Stable Galerkin Finite Element Formulation for the Simulation of Electromagnetic Flowmeter

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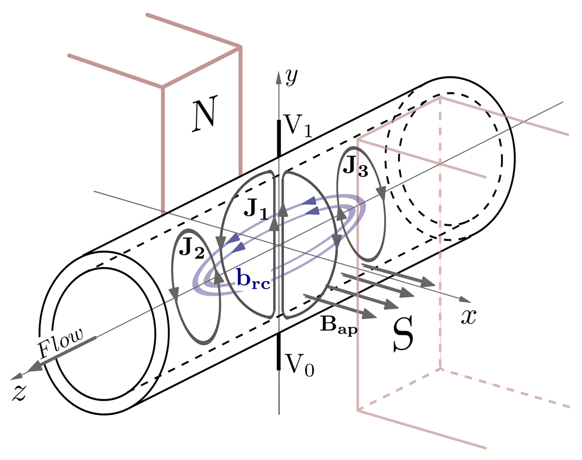
Introduction



- Electromagnetic flowmeter is extensively employed for the measurement of liquid-metal flow rate in the fast breeder reactors.
- Reliable measurement is essential for the control and safe operation of the reactor
- Experimental calibration of electromagnetic flowmeter is extremely difficult and theoretical approach is preferred
- The governing equations are,

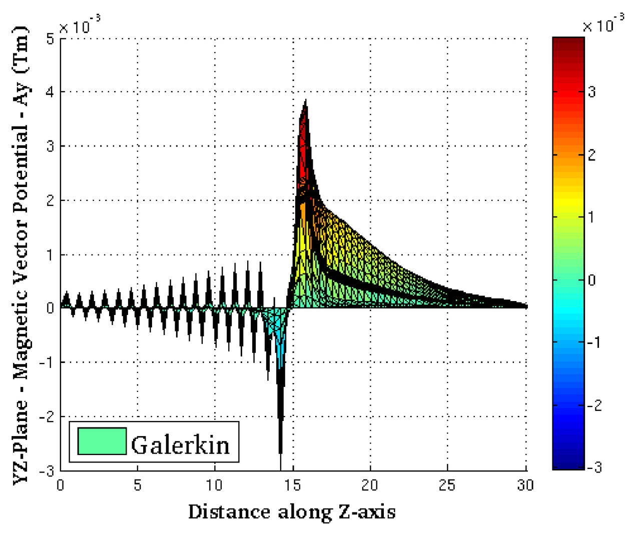
$$\sigma\nabla\phi \;-\; (\nabla\cdot\frac{1}{\mu}\nabla)\mathbf{A} - \sigma\,\mathbf{u}\times\nabla\times\mathbf{A} = \sigma\,\mathbf{u}\times\mathbf{B_{ap}}$$

$$\nabla\cdot(\sigma\nabla\phi) - \nabla\cdot(\sigma\,\mathbf{u}\times\nabla\times\mathbf{A}) = \nabla\cdot(\sigma\,\mathbf{u}\times\mathbf{B_{ap}})$$

where ϕ is the scalar potential arising out of the current flow, $\mathbf{A}$ is the magnetic vector potential associated with reaction magnetic field $\mathbf{b_{rc}}$, $\mathbf{B_{ap}}$ is the applied magnetic field, $\mathbf{u}$ is the velocity of liquid metal, μ is the magnetic permeability, σ is the electrical conductivity.

- Galerkin finite element method (GFEM) is a ready choice to solve the governing equations. Only in very limited literature whole 3D version of the problem is simulated using GFEM [6]

Galerkin Scheme - Numerical Instability



- GFEM is known to suffer from numerical oscillations when $Pe = \mu\sigma|\mathbf{u}|\Delta z/2 > 1$. ($\Delta z$ is the element length along the flow direction)
- As a remedial measure Streamline upwind/Petrov Galerkin (SU/PG) scheme is suggested in the allied literature [1] [2].
- SU/PG scheme introduces boundary error [4] [5] and non-physical current in the solution [8]
- In addition, SU/PG scheme needs calculation of stabilization parameter and requires more calculation for higher order elements.
- Scope of the work: To arrive at a 'Stable Galerkin Finite Element Formulation for Electromagnetic Flowmeter Analysis'

