Reverse Channel Training in Multiple Antenna TDD Systems

Bharath B. N.

Advisor
Dr. Chandra R. Murthy

Signal Processing for Communications Lab,
ECE Department, Indian Institute of Science
Bangalore-560012
Jan. 23, 2013
1 Motivation

2 RCT for SIMO Channels

3 RCT for MIMO Channels

4 Conclusions
System Model

- **Motivation**
  - RCT for SIMO Channels
  - RCT for MIMO Channels
  - Conclusions

---

**System Model**

![System model diagram](Image)

**Figure**: System model of a reciprocal SISO system

1. **Input-output equation (Forward-link)**
   
   \[ y_B = h x_A + w_B \]

2. **Input-output equation (Reverse-link)**
   
   \[ y_A = h^* x_B + w_A \]

---

Bharath B. N. | Reverse Channel Training in Multiple Antenna TDD Systems
Conventional Reverse Channel Training (RCT)

Training method
- Training sequence
  \[ x_{B,\tau} = \sqrt{P_{B,\tau}} \cdot 1 \]
  where \( P_{B,\tau} \) is the training power

- Received training signal at node A
  \[ y_{A,\tau} = \sqrt{P_{B,\tau}} h^* + w_A \]

Estimation of \( |h| \)
- Natural estimate
  \[ |\hat{h}|_{(conv)} \triangleq \frac{|y_{A,\tau}|}{\sqrt{P_{B,\tau}}} = \left| h + \frac{w_A}{\sqrt{P_{B,\tau}}} \right| \]
Proposed RCT

1 Proposed RCT method
   
   - Proposed RCT sequence
     \[ x_{B,\tau}^{(\text{prop})} = \sqrt{P_{B,\tau}} e^{-j\theta} \]
   
   - Received training signal at Node A
     \[ y_{A,\tau} = \sqrt{P_{B,\tau}} e^{-j\theta} h + w_A = \sqrt{P_{B,\tau}} |h| + w_A \]

2 Estimation method
   
   - Proposed estimation
     \[ |\hat{h}|_{(\text{prop})} \triangleq \frac{|\Re\{y_{A,\tau}\}|}{\sqrt{P_{B,\tau}}} = |h| + \frac{\Re\{w_A\}}{\sqrt{P_{B,\tau}}} \]
Conventional Versus Proposed

**Metric:** Mean Square Error (MSE)

Figure: MSE versus training power in the reverse-link in a TDD-SISO system.
Road Map of the Thesis

- Diversity Multiplexing gain Tradeoff (DMT) performance of channel dependent Reverse Channel Training (RCT) in a TDD-SIMO system
- Design of novel channel dependent RCT for a Spatial Multiplexing (SM) based TDD-MIMO system that is optimized using
  - MSE as a metric
  - a capacity lower bound as a metric

Main Message
- Exploit CSI while designing the RCT sequence
- The training sequence should be designed so as to convey only the part of the CSI required for data transmission
Reciprocal SIMO Channel

Figure: System model of a reciprocal SIMO with coherence time $L_c$

- Input-output equations

  $y_B = h x_A + w_B$
  $y_A = h^H x_B + w_A$

- Flat fading channel with $h \sim \mathcal{CN}(0, I_{n_B})$

- Question: Which RCT sequence enables an “efficient” estimate of CSI at node A?
A Quick Introduction to DMT

Diversity order (a proxy for probability of error)

The diversity order is defined as

\[ d \triangleq - \lim_{\text{SNR} \to \infty} \frac{\log P_{out}}{\log \text{SNR}} \]

where \( P_{out} \triangleq \Pr\{\text{Capacity} < R_{\bar{P}}\} \)

Multiplexing gain (a proxy for data rate)

Multiplexing gain is defined as

\[ g_{m} \triangleq \lim_{\text{SNR} \to \infty} \frac{R_{\text{SNR}}}{\log \text{SNR}} \]

where \( R_{\text{SNR}} \) is the data rate
SIMO: Infinite Diversity Order

- Infinite diversity order is achieved using the following power control scheme at node A

\[ P(\sigma) \triangleq \frac{c\bar{P}}{\sigma^2} \]

where \( \sigma \triangleq \|h\|_2 \) and \( c > 0 \) is chosen to satisfy an average power constraint

- It is sufficient to know \( \sigma \) at node A!

