfer (EXIT) behavior of a factor graph based message passing algorithm for detection in large multiple-input multiple-output (MIMO) systems with tens to hundreds of antennas. The EXIT curves of a joint detection-decoding receiver are obtained for low density parity check (LDPC) codes of given degree distributions. From the obtained EXIT curves, an optimization of the LDPC code degree profiles is carried out to design irregular LDPC codes matched to the large-MIMO channel and joint message passing receiver. With low complexity joint detection-decoding, these codes are shown to perform better than off-the-shelf irregular codes in the literature by about 1 to 1.5 dB at a coded BER of 10^{-5} in 16×16, 64×64 and 256×256 MIMO systems.**

***Index Terms*—Large-MIMO systems, EXIT chart, degree profile optimization, message passing, joint detection-decoding.**

I. Introduction

LARGE-SCALE MIMO systems with tens to hundreds of antennas have recently stirred a lot of interest, mainly due to their potential to practically achieve the theoretically predicted benefits of MIMO in terms of very high spectral efficiencies/sum rates and increased reliability/power efficiency, through the exploitation of large spatial dimensions [1]. Low-complexity receiver algorithms for such large-MIMO systems are becoming increasingly common [2]-[5]. Message passing on graphical models is one approach that has been shown to be attractive for detection in large-MIMO systems [4],[5] and SISO-ISI channels [6]. Achieving near-capacity performance in large-MIMO systems is of interest. Low density parity check (LDPC) codes are known to achieve capacity on AWGN channels [7]. Typically, irregular LDPC codes are known to perform better than regular LDPC codes which have constant variable and check node degrees [8]. A way to construct irregular LDPC codes is to optimize the degree distribution using either density evolution [9] or EXIT charts [10]. In this letter, we design irregular LDPC codes for large-MIMO systems using EXIT chart approach. One requirement in LDPC code design using EXIT chart approach is the knowledge of the EXIT characteristics of the detector of interest on a given channel. For simple detectors and channels, the EXIT behavior can be analytically characterized in closed-form [11]. When such analytical characterization is not tractable, one resorts to Monte Carlo simulations to obtain the EXIT curves [10], [12].

In this letter, we consider a factor graph (FG) based message passing detector which scales very well in complexity and achieves near-optimal performance in large-MIMO systems [5]. The detector uses a scalar Gaussian approximation of the interference (GAI) which is instrumental in achieving very low complexity (*linear complexity in* n_t). We first characterize the EXIT characteristics of the detector, and then obtain the EXIT curves for the combination of the detector and the decoder using the knowledge of the EXIT characteristics of the detector. Since the detector is based on message passing on a factor graph, it is natural and beneficial to integrate it with message passing LDPC decoder through a *single joint graph*. We construct one such joint graph on which we define messages and carryout message passing to do joint detection-decoding. We construct LDPC codes by matching the EXIT chart of the combined detector-decoder with the EXIT chart of the LDPC check node set. The constructed irregular LDPC codes are shown to outperform regular LDPC codes as well as off-the-shelf irregular LDPC codes available in the literature (e.g., irregular codes from [8] and IEEE 802.16 WiMax standard). Though the EXIT chart approach is a known approach in the literature, what makes our contribution interesting is the demonstration of the ability to generate EXIT curves for the joint message passing detector with GAI for large number of antennas and exploit it for the purpose of designing efficient LDPC codes for large-MIMO systems. To our knowledge, irregular LDPC code designs for large-MIMO systems have not been reported before.

II. System Model

Consider a point-to-point spatially multiplexed MIMO system with n_t transmit and n_r receive antennas, where n_t, n_r are large (tens to hundreds[1]). A sequence $\mathbf{u}$ of k information bits is encoded by an LDPC code into a codeword $\mathbf{b}$ of n coded bits, so that the rate of the code is $R = \frac{k}{n}$ and the number of parity bits in a codeword is $m = n - k$. These coded bits are then QPSK modulated. n_t modulated symbols are sent in one channel use on n_t transmit antennas using spatial multiplexing. So the number of channel uses needed to send all n coded bits is $L = \lceil \frac{n}{2n_t} \rceil$. Let $\mathbf{x}^{(l)}$, $l = 1, \cdots, L$, denote the n_t-sized modulated symbol vector sent in the lth channel use. Let $\mathbf{H}^{(l)} \in \mathbb{C}^{n_r \times n_t}$ denote the channel gain matrix in the lth channel use, whose entries are assumed to be i.i.d. Gaussian with zero mean and unit variance. The received vector in the lth channel use, $\mathbf{y}^{(l)}$, is