Proposed Approach

- Classically, numerical stability of the FEM solution is analyzed with the 1D version of the problem [3] [9]. FEM equations for a regular grid takes the form of difference equation, which is employed for the required analysis
- Following the same, 1D version of the flowmeter governing equation:

$$-\frac{d^2A_y}{dz^2} + \mu\sigma u_z\frac{dA_y}{dz} = \mu\sigma u_z B_x$$

where, A_y is the y component of the vector potential, u_z is the velocity of the liquid metal along the $z-$ direction and B_x is the input magnetic field.

- The resulting, FEM difference equation:

$$(-1-Pe)A_{y(n-1)} + 2A_{y(n)} + (-1+Pe)A_{y(n+1)} = 2Pe\Delta z\Big(\frac{B_{x(n-1)} + 4B_{x(n)} + B_{x(n+1)}}{6}\Big)$$

- In this work, the Z-transform approach is proposed so as to bring tools from control systems theory. Accordingly when $Pe >> 1$, the relation between A_y and the input field B_x can be written as,

$$\frac{A_y}{B_x} \simeq \frac{\Delta z}{3}\,\frac{(Z+0.27)(Z+3.73)}{(Z\;-\;1)\,(Z\;+\;1)}$$

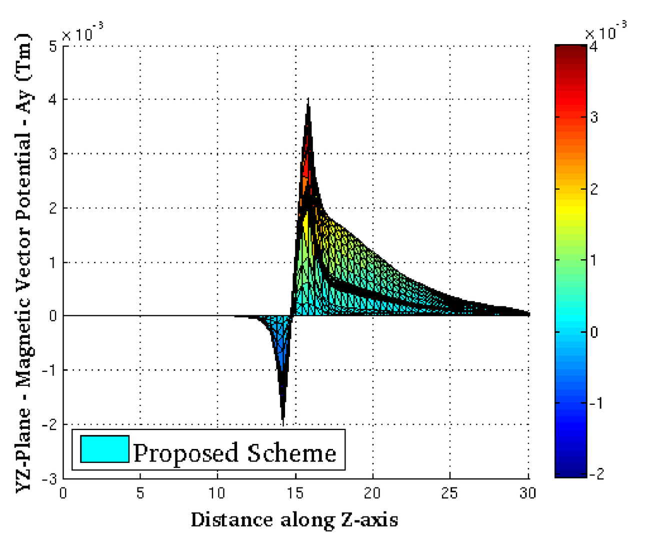
Pole at '-1' is responsible for the numerical oscillations

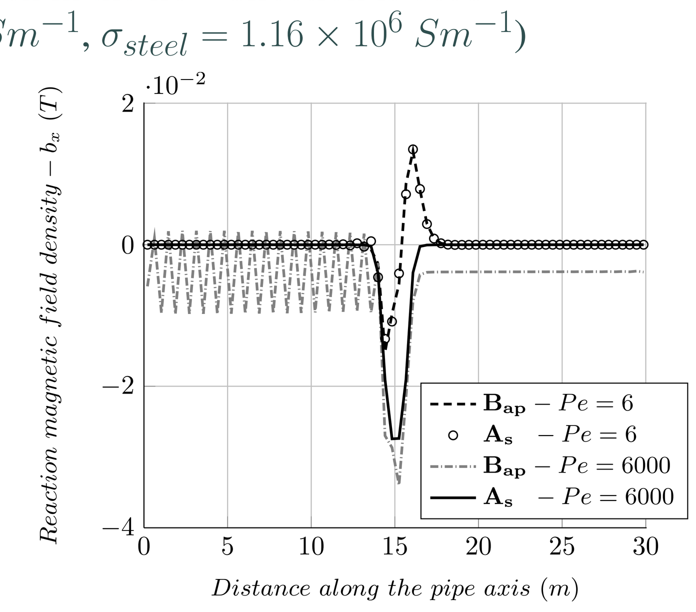
- **Proposed approach:** To seek re-formulation of the RHS so as to introduce necessary zeros
- **Scheme-1:** Input field on the RHS is restated in terms of magnetic vector potential [7]