- **Question**: How to acquire an estimate of \( \sigma \) at node A?
  - Conventional method
  - Proposed channel-dependent training
Conventional Training (Phase I)

Figure: Phase I of the conventional channel-agnostic training

Here $x_1 \triangleq \sqrt{\frac{P_{LB,\tau}}{n_B}} \cdot (1, 0, \ldots, 0)^T$
Conventional Training (Phase $n_B$)

**Figure:** Here $\mathbf{x}_{B,\tau} \triangleq (x_1, \ldots, x_{n_B}) \triangleq \sqrt{\frac{\bar{P}_{L_B,\tau}}{n_B}} I_{n_B \times n_B}$, $\|\mathbf{x}_{B,\tau}\|_2^2 = \bar{P}_{L_B,\tau}$ and $\hat{\mathbf{h}} \triangleq (\hat{h}_1, \ldots, \hat{h}_{n_B})$

1. Conventional estimate of $\|\mathbf{h}\|_2$ is given by $\|\hat{\mathbf{h}}\|_2$
2. Requires $n_B$ symbol duration for training!
DMT of the Conventional RCT Scheme

**Theorem**

(StegerSabharwal2008) For a SIMO channel with perfect CSIR and imperfect CSIT obtained using channel agnostic training the following DMT is achievable

\[ d(g_m) = n_B \left( n_B + 1 - \frac{g_m}{\alpha} \right), \quad 0 \leq g_m \leq \alpha \]

where \( \alpha \triangleq \frac{L_c - L_{B,\tau}}{L_c}, \quad L_{B,\tau} \geq n_B \) is the training overhead

1. Assumes a genie aided receiver: \( P(\hat{\sigma}) \) is known at node A

**Observation**

The training overhead reduces the achievable DMT! Cannot achieve nonzero \( g_m \) if \( n_B > L_c \). Might need to switch off antennas
RCT and Estimation Method

- Proposed RCT

\[ x_{B,T} = \sqrt{\tilde{P} L_{B,T}} v, \]

where \( v \equiv \frac{h}{\| h \|_2} \)
RCT and Estimation Method

- Proposed RCT

\[ \mathbf{x}_{B,T} = \sqrt{\bar{P}L_{B,T}} \mathbf{v}, \]

where \( \mathbf{v} \triangleq \frac{\mathbf{h}}{\|\mathbf{h}\|_2} \)

- Minimum training duration is *only one* symbol!
RCT and Estimation Method

- Proposed RCT

\[ x_{B,\tau} = \sqrt{\bar{P}L_{B,\tau}} \mathbf{v}, \]

where \( \mathbf{v} \triangleq \frac{\mathbf{h}}{\|\mathbf{h}\|_2} \)

- Minimum training duration is \textit{only one} symbol!

- Received training signal at node A

\[ \hat{\sigma} \triangleq \frac{\Re\{y_{A,\tau}\}}{\sqrt{\bar{P}L_{B,\tau}}} = \sigma + \frac{\Re\{w_{A,\tau}\}}{\sqrt{\bar{P}L_{B,\tau}}}, \]

where \( \sigma \triangleq \|\mathbf{h}\|_2 \), \( L_{B,\tau} \) is the training duration, \( w_{A,\tau} \) is the AWG training noise at node A and \( \bar{P} \) is the training power.
Power Control Scheme

- Power-controlled data transmission

\[ y_{B,d} = \sqrt{\mathcal{P}(\hat{\sigma})} h x_{A,d} + w_{B,d}, \]

where \( x_{A,d} \sim \mathcal{CN}(0, 1) \), \( w_{B,d} \in \mathbb{C}^{r \times 1} \) is a \( \mathcal{CN}(0, I_n) \) r.v.

