Zero-Crossings Based Spectrum Sensing Under Noise Uncertainties

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Motivation
System model
Weighted Zero-Crossings based Detector (WZCD)
Robustness to noise uncertainties: parameter, model
Simulation results
Spectrum Sensing

- In most of the CR-SS literature,

\[ \mathcal{H}_0 : (\text{signal absent}) \ Y_i = n_i \]
\[ \mathcal{H}_1 : (\text{signal present}) \ Y_i = h_i s_i + n_i, \ i = 1, 2, \ldots, M \]

- Classical Goodness-of-Fit Test formulation

\[ \mathcal{H}_0 : Y_i \sim f_N, \ i \in \mathcal{M} \]

- Threshold: chosen s.t. for \( \alpha_f \in [0, 1] \),

\[ p_f \triangleq \mathbb{P}\{\text{reject } \mathcal{H}_0 | \mathcal{H}_0 \text{ is true}\} \leq \alpha_f. \]
Receiver noise in communication systems $\sim \mathcal{N}(0, \sigma_G^2)$
Tests against Gaussianity should suffice?!
Receiver noise in communication systems $\sim \mathcal{N}(0, \sigma^2_G)$

Tests against Gaussianity should suffice?!

Bad assumption to begin with!
Receiver noise in communication systems $\sim \mathcal{N}(0, \sigma_G^2)$: bad assumption!

In signal processing for telecommunication systems, under $\mathcal{H}_0$: [Middleton1999]

$$Y_i = Y_i^G + Y_i^A + Y_i^B$$

Gaussian Class A Class B

$$(1 - \epsilon)fG + \epsilon fI S\alpha S(\alpha)$$

Approximations due to [Vatsola1984] and [Middleton1999] (and earlier works of Middleton)
As seen earlier,

\[ Y_i^{(G)} + Y_i^{(A)} \sim (1 - \epsilon)f_G + \epsilon f_I, \text{ with } 0 < \epsilon \ll 1. \]

- \[ f_G \overset{d.}{=} \mathcal{N}(0, \sigma^2_G) \]
- \[ f_I \overset{d.}{=} \mathcal{N}(0, \sigma^2_I) \triangledown \text{[Vatsola1984], [AazhangPoor1987], or} \]
- \[ f_I \overset{d.}{=} \mathcal{L}(0, \sigma^2_I) \triangledown \text{[MillerThomas1976], with } \sigma^2_I / \sigma^2_G \in (10, 100). \]

\[ Y_i^{(B)} \sim S\alpha \mathcal{S}(\alpha) \]

- Characteristic function of the \( S\alpha \mathcal{S}(\gamma_0, \alpha) \) distribution

\[ \Phi_B(w, \gamma_0, \alpha) = \exp(-\gamma_0|w|^\alpha), \quad \gamma_0 > 0, \ 0 < \alpha \leq 2. \quad (1) \]

- No closed form for PDF; except for the cases \( \alpha = 2 \) (Gaussian), \( \alpha = 1 \) (Cauchy) and \( \alpha = 0.5 \) (Lévy)

- Should not ignore the Gaussian component! (opposed to [Chavali2012], and references therein)
Noise Uncertainties

- Noise Model Uncertainty (NMU)
  - Uncertainty in knowledge of $f_N \Rightarrow$ either class A, B or both, and PDF of $f_I$

- Noise Parameter Uncertainty (NPU)
  - Uncertainty in the parameter set $(\sigma^2_G, \sigma^2_I, \epsilon, \alpha)$
An Existing Technique

- Proposed by [Shen et al. 2011], called the Blind Detector (BD).
- Robust to the classical noise variance uncertainty, when \( f_N \sim \mathcal{N}(0, \sigma^2_G) \)
- \( M \) observations are divided into \( n \) windows of \( m \) samples each. Choose \( n \) to be small.
- Calculate sample mean and sample variances from each window. Their ratio is student-t distributed with parameter \( m - 1 \).
- Over the obtained \( n \) samples, run an Anderson-Darling GoFT.
Weighted Zero-Crossings based Sensing

- In an earlier work, [KedemSlud1982] have studied for the Gaussian case (both i.i.d. and correlated)
- Consider the first $k$ order difference operators:

$\nabla Y_i \triangleq Y_i - Y_{i-1}$

$\nabla^2 Y_i = \nabla(\nabla Y_i) = Y_i - 2Y_{i-1} + Y_{i-2}$

$\vdots$

$\nabla^k Y_i = \sum_{j=0}^{k} \binom{k}{j} (-1)^j Y_{i-j}, \quad i \geq k + 1.$  \hspace{1cm} (2)
Weighted Zero-Crossings based Sensing

The $k^{\text{th}}$ order ZC is said to occur, if the sign of $\nabla^{k-1} Y_i$ is different from that of $\nabla^{k-1} Y_{i+1}$

$$\Delta_{j,M} \triangleq \begin{cases} D_{1,M}, & j = 1, \\ D_{j,M} - D_{j-1,M}, & j = 2, \ldots, k - 1 \\ (M - 1) - D_{k-1,M}, & j = k, \end{cases}$$