$$\mathbf{y}^{(l)} = \mathbf{H}^{(l)}\mathbf{x}^{(l)} + \mathbf{w}^{(l)}, \tag{1}$$

[1]One interesting application of such large-MIMO systems is to provide high-speed wireless backhaul link between base stations. Large number of antennas can be mounted in the base stations to carry the backhaul traffic.

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where $\mathbf{w}^{(l)}$ is the noise vector whose entries are modeled as i.i.d. $\mathbb{C}\mathcal{N}(0, \sigma_n^2)$. We work with the real-valued system model corresponding to (1), given by $\mathbf{y}_r^{(l)} = \mathbf{H}_r^{(l)}\mathbf{x}_r^{(l)} + \mathbf{w}_r^{(l)}$, where

$$\mathbf{y}_r^{(l)} = \begin{bmatrix} \Re(\mathbf{y}^{(l)}) \\ \Im(\mathbf{y}^{(l)}) \end{bmatrix}, \quad \mathbf{x}_r^{(l)} = \begin{bmatrix} \Re(\mathbf{x}^{(l)}) \\ \Im(\mathbf{x}^{(l)}) \end{bmatrix},$$

$$\mathbf{H}_r^{(l)} = \begin{bmatrix} \Re(\mathbf{H}^{(l)}) & -\Im(\mathbf{H}^{(l)}) \\ \Im(\mathbf{H}^{(l)}) & \Re(\mathbf{H}^{(l)}) \end{bmatrix}, \quad \mathbf{w}_r^{(l)} = \begin{bmatrix} \Re(\mathbf{w}^{(l)}) \\ \Im(\mathbf{w}^{(l)}) \end{bmatrix},$$

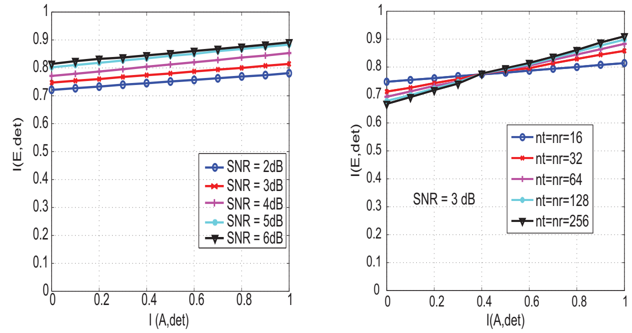
and $\Re(.)$ and $\Im(.)$ represent the real and imaginary parts, respectively. The receiver observes $\mathbf{y}_r^{(l)}$ and with the knowledge of $\mathbf{H}_r^{(l)}$ and the LDPC code matrix, jointly performs detection and decoding, i.e., marginalizes $p(\mathbf{x}_r^{(l)} \mid \mathbf{y}_r^{(l)}, \mathbf{H}_r^{(l)})$. Specifically, we consider a factor graph (FG) based soft-output MIMO detector with Gaussian approximation of interference (GAI) [5], and a joint detection-decoding scheme on a joint factor graph. The FG-GAI approach is attractive for large-MIMO systems because its complexity scales linearly with n_t and it achieves near-optimal performance for large n_t. Our motivation here is to achieve near capacity performance by designing appropriate LDPC codes matched to the large-MIMO channel and the message passing receiver.