$$\frac{A_y}{A_{sy}} \simeq -\frac{(Z\;-\;1)\,(Z\;+\;1)}{(Z\;-\;1)\,(Z\;+\;1)}$$

Simulation Results for Scheme-1

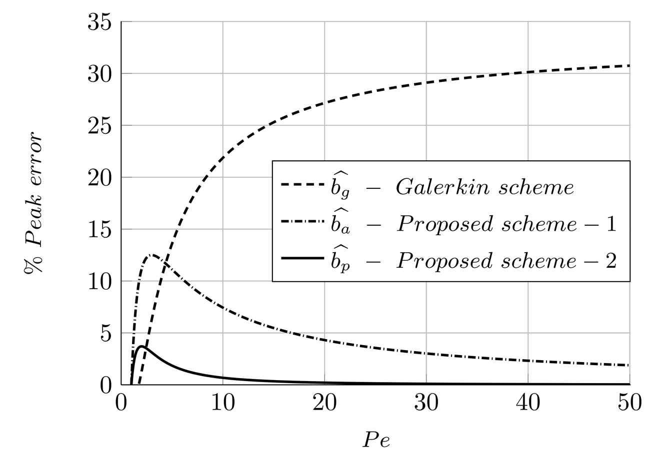
- 33598 brick elements with graded structured mesh in the flow direction is used ($\mu = 4\pi\times10^{-7}Hm^{-1}$, $\sigma_{sodium} = 7.21\times10^{6}\;Sm^{-1}$, $\sigma_{steel} = 1.16\times10^{6}\;Sm^{-1}$)





- In this scheme, the variation of input field only along the flow direction is considered, which is generally true for electromagnetic flowmeters