- Power control scheme

\[ \mathcal{P}(\hat{\sigma}) = \begin{cases} \bar{P}^l & \hat{\sigma} \leq \theta \bar{P} \quad \text{(bad channel)} \\ \kappa \bar{P} \times \Phi(\hat{\sigma}^{2s}) & \hat{\sigma} > \theta \bar{P} \quad \text{(good channel)} \end{cases} \]

where \( s \geq 1, \theta \bar{P} \triangleq \frac{1}{\bar{P}^n}, n > 0 \) and

\[ \Phi(\hat{\sigma}^{2s}) \triangleq \frac{\exp \left( \frac{RL_c}{L_c - L_{B,\tau}} \right) - 1}{\hat{\sigma}^{2s}} \quad \text{(Channel inversion)} \]
Power Control Scheme

- Power-controlled data transmission

\[ y_{B,d} = \sqrt{\mathcal{P}(\hat{\sigma})} h x_{A,d} + w_{B,d}, \]

where \( x_{A,d} \sim \mathcal{CN}(0, 1) \), \( w_{B,d} \in \mathbb{C}^{r \times 1} \) is a \( \mathcal{CN}(0, I_{n_B}) \) r.v.

- Power control scheme

\[ \mathcal{P}(\hat{\sigma}) \triangleq \begin{cases} \bar{P}^l & \hat{\sigma} \leq \theta_{\bar{P}} \text{ (bad channel)} \\ \kappa_{\bar{P}} \times \Phi(\hat{\sigma}^{2s}) & \hat{\sigma} > \theta_{\bar{P}} \text{ (good channel)} \end{cases} \]

where \( s \geq 1, \theta_{\bar{P}} \triangleq \frac{1}{\bar{P}^n}, n > 0 \) and

\[ \Phi(\hat{\sigma}^{2s}) \triangleq \exp \left( \frac{RL_c}{L_c - L_{B,\tau}} \right) - 1 \text{ (Channel inversion)} \]

- Can choose \( \kappa_{\bar{P}}, n \) and \( l \) such that \( \mathbb{E}\mathcal{P}(\hat{\sigma}) = \bar{P} \)
DMT Achieved using the Proposed RCT

**Theorem**

Using the proposed RCT and power-control scheme with perfect CSIR and a genie-aided receiver (node B), an achievable diversity order as a function of multiplexing gain $g_m$ is given by

$$d(g_m) = n_B \left( \min\{l, n_B + 1\} - \frac{g_m}{\alpha} \right),$$

where $0 \leq l \leq n_B + 1$, $1 \leq s < r$, $0 \leq g_m < \alpha$, and $\alpha \triangleq \frac{L_c - L_B \cdot \tau}{L_c}$.

- **DMT of proposed RCT and power-control scheme:**
  $$d(g_m) = n_B \left( n_B + 1 - \frac{g_m}{\alpha} \right) \quad \text{where} \quad \alpha \triangleq \frac{L_c - 1}{L_c}.$$

- **DMT of Conventional RCT and power-control scheme:**
  $$d(g_m) = n_B \left( n_B + 1 - \frac{g_m}{\alpha_{\text{conv}}} \right), \quad \alpha_{\text{conv}} \triangleq \frac{L_c - n_B}{L_c}.$$

- Diversity order does not saturate with $n_B$!
Conventional Versus Proposed

**Figure:** DMT for a $5 \times 1$ SIMO system with $L_c = 20$
Story So Far

- We assumed
  1. Perfect CSI at node B
  2. Genie-aided receiver
- And showed that a diversity order of $n_B \left( n_B + 1 - \frac{g_m}{\alpha} \right)$ is achievable with fixed-power channel-dependent training, $0 \leq g_m < \alpha$
Story So Far

- We assumed
  1. Perfect CSI at node $B$
  2. Genie-aided receiver
- And showed that a diversity order of $n_B \left( n_B + 1 - \frac{g_m}{\alpha} \right)$ is achievable with fixed-power channel-dependent training, $0 \leq g_m < \alpha$

Next Question

What can we say about the DMT if CSI is estimated at node $B$ and the genie stopped helping us?
Phase I: Forward Training

Figure: Phase I: Node B computes an estimate of $h$

- $L_{A,\tau_1}$ symbol duration is spent on forward training
Phase II: Reverse Training

\[ x_B,\tau = \sqrt{PL_{B,\tau}} \frac{\hat{h}}{\|\hat{h}\|_2} \]

\[ \hat{h} = h - \tilde{h} \]

