Crowdfunding Public Projects with Provision Point: A Prediction Market Approach

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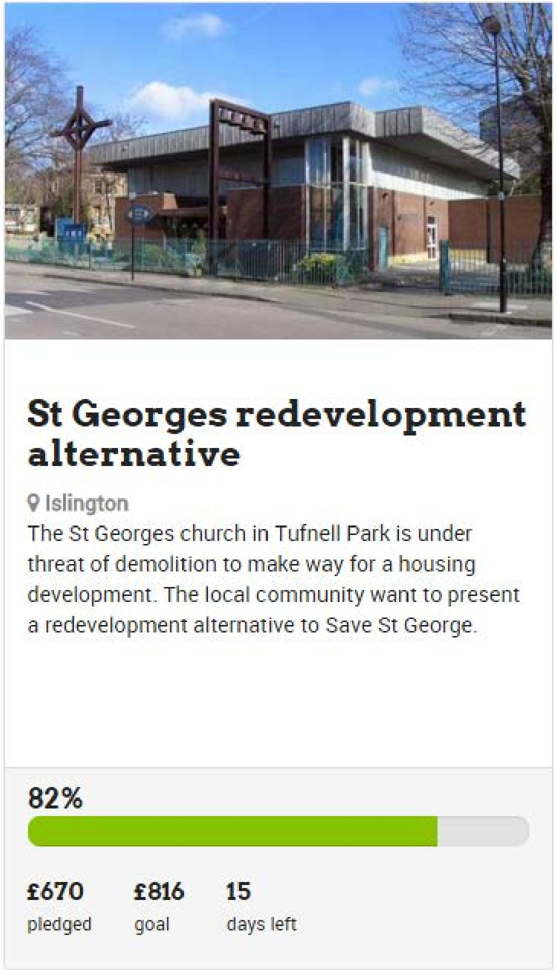
Crowdfunding: Private Provisioning of Public Goods

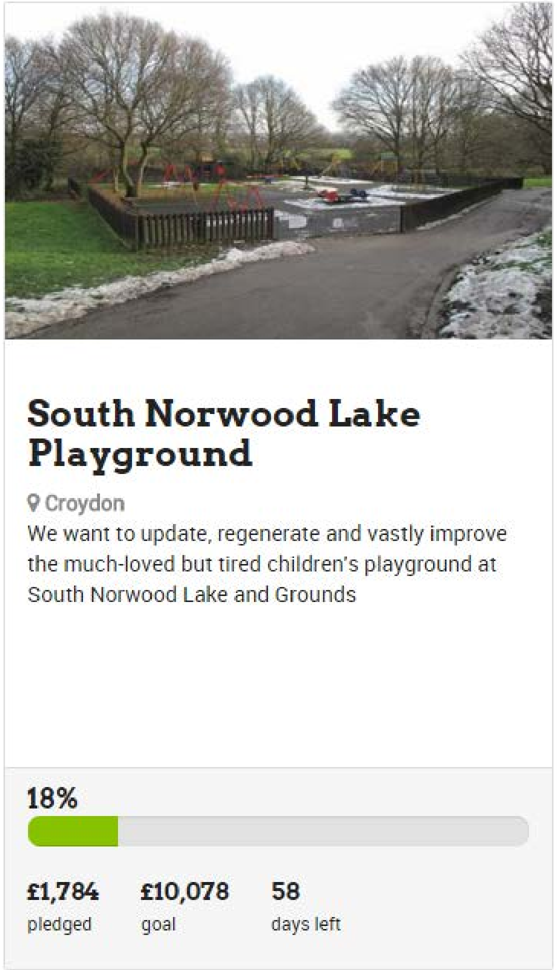
Crowdfunding Process

1. Requester posts public project (non-excludable)
2. Agents arrive & observe (i) target (provision point), (ii) deadline & (iii) pending amount.
3. Agents contribute (or not)
4. Requester executes project or refunds.

Motivation for Mechanism Design

- Agent's true value for the project is private information.
- Strategic agents can freeride (No / Low contribution).
- Strategic agents can delay contribution.
- Project may not be funded even if everyone values it!
- Mechanism Design: Induce a game such that agents contribute





Related Work

[Bagnoli & Lipman '89] : Provision Point Mechanism

a) Simultaneous move game
b) Multiple Equilibria; Project not funded at several.

