# A Framework for Designing Multihop Energy Harvesting Sensor Networks

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## Use case scenarios

Integral part of cyber-physical systems, for example

- Urban sensing systems
- Integrated environment monitoring
- Industrial automation
- Civilian surveillance

## Ingredients of multihop sensor networks

- **Sensor nodes:** equipped with sensor modules, finite battery, finite storage, an energy harvesting device and a single antenna radio.
- **Gateway nodes:** larger nodes equipped with a wireless interface for communications with the WSN, and a wired interface for communications with the controlling station.
- **Adhoc architecture:** offers a range of benefits, including reliability, robustness, quick and easy network deployment, energy efficient network operations etc.

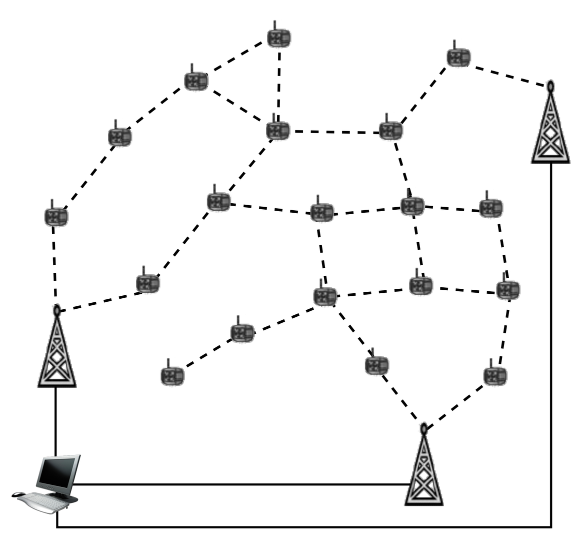

Figure 1: A multihop energy harvesting wireless sensor network with multiple gateway (sink) nodes; the dotted lines represent the wireless links.

## A utility function

- Time is divided into slots of length $\sigma$.
- $d_i(t)$ : fraction of time sensor node is sensing the environment in the $t^{\text{th}}$ slot.
- Let $\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T} d_i(t) = d_i$ - fraction of time sensor node $i$ senses. We define the utility as $\sum_{i\in\mathcal{N}} U_i(d_i)$; $U_i$'s are increasing concave function.
- We use this utility function to compare and contrast different deployment scenarios

## Long-term time-averaged system

- In such WSNs, typically, the goal is to come up with optimal decision rules $\{\mathbf{d}(t), \mathbf{Y}(t), \mathbf{a}(t), t \geq 1\}$; usually posed as **Markov decision process (MDP)**.
- However, in our setting, the reward depends on the long-term time-averaged quantities $\{d_1, d_2, \cdots, d_n\}$.
- This enables us to look at the long-term time-averaged system.
- It can be shown that the long-term time-averaged system under consideration should satisfy the following energy constraint

$$e^s \cdot d_j + \sum_{k\in\mathcal{N}} e \cdot \left( \sum_{l\in\mathcal{O}(j)} y_{kl} + \sum_{l\in\mathcal{I}(j)} y_{kl} \right) \leq e_j^h \, \forall j \in \mathcal{N} \tag{1}$$

## A long-term time-averaged optimization problem

$$P_1 : \max_{\{\mathbf{a}\geq\mathbf{0},\ \mathbf{Y}\geq\mathbf{0},\ \mathbf{d}\in[0,1]^{|\mathcal{N}|}\}} \sum_{j\in\mathcal{N}} U_j(d_j)$$

Subject to:

$$\sum_{l\in\mathcal{O}(j)} y_{jl} = r^s \cdot d_j, \quad \sum_{l\in\mathcal{I}(j)} y_{jl} = 0 \quad \forall j \in \mathcal{N} \tag{2}$$

no accumulation at the sources

$$\sum_{s\in\mathcal{S}} \sum_{l\in\mathcal{I}(s)} y_{jl} = r^s \cdot d_j \quad \forall j \in \mathcal{N} \tag{3}$$

no packet drops

$$\sum_{l\in\mathcal{I}(j)} y_{kl} = \sum_{l\in\mathcal{O}(j)} y_{kl} \quad \forall j \in \mathcal{N}, \forall k \in \mathcal{N} \setminus \{j\} \tag{4}$$

flow conservation

$$e^s \cdot d_j + \sum_{k\in\mathcal{N}} e \cdot \left( \sum_{l\in\mathcal{O}(j)} y_{kl} + \sum_{l\in\mathcal{I}(j)} y_{kl} \right) \leq e_j^h \, \forall j \in \mathcal{N} \tag{5}$$

rate of energy consumption $\leq$ rate of energy harvesting

$$\sum_{k\in\mathcal{N}} y_{kl} \leq (\mathbf{M}\cdot\mathbf{a})_l \, \forall l \in \mathcal{L} \tag{6}$$