III. EXIT Chart Analysis

EXIT chart analysis was first introduced in [11] for analyzing iterative decoders. EXIT function of a decoder describes the relation between the input *a priori* information and the output extrinsic information. Let I_A be the average mutual information between the coded bits and the *a priori* input (*a priori* information), and I_E be the average mutual information between the coded bits and the extrinsic output. The function $f(I_A) = I_E$ is the EXIT function of the decoder. This function $f(.)$ completely characterizes the information transfer in the decoder. We proceed by first obtaining the EXIT curves for the FG-GAI based detector. This is then utilized to obtain the EXIT curves of the joint detector-decoder.

A. EXIT Characteristics of the FG-GAI Detector

The FG-GAI detector is introduced and explained in detail in [5]. In brief, the detector has two set of nodes, namely, the observation nodes and the variable nodes. The observation nodes are initialized with $\mathbf{y}_r^{(l)}$, and after a few iterations of message passing, the soft values or log-likelihood ratios (LLR) of the transmitted $\mathbf{x}_r^{(l)}$ are obtained from the variable nodes. In computing the messages, the interference from other antennas are approximated to be Gaussian, whose mean and variance are obtained in closed-form and used in the message computation. Denoting the extrinsic information at the output of the FG-GAI detector by $I_{E,det}$ and the *a priori* information at the input of the detector by $I_{A,det}$, and the average signal-to-noise ratio (SNR) at the receiver by γ, we have $I_{E,det} = f(\gamma, n_t, n_r, I_{A,det})$. To obtain the EXIT curve, the input *a priori* LLR value A is modeled as $A \sim \mathcal{N}(x\mu_A, \sigma_A^2)$ such that $\mu_A = \frac{\sigma_A^2}{2}$ and $x \in \{+1, -1\}$. $I_{A,det}$ can be written as $I_{A,det}(\sigma_A) = I(X, A) \triangleq J(\sigma_A)$, as defined in Eq. (12) in [11]. Therefore, $\sigma_A = J^{-1}(I_{A,det})$. Similarly, the output extrinsic information $I_{E,det}$ is given by



(a) Different SNRs (b) Different $n_t = n_r$

Fig. 1. $I_{E,det}$ vs $I_{A,det}$ EXIT curves of FG-GAI detector in large-MIMO systems.

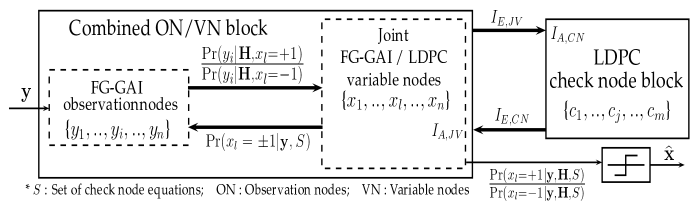


Fig. 2. Transfer of mutual information in joint detector-decoder.

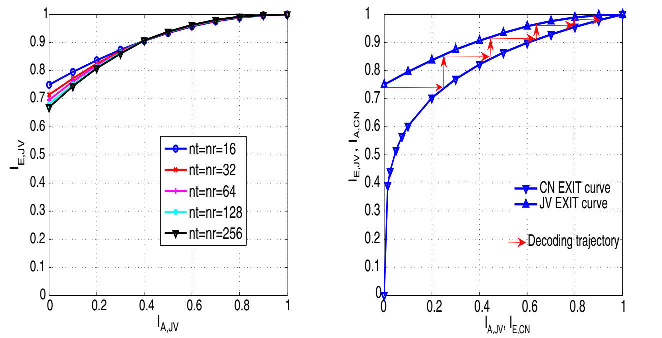
$$I_{E,det}(\sigma_A) = \frac{1}{2}\sum_{x=-1,1}\int_{-\infty}^{+\infty} P_E(z|X=x) \times \log_2 \frac{2P_E(z|X=x)}{P_E(z|X=1) + P_E(z|X=-1)} dz. \quad (2)$$

The above extrinsic information depends on the detector used. For the considered system and FG-GAI detector, an analytical evaluation of $I_{E,det}$ in (2) is a difficult task. For simplicity, we resort to the computation of $P_E(z|X = x)$ through Monte Carlo simulations. Thus, the EXIT curve is computed by simulating equation (2) in conjunction with the simulation of the FG-GAI detection algorithm for various values of received SNR. In Fig. 1(a), we plot the EXIT curves obtained for the FG-GAI detector in a 16×16 MIMO system for different SNR values. In Fig. 1(b), we plot the EXIT curves of the FG-GAI detector for $n_t = n_r = 16, 32, 64, 128, 256$ MIMO configurations at 3 dB SNR.