[Zubrickas '14] : Provision Point Mechanism with Refund

a) Simultaneous move game
b) Set of equilibria at which project is funded.

[Our work] : Provision Point Mechanism with Securities

a) Sequential game
b) Set of subgame perfect equilibria: project funded.
c) Agents contribution proportional to their value
d) Agents contribute as soon as they arrive

[Hanson'03], [Chen & Pennock '10] : Prediction Markets

a) Software agents trading securities for predictions.
b) Scoring Rule ← →Cost Function
c) Specially suited for thin markets.

Our Work: Intuition

Incentivizes agents to contribute by offering them a bonus greater than their contribution.

Bonus paid out iff the project is not funded.

Ensures that project is funded at equilibrium.

Novel Idea: Use prediction markets for bonus!

Problem Setup

St Georges redevelopment alternative

Islington

The St Georges church in Tufnell Park is under threat of demolition to make way for a housing development. The local community want to present a redevelopment alternative to Save St George.

82%

£670 pledged £816 goal 15 days left

South Norwood Lake Playground

Croydon

We want to update, regenerate and vastly improve the much-loved but tired children's playground at South Norwood Lake and Grounds

18%

£1,784 pledged £10,078 goal 58 days left

Table 1: Key Notation

Symbol	Definition
T	Time at which fund collection ends
h^t	Amount that remains to be funded at t; h^0 is the target amount
$i \in \{0, 1, \ldots, n\}$	Agent id; $i = 0$ refers to the requester
$\theta_i \in \mathbb{R}_+$	Agent i's value for the project
$x_i \in \mathbb{R}_+$	Agent i's contribution to the project
$a_i \in [0, T]$	Time at which agent i arrives at the platform
$t_i \in [a_i, T]$	Time at which agent i makes a contribution towards the project
$\psi_i = (x_i, t_i)$	Strategy of agent i
$\vartheta \in \mathbb{R}_+$	Net value for the project
$\chi \in \mathbb{R}_+$	Net contribution for the project
$k \in \{0, 1\}$	Project provisioning decision

$$u_i(\psi; \theta_i) = \mathcal{I}_{\chi \geq h^0} \times (\theta_i - x_i) + \mathcal{I}_{\chi < h^0} \times (r_i^{t_i} - x_i)$$

Provision Point Mechanism with Securities (PPS)

Prediction Market in Crowdfunding

- Binary Event (At deadline, project funded or not?)
- Positive securities pay $1 if project funded.
- Negative securities pay $1 if project is not funded.
- Software agent always accepts trades.
- Price determined as the first order derivative of a cost function.

$$C_{LMSR}(\mathbf{q}) = b\ln(\exp(q_{\omega_0}/b) + \exp(q_{\omega_1}/b))$$

$$\begin{aligned} C_0(q^t) &= b\ln(1+\exp(q^t/b)) \\ \texttt{Cost}(r^t|q^t) &= C_0(q^t + r^t) - C_0(q^t) \\ &= b\ln\left(\frac{1+\exp(\frac{q^t+r^t}{b})}{1+\exp(\frac{q^t}{b})}\right) \end{aligned}$$

Complex Prediction Market in PPS: Issue only Negative securities

- Number of securities issued to an agent depend on
  - Quantum of his contribution
  - Timing of his contribution
- Sponsor* pays out only if project is not funded.

$$r_i^{t_i} = b\ln\left(\exp\left(\frac{x_i}{b} + \ln(1+\exp(\frac{q^{t_i}}{b}))\right) - 1\right) - q^{t_i}$$