Figure: Phase II: Node A computes an estimate of \( \sigma \)

- Estimate of \( \|h\|_2 \) (\( \hat{v} \triangleq \frac{\hat{h}}{\|\hat{h}\|_2} \))

\[ \hat{\sigma} \triangleq \frac{\Re\{y_{A,\tau}\}}{\sqrt{PL_{B,\tau}}} = \sigma \Re\{\hat{v}^H\hat{v}\} + \frac{\Re\{w_{A,\tau}\}}{\sqrt{PL_{B,\tau}}} \]
Phase III: Power-controlled Forward Training

- The power control used for data transmission is $\mathcal{P}(\hat{\sigma})$
- But node $B$ does not have the knowledge of $\mathcal{P}(\hat{\sigma})$
Phase III: Power-controlled Forward Training

- The power control used for data transmission is $P(\hat{\sigma})$
- But node $B$ does not have the knowledge of $P(\hat{\sigma})$

Figure: Phase III: Node $B$ computes an estimate of the composite channel $h \sqrt{P(\hat{\sigma})}$

$L_{A,T_2}$ symbol duration is spent on forward training
Main Result II: DMT Achieved

Theorem

For a SIMO system with $n_B$ antennas at node B and for the proposed three-way training scheme, an achievable DMT is given by

$$d(g_m) = n_B \left( \min\{l, s\} + 1 - \frac{g_m}{\alpha} \right),$$

where $0 \leq l \leq n_B$, $1 \leq s < n_B$, $0 \leq g_m < \alpha$, and

$$\alpha \triangleq \frac{L_c - L_{B, \tau} - L_{A, \tau_1} - L_{A, \tau_2}}{L_c}.$$

- Proposed RCT and power-control scheme with $l = n_B$:
  $$d(g_m) = n_B \left( n_B + 1 - \frac{g_m}{\alpha} \right),$$
  where $\alpha \triangleq \frac{L_c - 1 - L_{A, \tau_1} - L_{A, \tau_2}}{L_c}$.

- Conventional RCT and power-control scheme:
  $$d(g_m) = n_B \left( n_B + 1 - \frac{g_m}{\alpha} \right),$$
  where $\alpha \triangleq \frac{L_c - n_B - L_{A, \tau_1} - L_{A, \tau_2}}{L_c}$.

- Diversity order does not saturate with $n_B$!
Can We do Better?

- Infinite diversity order is achieved using the power control

\[ \mathcal{P}(\sigma) = \frac{c\bar{P}}{\sigma^2} \]
Can We do Better?

- Infinite diversity order is achieved using the power control
  \[ P(\sigma) = \frac{c\bar{P}}{\sigma^2} \]
- Procedure for estimating the power control
Can We do Better?

- Infinite diversity order is achieved using the power control
  \[ P(\sigma) = \frac{c\bar{P}}{\sigma^2} \]

- Procedure for estimating the power control
  - Estimate \( \sigma \)
Can We do Better?

- Infinite diversity order is achieved using the power control
  \[ P(\sigma) = \frac{c\bar{P}}{\sigma^2} \]

- Procedure for estimating the power control
  - Estimate \( \sigma \)
  - Compute \( P(\sigma) \) using the estimate
Can We do Better?

- Infinite diversity order is achieved using the power control

\[ P(\sigma) = \frac{c\bar{P}}{\sigma^2} \]

- Procedure for estimating the power control
  - Estimate \( \sigma \)
  - Compute \( P(\sigma) \) using the estimate

- Can we directly convey \( P(\sigma) \)?
Can We do Better?

- Infinite diversity order is achieved using the power control

\[ P(\sigma) = \frac{cP}{\sigma^2} \]

- Procedure for estimating the power control
  - Estimate \( \sigma \)
  - Compute \( P(\sigma) \) using the estimate

- Can we directly convey \( P(\sigma) \)?