B. Analysis of the Joint Detection-Decoding Receiver

Let d_v and d_c denote the variable node and check node degrees of the LDPC decoder, respectively. $I_{E,VN}(\gamma, d_v, I_{A,VN})$ is the extrinsic information at the variable nodes of the LDPC decoder, where $I_{A,VN}$ is the a priori information at the variable nodes. Likewise, $I_{A,CN}(d_c, I_{E,CN})$ and $I_{E,CN}$ are the a priori and extrinsic information at the check nodes of the LDPC decoder. For the joint detector-decoder, we use a single graph that combines the variable nodes of the FG-GAI MIMO detector and the variable nodes of the LDPC decoder. Each node in the variable nodes set of the detector and decoder represents a coded bit. The edges from this combined variable nodes set are appropriately interleaved and connected to the check nodes set of the decoder [13]. The EXIT curve of the combined detector-decoder is evaluated as [10]

$$I_{E,JV}(\gamma, d_v, I_{A,JV}, I_{E,det}) = J\left(\sqrt{(d_v - 1)(J^{-1}(I_{A,JV}))^2 + (J^{-1}(I_{E,det}))^2}\right), \quad (3)$$



(a) $d_v = 3$, different $n_t = n_r$ (b) $d_v = 3$, $d_c = 6$, 16×16 MIMO

Fig. 3. (a) EXIT curves, and (b) mutual information trajectory of joint detector-decoder for large-MIMO systems at SNR = 3 dB.

where $I_{E,JV}$ is the extrinsic information and $I_{A,JV}$ is the a priori information at the block formed by the combination of the variable nodes of the FG-GAI detector and LDPC decoder. Figure 2 illustrates the transfer of mutual information between the combined observation node/variable node (ON/VN) block and the LDPC check node block of the joint detector/decoder.

We evaluated the EXIT curves for the combined detector-decoder for $n_t = n_r =$16, 32, 64, 128, 256 MIMO systems at 3 dB SNR using the EXIT curves of the FG-GAI detector and the LDPC variable nodes; the results are shown in Fig. 3(a). The decoding trajectory for $d_v = 3$, $d_c = 6$, $n_t = n_r = 16$ and 3 dB SNR is plotted in Fig. 3(b). It is necessary that the EXIT curve of the check nodes set lies below the EXIT curve of the combined variable nodes set for the decoder to converge, which must be satisfied by the code design.

IV. LDPC Code Design

To approach the capacity of the channel, the EXIT curves of the check nodes set and the combined variable nodes set should be matched [14]. To match these curves, the degree distribution of both the variable nodes (d_v) and the check nodes (d_c) can be varied. Alternately, only d_v can be varied for a fixed d_c [10]. We adopt the former approach for our problem. The LDPC codes so obtained will have non-uniform degree distribution; such codes are referred to as irregular LDPC codes. We first explain the design methodology that varies d_v for a fixed d_c. Then we proceed to explain the adopted methodology that varies d_v and d_c.

Let $\{d_v^i\}$ and $\{p_v^i\}$, respectively, denote the variable node degrees and the probabilities of occurrence of each d_v^i, $i = 1, \cdots, D_v$, where D_v is the number of variable node degrees considered for the design. An example can be as follows: $D_v = 4$ and $\{d_v^1, d_v^2, d_v^3, d_v^4\} = \{2, 4, 5, 7\}$. The average variable node degree is then given by

$$\bar{d_v} \;=\; \mathbb{E}(d_v) \;=\; \sum_{i=1}^{D_v} p_v^i d_v^i \;=\; \mathbf{p}_v^T \mathbf{d}_v, \tag{4}$$