*PPS needs a sponsor to offer a refund bonus

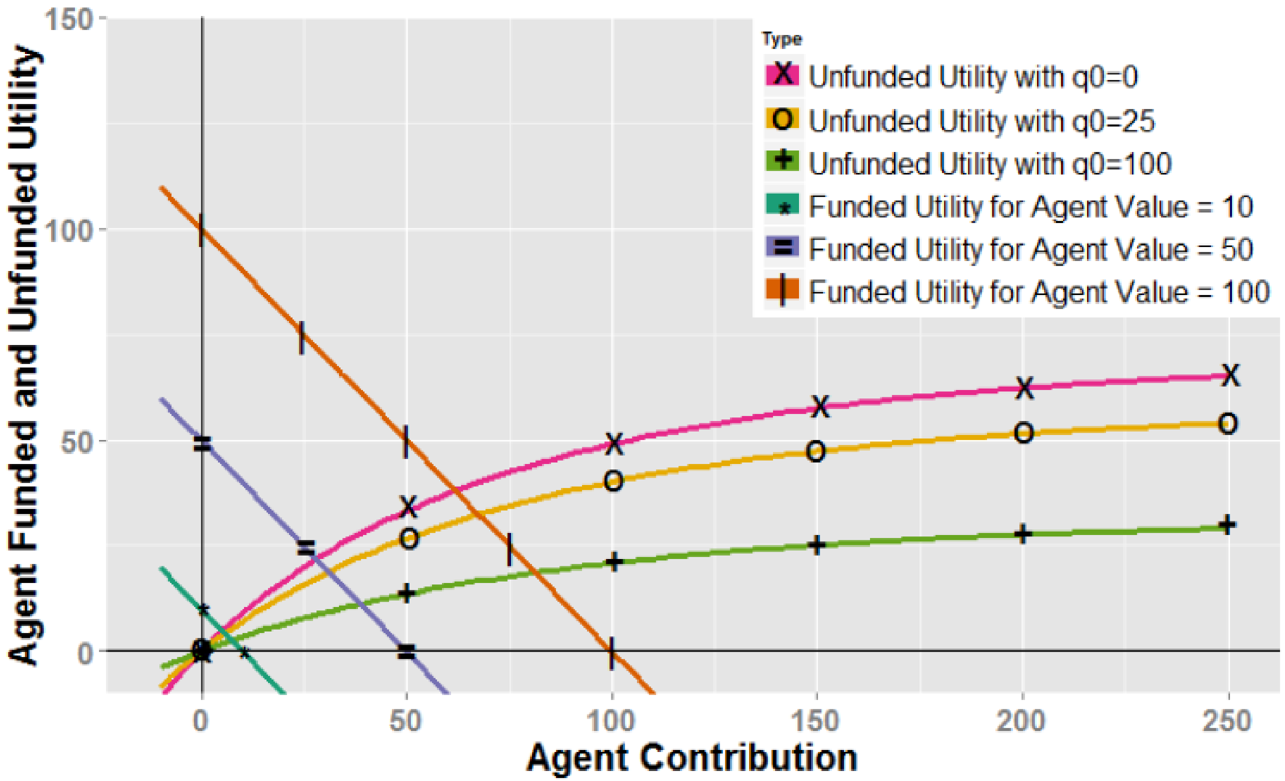
Logarithmic Market Scoring Rule based PPS

Funded Utility

- Monotonically decreases with contribution

Unfunded Utility

- Monotonically increases with contribution
- Monotonically decreases with outstanding securities (time)



$$u_i(\psi;\theta_i) \quad = \quad \mathcal{I}_{\chi \geq h^0} \times (\theta_i - x_i) + \mathcal{I}_{\chi < h^0} \times (r_i^{t_i} - x_i)$$

Key Result: Project gets funded at equilibrium.

If

- Net value of the project > Cost of the project
- $b \in (0, (\vartheta - h^0)/ \ln 2)$

Then

- Project is funded at Equilibrium
- Equilibrium is subgame perfect (sequential game)
- Each agent contributes in proportion to his true value
- Each agent contributes as soon as he arrives
- Agents have an incentive to contribute early.

$$x_i^* \leq C_0(\theta_i + q^{a_i}) - C_0(q^{a_i}) = b \ln \left(\frac{1 + \exp\left(\frac{\theta_i + q^{a_i}}{b}\right)}{1 + \exp(\frac{q^{a_i}}{b})} \right)$$