rate of flow on link $\leq$ effective link capacity

$$\sum_{I} a_I \leq 1 \tag{7}$$

two different MIS cannot be active simultaneously

- $\mathcal{N}$ set of sensor nodes
- $d_j$ average fraction of time node $j$ senses
- $r^s$ maximum rate at which sensor node can generate data
- $\mathcal{S}$ set of sink/gateway nodes
- $\mathcal{L}$ set of wireless links
- $\mathbf{M}$ matrix representing the collection of maximal independent sets (MIS)
- $\mathbf{a}$ schedule vector; average fraction of time each MIS is scheduled
- $y_{jl}$ average rate of flow of node $j$'s traffic, on the wireless link $l$
- $e_j^h$ rate at which node $j$ harvests energy
- $e$ energy needed to transmit or receive data at unit rate
- $e^s$ rate of energy consumed for sensing
- $\mathcal{O}(j)$ the set of directed links that originate at node $j$
- $\mathcal{I}(j)$ the set of directed links that terminate at node $j$
- $U_j(\cdot)$ concave twice differentiable function

## An alternate formulation

- Computing matrix $\mathbf{M}$ in an arbitrary graph is a well-known *NP-hard* problem.
- Since problem $P_1$ satisfies *Slater's condition*, it has no duality gap. Therefore, we can optimally solve problem $P_1$, by solving its dual problem. However, to solve the dual problem of $P_1$, we need to find a *maximum weighted matching* in the *directed graph* $\mathcal{G}$. Complexity of computing a *maximum weighted matching* in a graph with *directed edges* remains unknown.
- Alternatively, we relax the MIS constraints into clique constraints. This relaxation allows us to handle the NP-hardness of the time-averaged problem $P_1$ while achieving the optimum value of problem $P_1$.
- For the primary inference model, under clique constraints, we obtain the following *necessary condition*

$$\sum_{l\in\mathcal{I}(j)\cup\mathcal{O}(j)} \frac{\sum_{k\in\mathcal{N}} y_{kl}}{c_l^0} \leq 1 \quad \forall j \in \mathcal{N} \tag{8}$$

where $c_l^0$ is the capacity of link $l \in \mathcal{L}$.

After replacing the MIS constraint in problem $P_1$ with the clique constraints, we obtain the following optimization problem

$$P_2 : \max_{\{\mathbf{c}\geq\mathbf{0},\ \mathbf{Y}\geq\mathbf{0},\ \mathbf{d}\in[0,1]^{|\mathcal{N}|}\}} \sum_{j\in\mathcal{N}} U_j(d_j)$$

Subject to: constraints (2), (3), (4), (5), (8) and $\sum_{k\in\mathcal{N}} y_{kl} \leq c_l \quad \forall l \in \mathcal{L}$

## The dual problem

Relaxing the capacity and energy constraints, we obtain the dual of problem $P_2$ as

$$\min_{\boldsymbol{\beta}\geq\mathbf{0},\boldsymbol{\gamma}\geq\mathbf{0}} D(\boldsymbol{\beta},\boldsymbol{\gamma})$$

where

$$D(\boldsymbol{\beta},\boldsymbol{\gamma}) = \max_{\mathbf{d},\mathbf{Y},\mathbf{c}} \left\{ \sum_{j\in\mathcal{N}} \left( U_j(d_j) + \beta_j \cdot \left( e_j - e^s d_j - \sum_{k\in\mathcal{N}} e \cdot \left( \sum_{l\in\mathcal{O}(j)} y_{kl} + \sum_{l\in\mathcal{I}(j)} y_{kl} \right)\right)\right) + \sum_{l\in\mathcal{L}} \gamma_l \left( c_l - \sum_{k\in\mathcal{N}} y_{kl} \right) \right\}$$

Subject to: constraints (2), (3), (4), (8) and $\mathbf{c} \geq \mathbf{0}, \quad \mathbf{Y} \geq \mathbf{0}, \quad \mathbf{d} \in [0,1]^{|\mathcal{N}|}$

## A solution approach

- This dual can be decomposed into the following sub-problems that can be solved independent of each other.
- **Scheduling subproblem**

$$\max_{\mathbf{c}\geq\mathbf{0}} \boldsymbol{\gamma}^T\mathbf{c} \quad \text{subject to constraint (8)}$$

- **Joint sensing fraction allocation and routing subproblem**

$$\max_{\mathbf{d},\mathbf{Y}} \left\{ \sum_{j\in\mathcal{N}} (U_j(d_j) - \beta_j e^s d_j) - \sum_{k\in\mathcal{N}} \sum_{l\in\mathcal{L}} \gamma_l \cdot y_{kl} - \sum_{k\in\mathcal{N}} \left( \sum_{j\in\mathcal{N}} \beta_j \cdot e \cdot \left( \sum_{l\in\mathcal{O}(j)} y_{kl} + \sum_{l\in\mathcal{I}(j)} y_{kl} \right)\right) \right\} \tag{9}$$

Subject to: constraints (2), (3), (4) and $\mathbf{Y} \geq \mathbf{0}, \mathbf{d} \in [0,1]^{|\mathcal{N}|}$