where $\mathbf{d}_v = [d_v^1 \; d_v^2 \; \cdots \; d_v^{D_v}]$, $\mathbf{p}_v = [p_v^1 \; p_v^2 \; \cdots \; p_v^{D_v}]$ and $\|\mathbf{p}_v\|_1 = 1$. Since the number of edges incident at the variable nodes is same as the number of edges incident at the check nodes, $n\bar{d_v} = (n-k)d_c$ for a fixed d_c. Thus, $\bar{d_v} = (1-R)d_c$. The probability that an edge is connected to a variable node of degree d_v^i is $p_{ve}^i = \frac{np_v^i d_v^i}{n\bar{d_v}}$. For a fixed d_c, let $\mathbf{D} = \frac{1}{(1-R)d_c} diag(d_v^1, \cdots, d_v^{D_v})$. Hence, $\mathbf{p}_{ve} \triangleq [p_{ve}^1 \; p_{ve}^2 \; \cdots \; p_{ve}^{D_v}] = \mathbf{D}\mathbf{p}_v$, and $\|\mathbf{p}_{ve}\|_1 = 1$. Since the EXIT curve of a mixture of codes is the same as the average of the individual codes, the effective EXIT curve of the mixture of codes with varying variable node degree is

$$I_{E,JV}^{eff}(\gamma, I_{A,JV}) \;=\; \sum_{i=1}^{D_v} p_{ve}^i \; I_{E,JV}(\gamma, d_v^i, I_{A,JV},). \tag{5}$$

Since a closed-form analytical expression for $I_{E,det}$, and hence for $I_{E,JV}$, is unavailable for our system, we evaluate $I_{E,JV}$ at N different points, using which (5) is written in the form

$$\mathbf{q}_{E,JV} \;=\; \mathbf{Q}_{E,JV} \; \mathbf{p}_{ve}, \tag{6}$$

where $\mathbf{q}_{E,JV}$ is a $N \times 1$ vector and $\mathbf{Q}_{E,JV}$ is a $N \times D_v$ matrix whose element in the gth row and ith column is $I_{E,JV}(\gamma, d_v^i, I_{A,JV}^g)$, where $I_{A,JV}^g$ is the gth $I_{A,JV}$ value, $g = 1, \cdots, N$. Our aim is to find a vector $\mathbf{p}_v$ such that $(\mathbf{Q}_{E,JV}\mathbf{D}\mathbf{p}_v - \mathbf{q}_{A,CN})$ is minimized. This ensures that the EXIT curve of the LDPC check nodes set match the EXIT curve of the joint detector-decoder, by varying only the variable node degrees in the LDPC code. That is, we need to find a vector $\mathbf{p}_v$ such that

$$\widehat{\mathbf{p}}_v = \underset{\mathbf{p}_v}{\text{argmin}}\{\mathbf{Q}_{E,JV}\mathbf{D}\mathbf{p}_v \quad - \mathbf{q}_{A,CN}\} \tag{7}$$

subject to $\|\mathbf{p}_v\|_1 = 1$, $\|\mathbf{D}\mathbf{p}_v\|_1 = 1$ and $p_v^i \geq 0$, where $\mathbf{q}_{A,CN}$ is the $N \times 1$ vector consisting of the N evaluated values of $I_{A,CN}$ for a fixed d_c. For a fixed d_c, (7) can be solved using well known quadratic programming optimization methods like interior point methods. We match the EXIT curves by varying the check node degrees also. For this, as we defined for the variable nodes, define $\{d_c^j\}$ and $\{p_c^j\}$, $j = 1, \cdots, D_c$ for the check node degrees so that the average check node degree is $\bar{d_c} = \mathbf{p}_c^T\mathbf{d}_c$, where $\mathbf{d}_c = [d_c^1 \; d_c^2 \; \cdots \; d_c^{D_c}]$, $\mathbf{p}_c = [p_c^1 \; p_c^2 \; \cdots \; p_c^{D_c}]$ and $\|\mathbf{p}_c\|_1 = 1$. The probability that an edge is connected to a check node of degree d_c^j is

$$p_{ce}^j = \frac{(n-k)p_c^j d_c^j}{(n-k)\bar{d_c}}, \quad \mathbf{p}_{ce} \triangleq [p_{ce}^1 \; p_{ce}^2 \; \cdots \; p_{ce}^{D_c}]. \tag{8}$$