**Answer:** Yes, using power-controlled RCT
Power-controlled RCT

- **Assumption:** Perfect CSI at node B
- Power-controlled RCT \((v \triangleq \frac{h}{\|h\|_2})\)

\[
x_{B,\tau} = \frac{\sqrt{P} L_{B,\tau} \sqrt{(r-1)(r-2)}}{\|h\|_2^2} v
\]

- \(x_{B,\tau}\) satisfies an average power constraint
- Received RCT signal at node A is

\[
y_{A,\tau} = \sqrt{P} L_{B,\tau} \frac{\sqrt{(r-1)(r-2)}}{\|h\|_2} + w_{A,\tau},
\]

- Use \(g_c \triangleq c_P \Re\{y_{A,\tau}\}\) as power control during data
  transmission where \(c_P \triangleq \sqrt{\frac{2P}{2(r-2)PL_{B,\tau}+1}}\)
Main Result III: Infinite Diversity Order

**Theorem**

For the proposed power-controlled RCT and power control, there exists an uncoded data transmission scheme that achieves an infinite diversity order for $0 \leq g_m < \alpha$, for all block lengths $L_d \geq 1$. In particular, the probability of error $P_e \leq \exp \left( -P^E \right)$, where

$$E = \frac{1}{2} \left(1 - \frac{g_m}{\alpha}\right), \quad 0 \leq g_m < \alpha$$
Recap

- Studied channel-dependent RCT and its DMT performance for a SIMO channel
- Demonstrated the performance benefits of the proposed RCT
Recap

- Studied channel-dependent RCT and its DMT performance for a SIMO channel
- Demonstrated the performance benefits of the proposed RCT
- Can we generalize the RCT to MIMO channels?
**System Model**

**SVD:** \( H = U\Sigma V^H \in \mathbb{C}^{n_B \times n_A} \)

**Perfect Knowledge of** \( H \)

**Figure:** System model of a \( n_B \times n_A \) reciprocal MIMO system. Here, the entries of \( H \) are assumed to be i.i.d. \( \mathcal{CN}(0, 1) \) random variables.
Reverse Channel Training

Training Seq. $X_{B,\tau} \in \mathbb{C}^{n_B \times L_{B,\tau}}$

$Y_{A,\tau} = H^H X_{B,\tau} + W_{A,\tau}$

Estimate of $V_m$

Goal:

Design of $X_{B,\tau}$ that enables *node A* to “efficiently” estimate $V_m \triangleq [v_1, \ldots, v_m]$
Why Estimate $V_m$?

**Figure:** Data is transmitted from node A to node B for a duration of $L_c - L_{B,\tau}$ symbols. Here, the entries of $x_{A,d}$ and $w_{B,d}$ are assumed to be i.i.d. $\mathcal{CN}(0, 1)$ random variables.

- Optimal in terms of capacity at high SNR
Conventional Training (Symbol I)

Figure: Symbol I of the conventional channel agnostic training
Conventional Training (Symbol II)

Figure: Symbol II of the conventional channel agnostic training
Conventional Training (Symbol $n_B$)

- **Motivation**
- **RCT for SIMO Channels**
- **RCT for MIMO Channels**
- **Conclusions**

Conventional Training ($\text{Symbol } n_B$)

Node A

$\hat{H} = [\hat{h}_1^H \hat{h}_2^H \hat{h}_{n_B}^H]$

$H \rightarrow SV D \rightarrow \hat{U} \hat{\Sigma} \hat{V}^H \rightarrow \hat{V}_m$

**Perfect Knowledge of $H$**

- **Figure**: Symbol $n_B$ of the conventional channel agnostic training

**Training overhead**

Requires $n_B$ symbols to convey $V_m$ to node A!
Proposed Reverse Channel Training

Proposed Estimation Method

- The proposed training sequence $X_{B,\tau} = \sqrt{P_{B,\tau}} L_{B,\tau} \phi_c UD$
  where $D = \text{diag}\{d_1, \ldots, d_m\}$ such that $\|D\|_F^2 = 1$, and the temporal power allocation $\phi_c$ satisfies $\mathbb{E}\phi_c \leq 1$

- Received training signal at node $A$:
  \[
  \bar{Y}_{A,\tau} \overset{\Delta}{=} \frac{Y_{A,\tau}}{\sqrt{P_{B,\tau}} L_{B,\tau}} = \sqrt{\phi_c} V \Sigma^H D + \frac{W_{A,\tau}}{\sqrt{P_{B,\tau}} L_{B,\tau}}
  \]