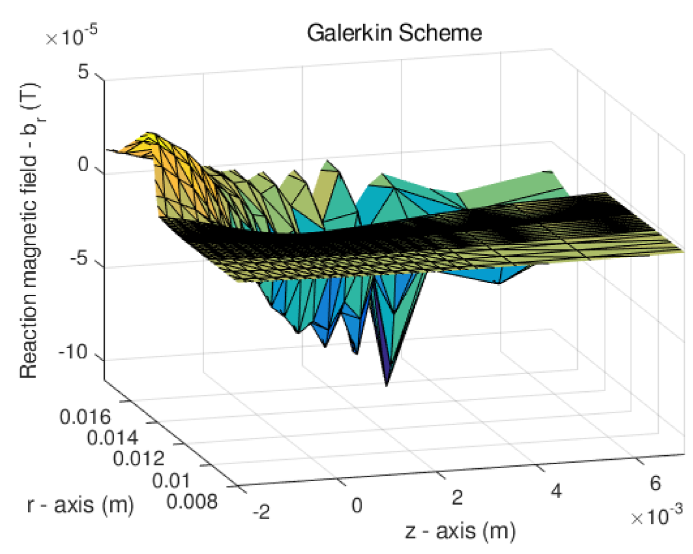
Proposed Scheme - 2

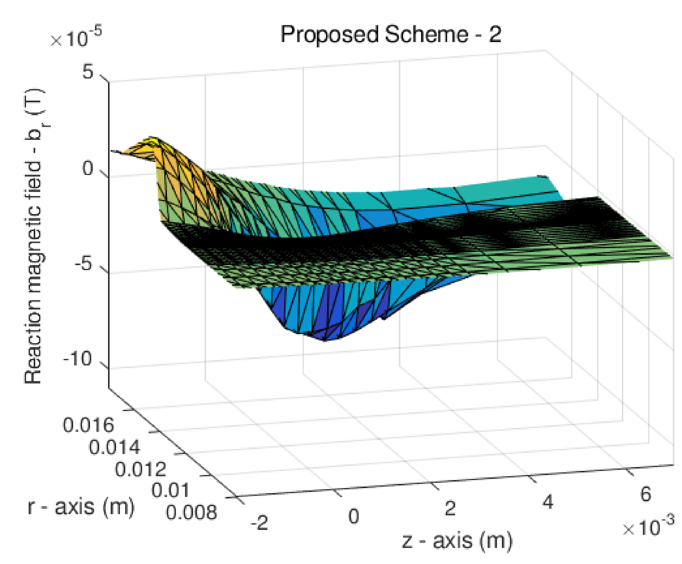


- Weighted nodal input magnetic field is considered where the required weights are constrained so as to be consistent, as well as, brings in necessary zero
- **Scheme 2:** $\dfrac{A_y}{B_x} \simeq \dfrac{\Delta z}{2}\,\dfrac{(Z+1)^2}{(Z\;-\;1)\,(Z+1)}$
- Performs better than 'scheme−1' - double zeros at '-1'.
- For both the schemes, extensive 1D and 2D Z-transform analysis has been performed to ascertain the characteristics of the numerical solution
- Performs well, even when the input magnetic field varies transverse to the flow direction

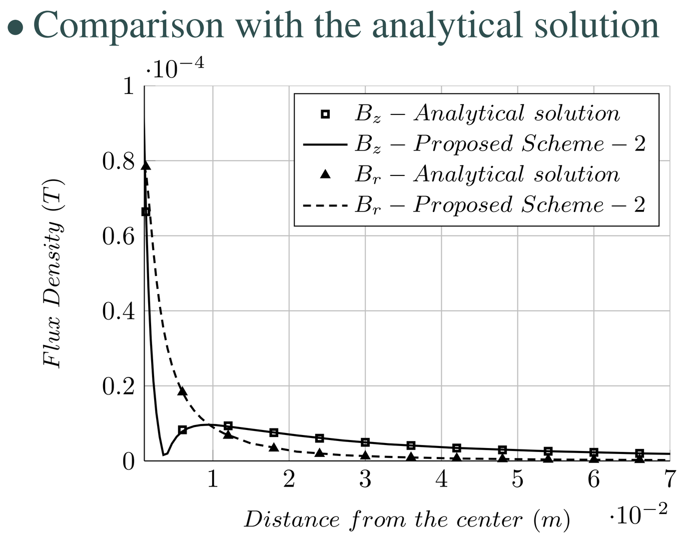
Application to other moving conductor problems

- TEAM-9 Standard test Problem Results: Scheme-2 gives stable results, while oscillations are found in the Galerkin scheme

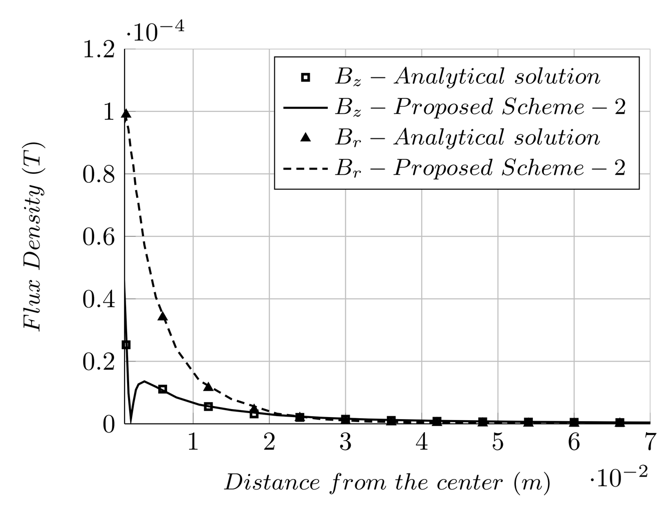




- Comparison with the analytical solution



$\mu_r = 1$



$\mu_r = 50$

- Scheme-2 results are matching well with the analytical solution of the TEAM-9 problem

Conclusions

- Theoretical evaluation of the sensitivity of electromagnetic flowmeter is a preferred choice for liquid metal flow measurement. Only numerical approach is feasible and GFEM is a ready choice. The GFEM suffer from numerical instability, when $Pe > 1$.
- Existing remedial measures in allied fields like SU/PG scheme gives non-physical solutions at the boundary.
- Two novel stable schemes have been proposed for graded regular mesh along the flow direction. Accurate results have been obtained for flowmeter and similar problems even at very high flow rates/velocity

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