Let $R' = 1 - R$. So $\bar{d_v}$ becomes $\bar{d_v} = R'\bar{d_c}$. To start the optimization, we fix either $\bar{d_v}$ or $\bar{d_c}$. The effective EXIT curve for varying check node degree is

$$I_{A,CN}^{eff}(I_{E,CN}) \;=\; \sum_{j=1}^{D_c} p_{ce}^j I_{A,CN}(d_c^j, I_{E,CN}). \tag{9}$$

Let $\mathbf{Q}_{A,CN}$ be a $N \times D_c$ matrix whose element in the gth row and jth column is $I_{A,CN}(d_c^j, I_{E,CN}^g)$, where $I_{E,CN}^g$ is the gth $I_{E,CN}$ value, $g = 1, \cdots, N$. We write (9) in the form

$$\mathbf{q}_{A,CN} \;=\; \mathbf{Q}_{A,CN}\mathbf{p}_{ce}, \tag{10}$$

where $\mathbf{q}_{A,CN}$ is a $N \times 1$ vector. Let $\mathbf{V} = diag(d_v^1, \cdots, d_v^{D_v})$ and $\mathbf{C} = diag(d_c^1, \cdots, d_c^{D_c})$. Then, $\mathbf{p}_{ve} = K_v\mathbf{V}\mathbf{p}_v$, $\mathbf{p}_{ce} = K_c\mathbf{C}\mathbf{p}_c$, where the scalars K_v and K_c are given by $K_v = \frac{1}{\bar{d_v}}$ (for fixed $\bar{d_v}$) and $= \frac{1}{R'\bar{d_c}}$ (for fixed $\bar{d_c}$), and $K_c = \frac{R'}{\bar{d_v}}$ (for fixed $\bar{d_v}$) and $\frac{1}{\bar{d_c}}$ (for fixed $\bar{d_c}$). We need to match $\mathbf{q}_{A,CN}$ and $\mathbf{q}_{E,JV}$. Thus, the optimization problem is to find $\mathbf{p}_v$ and $\mathbf{p}_c$ such that $\{K_v\mathbf{Q}_{E,JV}\mathbf{V}\mathbf{p}_v - K_c\mathbf{Q}_{A,CN}\mathbf{C}\mathbf{p}_c\}$ is minimized, subject to $\|\mathbf{p}_v\|_1 = 1$, $\|\mathbf{p}_c\|_1 = 1$, $\|\mathbf{V}\mathbf{p}_v\|_1 = \frac{1}{K_v}$, $\|\mathbf{C}\mathbf{p}_c\|_1 = \frac{1}{K_c}$, $p_c^i \geq 0$ and $p_v^i \geq 0$. Let $\mathbf{p} \triangleq \begin{bmatrix} \mathbf{p}_v \\ \mathbf{p}_c \end{bmatrix}$. The optimization problem can then be reduced to

$$\widehat{\mathbf{p}} \;=\; \underset{\mathbf{p}}{\text{argmin}}\{[K_v\mathbf{Q}_{E,JV}\mathbf{V} \quad - K_c\mathbf{Q}_{A,CN}\mathbf{C}]\,\mathbf{p}\} \tag{11}$$

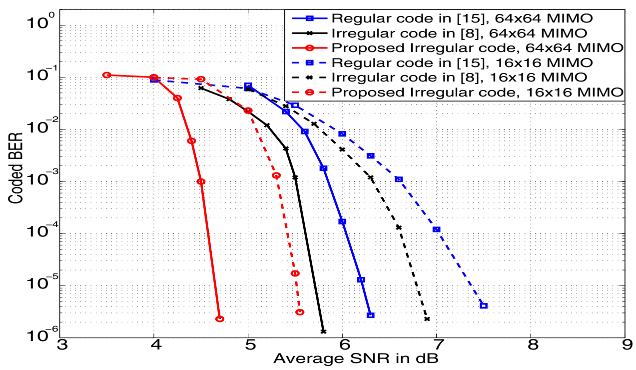


Fig. 4. Coded BER of rate-1/2 LDPC codes with $n = 4000$ for 16×16 and 64×64 MIMO systems with the joint detector-decoder.