- The solution to the above problem is as follows

$$d_j(\boldsymbol{\beta},\boldsymbol{\gamma}) = \left[ U'^{-1} \left( \beta_j \cdot e^s + r^s \cdot c_j^{lcp}(\boldsymbol{\beta},\boldsymbol{\gamma}) \right) \right]^+$$

where $c_j^{lcp}$ is the cost of least-cost path and is given as

$$c_j^{lcp} = \arg\min_{s\in\mathcal{S}} \min_{P\in\mathcal{P}_{js}} \left( \sum_{l\in P\cap\mathcal{L}} \gamma_l + 2e \cdot \sum_{k\in P\cap\mathcal{N}} \beta_k \right)$$

- Let $\mathbf{p} = [\boldsymbol{\beta},\boldsymbol{\gamma}]^T$ denote the price vector. Then, the price vector can be updated using the projected subgradient method as follows

$$\mathbf{p}[m+1] = [\mathbf{p}[m] - \delta \cdot \mathbf{g}(\mathbf{p}[m])]^+$$

### Proposition

The optimum values of problems $P_1$ and $P_2$ are equal.

## Numerical evaluation

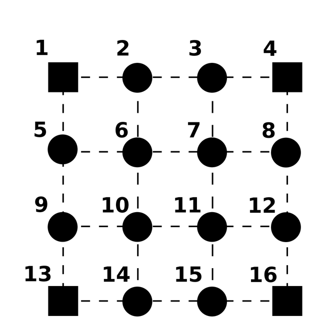


Figure 2: Network $G_1$

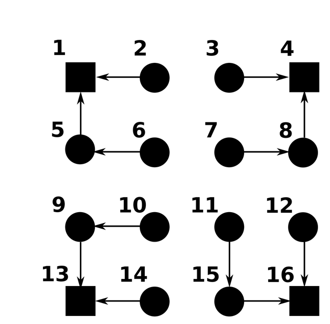


Figure 3: Optimal routes

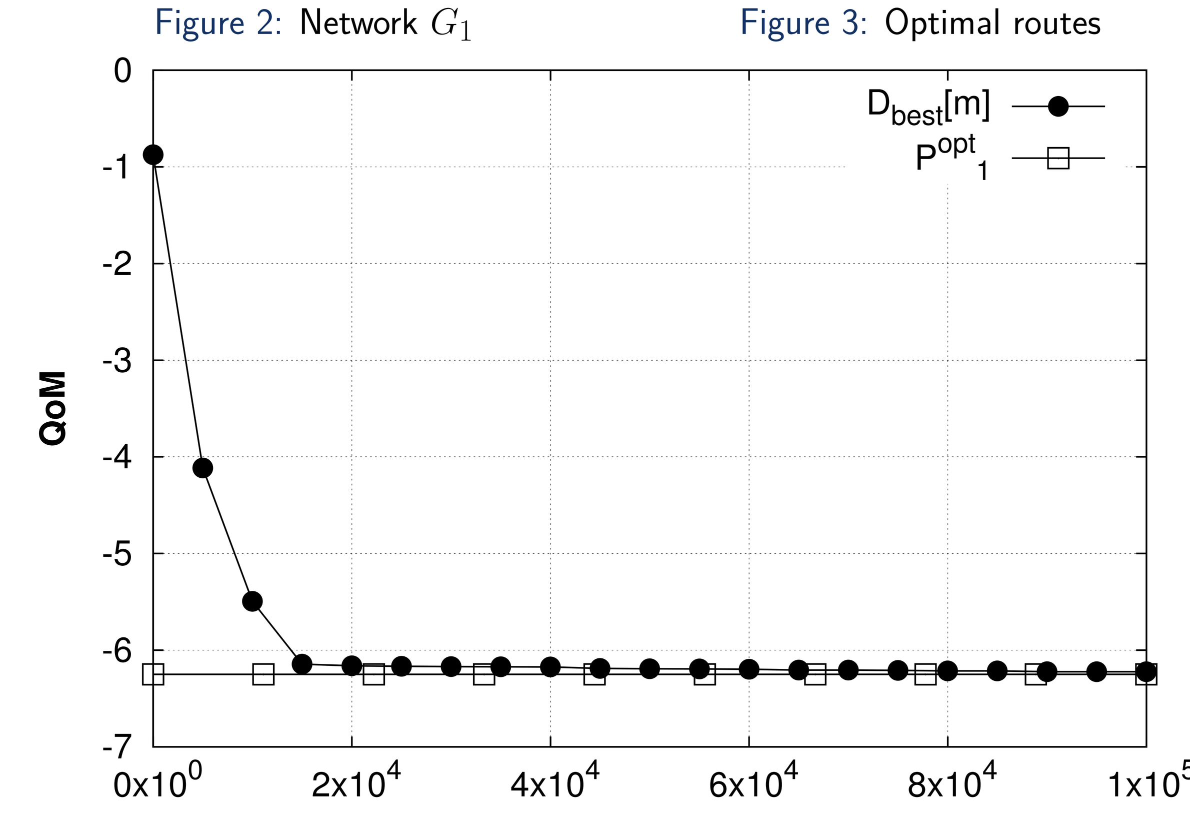


Figure 4: Plot of dual objective achieved by the projected sub-gradient update.