(3)

$$\mu_{j,M} \triangleq \mathbb{E} \Delta_{j,M}, j = 1, \ldots, k,$$

(4)

For a given set of weights $w_j$, a $\Psi^2_w$ Statistic-based Detector ($\Psi_w$SD) is given by

$$\Psi^2_w \sim \mathcal{H}_0 \leq \tau^\Psi_w,$$

(5)
We consider the following cases for comparison with BD

- Equal and unit weights: [KedemSlud1982] $\Psi_1^2 \sim \chi^2_3(11)$. Choose the threshold $\tau_{1\Psi}$ such that

$$Q_{3/2} \left( \sqrt{11}, \sqrt{\tau_{1\Psi}} \right) = \alpha_f,$$  \hspace{1cm} (6)

- Exponential weights: $\Psi_2^2 w \sim \mathcal{F}(17.5, 7)$. Choose the threshold $\tau_{m\Psi SD}$ such that

$$1 - \mathcal{I} \left( \frac{17.5 \tau_{m\Psi SD}}{17.5 \tau_{m\Psi SD} + 7}, 8.75, 3.5 \right) = \alpha_f,$$  \hspace{1cm} (7)

- Note that the statistic and the threshold are independent of variance of $f_N$!
Robustness to NPU and NMU: Intuition

- When \( f_I \sim \mathcal{N}(0, \sigma^2_G) \), no problem at all.
- The PDF of any member of the \( S_\alpha S \) family, for \( 1 \leq \alpha \leq 2 \) can be written as [West1987]

\[
p(X) = \int_0^\infty \left( \frac{1}{\sigma} \right) g \left( \frac{X}{\sigma} \right) h(\sigma) \, d\sigma,
\]

where \( g(\cdot) \) is the standard Gaussian PDF, and \( h : \mathbb{R}^+ \rightarrow \mathbb{R}^+ \) is some function (can as well be a PDF).
- West extends this result to exponential family, which includes the Laplace distribution.
More Advantages

- For large $M$, the number of zero-crossings for any symmetric distribution is $M/2$.
- Works with distributions with infinite variance.
- Works with distributions with infinite mean! ⇒ works as long as the median exists.
- Given that it is a GoFT, can be used with any signal and fading models.
- Computational complexity: same as the energy detector, and less intense that the blind detector.
- One disadvantage
Primary, and Fading models for simulations

- Primary signal models
  - Model 1: Constant primary
  - Model 2: Sinusoid primary

- Fading models
  - Rayleigh fading (i.i.d., and first order ARMA correlated)
Figure: Detection of primary model 1 under Rayleigh fading, with Gaussian + $S\alpha S$ model.
Figure: Detection of primary model 2 under Rayleigh fading, with Gaussian + SαS model.
Figure: Detection of primary models 1 and 2 under Rayleigh fading, with $\epsilon$-mixture model, $\epsilon = 0.05$, and $f_I \sim \mathcal{N}(0, \sigma_i^2)$. 
Figure: Detection of primary models 1 and 2 under Rayleigh fading, with $\epsilon$-mixture model, $\epsilon = 0.05$, and $f_\mathcal{I} \sim \mathcal{L}(\sigma_1^2)$. 
Figure: Detection of primary models 1 and 2 under pure Gaussian noise, with noise variance uncertainty $= 3dB$, $M = 300$, $\alpha_f = 0.05$. Average $p_f$ obtained through simulations for BD, $\Psi$SD and $m\Psi$SD are 0.0498, 0.05, and 0.0501, respectively.
**Figure:** Detection of primary models 1 and 2 under first order AR correlated fading (with $\rho = 0.5$) and pure Gaussian Noise, with noise variance uncertainty $= 3\, dB$, $M = 300$, $\alpha_f = 0.05$. 
Figure: Detection of primary model 1 under Gaussian + class A + class B noises, with noise variance uncertainty = 3dB, $M = 300$, $\alpha_f = 0.05$, $\epsilon = 0.05$, $f_I \sim \mathcal{N}(0, 100\sigma_G^2)$. 

\begin{itemize}
  \item BD, $p_d$
  \item BD, $p_f$
  \item $\Psi_1^S$, $p_d$
  \item $\Psi_1^S$, $p_f$
  \item $\Psi_e^S$, $p_d$
  \item $\Psi_e^S$, $p_f$
\end{itemize}
Figure: Detection of primary model 2 under Gaussian + class A + class B noises, with noise variance uncertainty $= 3dB$, $M = 300$, $\alpha_f = 0.05$, $\epsilon = 0.05$, $f_I \sim \mathcal{N}(0, 100\sigma^2_G)$. 
**Figure:** Optimal threshold calculation under Gaussian + class A + class B noises, with noise variance uncertainty = 3dB, \( M = 300, \alpha_f = 0.05, \epsilon = 0.05, f_I \sim \mathcal{N}(0, 100\sigma^2_G) \).