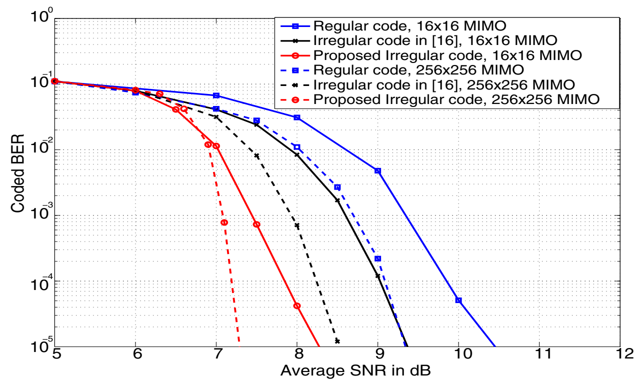


Fig. 5. Coded BER of rate-3/4 LDPC codes with $n = 2048$ for 16×16 and 256×256 MIMO systems with the joint detector-decoder.

subject to the linear constraint

$$\begin{bmatrix} 1 \; 1 \; \cdots \; 1 & \mathbf{0} \\ \mathbf{0} & 1 \; 1 \; \cdots \; 1 \\ d_v^1 \; d_v^2 \; \cdots \; d_v^{D_v} & \mathbf{0} \\ \mathbf{0} & d_c^1 \; d_c^2 \; \cdots \; d_c^{D_c} \end{bmatrix} \mathbf{p} = \begin{bmatrix} 1 \\ 1 \\ \frac{1}{K_v} \\ \frac{1}{K_c} \end{bmatrix},$$

and each element of $\mathbf{p}$ is ≥ 0. The solution $\widehat{\mathbf{p}}$ is the required degree profile distribution for the irregular LDPC code.

V. Results and Discussions

The coded bit error rate (BER) performance of the proposed irregular LDPC codes are evaluated through simulations. An example irregular LDPC code constructed to match the EXIT curves at an average receiver SNR of 5 dB for 16×16 MIMO system has the degree profile distribution shown in Table I. We perform QPSK modulation of the coded bits and use joint message passing based detection and decoding. The number of iterations of the joint detection-decoding is restricted to a maximum of 15. The simulations are carried out for 16×16, 64×64, and 256×256 MIMO systems. We compare the performance of the proposed irregular LDPC codes with that of i) the (3,6)-regular LDPC code in [15], ii) the irregular LDPC code in [8], and iii) the irregular LDPC code in WiMax standard [16].

Figure 4 shows the coded BER performance of i) the proposed irregular code, ii) the regular code in [15], and iii) the irregular code in [8] for 16×16 and 64×64 MIMO configurations with rate-1/2 codes and block length $n = 4000$. From Fig. 4, it is seen that in 16×16 MIMO, the proposed irregular LDPC code has a performance improvement of about 1.8 dB over the regular code in [15] and 1.2 dB over the irregular code in [8], at a coded BER of 10^{-5}. Likewise, the performance improvement in 64×64 MIMO is 1.6 dB over the regular code in [15] and 1 dB over the irregular code in [8]. Figure 5 shows a performance comparison between the proposed LDPC codes and the irregular LDPC code in WiMax standard [16] for rate-3/4 and $n = 2048$ in 16×16 and 256×256 MIMO. Here again, it is observed that the proposed irregular LDPC codes perform better by about 2 dB over their regular counterparts and by 1 dB over the irregular code specified in WiMax, at 10^{-5} coded BER. These performance improvements are due to the matching of the EXIT charts of the combined detector-decoder and the LDPC check node set in the proposed design.

TABLE I
Degree Profile Distribution of LDPC code for 16×16 MIMO.

d_v^i	2	8	10	16	18	24
p_v^i	0.3928	0.1990	0.2160	0.0505	0.1244	0.0173

d_c^i	8	12	24	64
p_c^i	0.6216	0.2072	0.0604	0.1108

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