- Estimate of the $k^{th}$ BF vector:
  \[
  \hat{v}_k = \frac{\bar{y}_{k,A,\tau}}{\|\bar{y}_{k,A,\tau}\|_2}, \quad 1 \leq k \leq m
  \]

where $\bar{y}_{k,A,\tau}$ is the $k^{th}$ column of $\bar{Y}_{A,\tau}$
Main Goal

RCT Sequence Design

- Reverse-link training:

\[ \tilde{Y}_{A,\tau} \triangleq \frac{Y_{A,\tau}}{\sqrt{P_{B,\tau}L_{B,\tau}}} = \sqrt{\phi_c} V \Sigma^H D + \frac{W_{A,\tau}}{\sqrt{P_{B,\tau}L_{B,\tau}}} \]

- Problem: Find \( D \) and \( \phi_c \) that optimizes a metric

Metric:

1. Mean Square Error (MSE)
2. Capacity Lower Bound
MSE as a metric

Problem statement:

\[ \min_{D, \phi_c} \mathbb{E} \| V_m - \hat{V}_m \|_F^2 \]
\[ \text{s.t. } \|D\|_F = 1, \mathbb{E} \phi_c \leq 1 \]

Approximate MSE:

\[ \mathbb{E} \| V_m - \hat{V}_m \|_F^2 \approx \frac{2n_A - 1}{2P_{B,\tau}L_{B,\tau}} \mathbb{E} \sum_{k=1}^{m} \frac{1}{\sigma_k^2 d_k^2 \phi_c} \]

Approx. is tight at high values of \( P_{B,\tau}L_{B,\tau} \)

Optimization problem:

\[ \min_{d_k, \phi_c} \mathbb{E} \sum_{i=1}^{m} \frac{1}{\sigma_k^2 d_k^2 \phi_c} \]
\[ \text{s.t. } \sum_{i=1}^{m} d_i^2 = 1, \mathbb{E} \phi_c \leq 1 \]
Solution: MSE as a metric

**Lemma**

The optimal $D$ and $\phi_c$ are given by

$$d_k = \sqrt{\frac{\sigma_k^{-1}}{\sum_{i=1}^m \sigma_i^{-1}}} \quad \text{and} \quad \phi_c = \frac{\sum_{i=1}^m \sigma_i^{-1}}{\mathbb{E} \sum_{i=1}^m \sigma_i^{-1}}$$

- The approximate MSE with $\phi_c = 1$ and optimal $D$ is

$$\mathbb{E} \| V_m - \hat{V}_m \|_F^2 \approx \frac{2n_A - 1}{2P_{B,\tau} L_{B,\tau}} \mathbb{E} \left( \sum_{i=1}^m \sigma_i^{-1} \right)^2$$

- The approximate MSE with the jointly optimal $D$ and $\phi_c$ is

$$\mathbb{E} \| V_m - \hat{V}_m \|_F^2 \approx \frac{2n_A - 1}{2P_{B,\tau} L_{B,\tau}} \left( \mathbb{E} \sum_{i=1}^m \sigma_i^{-1} \right)^2$$
Simulation Results: MSE as a metric

Figure: MSE versus training power for a $3 \times 4$ MIMO system with $m = 3$
Approximate Capacity Lower Bound

- Approximate capacity lower bound:
  \[
  C_a = \frac{L_c - L_{B,\tau}}{L_c} \mathbb{E} \log \left| l_m + \frac{P_{A,d}}{m} \frac{\sum_m \sum_m^H}{\sigma_{\text{eff}}^2 + 1} \right|
  \]

  with \( \sigma_{\text{eff}}^2 \triangleq \frac{P_{A,d}}{P_{B,\tau} L_{B,\tau} m^2} \sum_{i=1}^m \frac{\beta_i}{d_i^2 \phi_c} \) and \( \beta_i \triangleq \frac{1}{2} + \sum_{j=1, j \neq i}^m \frac{\sigma_j^2}{\sigma_i^2} \)

- The approximation is obtained using
  1. worst case noise theorem [Hassibi and Hochwald, TIT-2003]
  2. ignoring the terms of the order \( \frac{1}{(P_{B,\tau} L_{B,\tau})^{3/2}} \) and higher