Necessary Conditions on Cost Functions

1. Path Independence — $\texttt{Cost}(\mathbf{r}|\mathbf{q}) = C(\mathbf{q}+\mathbf{r}) - C(\mathbf{q})$
2. Continuous & Differentiable — $p_{\omega_j} = \partial C(\mathbf{q})/\partial(q_{\omega_j}) \geq 0 \quad \forall \omega_j \in \Omega$
3. Information Incorporation — $C(\mathbf{q}+2\mathbf{r}) - C(\mathbf{q}+\mathbf{r}) \geq C(\mathbf{q}+\mathbf{r}) - C(\mathbf{q})$.
4. No Arbitrage — $\exists \omega_j \in \Omega$ such that $C(\mathbf{q}+\mathbf{r}) - C(\mathbf{q}) > \mathbf{r} \cdot \pi_{\omega_j}$
5. ~~Expressiveness~~ — $\forall \mathbf{p} \in \Delta_{|\Omega|}, \exists \mathbf{q} \in \mathbb{R}^{|\Omega|}$ s.t. $\nabla C(\mathbf{q}) = \mathbb{E}_{\omega \sim \mathbf{p}}[\pi(\omega)]$.
6. Bounded Loss — $\sup_{\mathbf{q}}[\max_{\omega_j}(q_{\omega_j}) - C(\mathbf{q})] < \infty$.
7. <u>Sufficient Liquidity</u> — $\forall q^{t_i}, \forall x_i < h^0, \quad \frac{\partial}{\partial x_i}(r_i^{t_i} - x_i) > 0 \Rightarrow \frac{\partial r_i^{t_i}}{\partial x_i} > 1.$

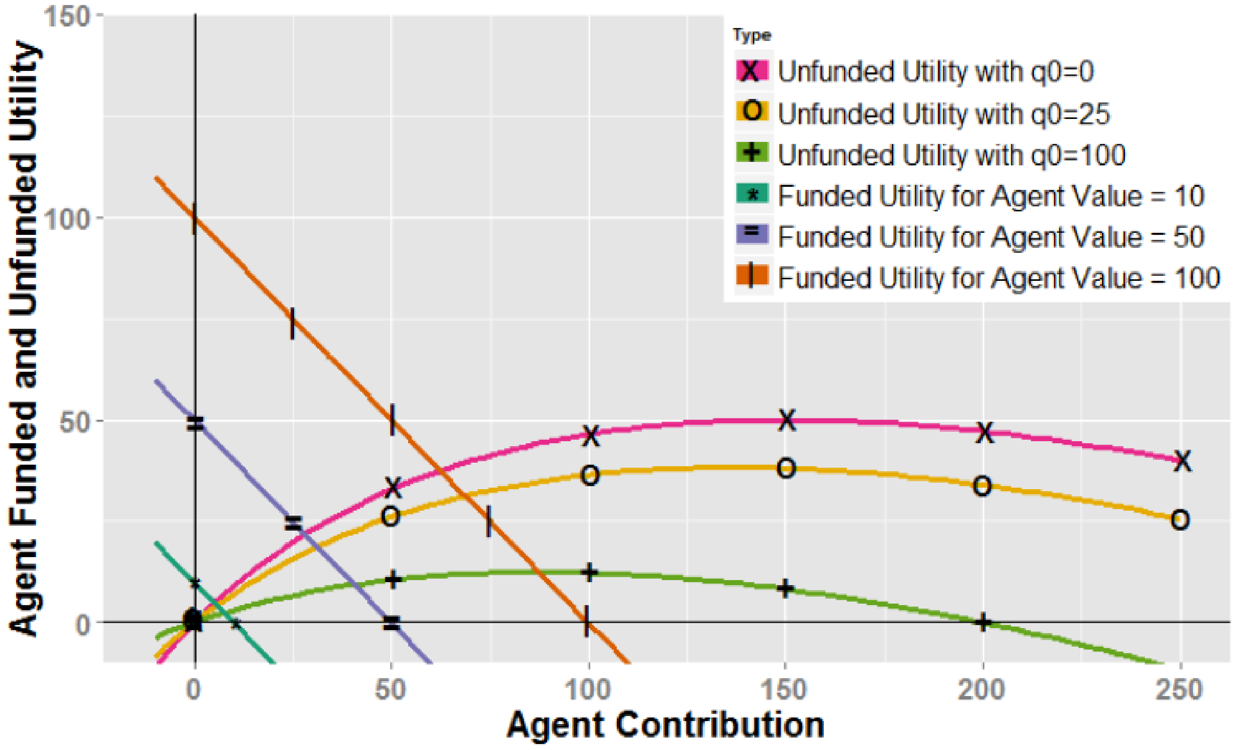
Quadratic Scoring Rule based PPS

Funded Utility

- Monotonically decreases with contribution

Unfunded Utility

- Monotonically increases with contribution
- Monotonically decreases with outstanding securities (time) : ***with appropriate parameterization.***



$$u_i(\psi;\theta_i) \;=\; \mathcal{I}_{\chi \geq h^0} \times (\theta_i - x_i) + \mathcal{I}_{\chi < h^0} \times (r_i^{t_i} - x_i)$$

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