- Problem Statement:
  \[
  \max_{D, \phi_c : \|D\|_F^2 = 1, \mathbb{E} \phi_c \leq 1} C_a
  \]
Solution: Capacity Lower Bound as a Metric

\textbf{Theorem}

For a given $L_{B,\tau}$, the optimal $D$ is given by

$$d_i = \sqrt{\frac{\sqrt{\beta_i}}{\sum_{j=1}^{m} \sqrt{\beta_j}}}, \quad 1 \leq i \leq m$$

where $\beta_i \triangleq \frac{1}{2} + \frac{\sum_{j=1, j \neq i}^{m} \sigma_j^2}{\sigma_i^2}$. Also, $\phi_c^*$ satisfies

$$\lambda = \mathcal{H}(\phi_c^*) \triangleq \left( \frac{1}{\tau + \phi_c^*} \right) \sum_{k=1}^{m} \frac{P_{A,d} \sigma_k^2 \tau}{(P_{A,d} \sigma_k^2 + m) \phi_c^* + m \tau}$$

where $\tau \triangleq \frac{P_{A,d}}{P_{B,\tau} L_{B,\tau} m^2} \left( \sum_{k=1}^{m} \sqrt{\beta_k} \right)^2$, and $\lambda$ is a Lagrange multiplier, chosen such that $\mathbb{E}\phi_c^* = 1$
Simulation Results: Capacity Lower Bound as a Metric

**Figure:** Capacity lower bound for a $3 \times 4$ MIMO system versus RCT power $P_{B,\tau}$, with $P_{A,d} = P_{B,\tau}$, $L_{B,\tau} = 3$ symbols, $L_c = 100$, and $m = 3$ modes.
Further Topics

Topics studied but not presented

- RCT for multi-user systems
- RCT for MIMO with two-way training
- RCT for MIMO with time correlated channels
Summary

- Studied DMT performance of channel dependent RCT in a TDD-SIMO system
- Proposed a novel channel dependent training for an SM based TDD-MIMO system that was optimized using
  - MSE as a metric
  - a capacity lower bound as a metric

Take-home lessons

- Exploiting CSI while designing the RCT sequence improves the performance
- The RCT should be designed so as to convey only the part of the CSI required for data transmission
- Power-controlled RCT, when feasible, significantly outperforms fixed power RCT
Publications I

Bharath, B. N., and Murthy, C. R.
Reverse Channel Training for Reciprocal MIMO Systems with spatial multiplexing.

Bharath, B. N., and Murthy, C. R.
Channel estimation at the transmitter in a reciprocal MIMO spatial multiplexing system.
In IEEE Int. Conf. on Acoustics, Speech and Sig. Proc. (ICASSP), Prague, Czech Republic (May 2011).

Bharath, B. N., and Murthy, C. R.
Channel estimation at the transmitter in a reciprocal MIMO spatial multiplexing system.

Bharath, B. N., and Murthy, C. R.
Channel training signal design for reciprocal multiple antenna systems with beamforming.
IEEE Trans. on Vehicular Technology, Accepted for publication (Aug. 2012).
Bharath, B. N., and Murthy, C. R.
On the DMT of TDD-SIMO systems with channel-dependent reverse channel training.
*IEEE Trans. on Commun.*, Accepted for Publication vol. 60, no. 11 (Nov. 2012), pp. 3332–3341.

Bharath, B. N., and Murthy, C. R.
Power controlled reverse channel training achieves an infinite diversity order in a TDD-SIMO system with perfect CSIR.
*IEEE Commun. letters, Accepted for publication* vol. 16, no. 11 (Nov. 2012), pp. 1800–1803.

Bharath, B. N., and Murthy, C. R.
Power controlled reverse channel training in a multi-user TDD-MIMO spatial multiplexing system with CSIR.

Prasad R., Bharath, B. N., and Murthy, C. R.
Joint data detection and dominant singular mode estimation in time varying reciprocal MIMO systems.
In *IEEE Int. Conf. on Acoustics, Speech and Sig. Proc. (ICASSP)* (Mar. 2010).

Thank You!

Questions?