A simple equivalent circuit analysis of rectangular folded-waveguide slow-wave structure

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1. Introduction

Rectangular folded-waveguide (RFW) slow-wave structures (SWS) became popular for millimeter-wave traveling-wave tubes (TWTs) for its potential for reasonable bandwidth along with its capability of high power operation and ease of fabrication [1,2]. However, analytical approaches available for the computation of slow-wave characteristics of a rectangular folded-waveguide slow-wave structure are limited: Hyun et al. [3], Carter [4] and Seong et al. [5] proposed approximate parametric analytical approaches that ignored the effects of the beam-hole, and thus have limited capabilities in analyzing the exact configuration of the structure. The present authors very recently proposed a rigorous and exact analysis of rectangular folded-waveguide SWS using conformal transformations [6]. Though rigorous, the analytical model based on conformal transformations is somewhat difficult to implement because of the complexity involved in it.

In this paper we propose a complete equivalent circuit model for a rectangular folded-waveguide slow-wave structure (RFW-SWS) for the analysis of the slow-wave characteristics including the effects of the presence of the beam-hole in the structure. The analysis was carried out following a recently proposed equivalent circuit approach by the authors used for analyzing a serpentine folded-waveguide slow-wave structure [7]. The analysis has been benchmarked against measurement and 3D electromagnetic modeling using MAFIA with close agreement for two typical RFW-SWSs, one operating at Ka-band and the other operating at Q-band. Subsequently, the analysis has been used for demonstrating the effect of the variation of the radius of the beam-hole on the RF interaction efficiency of the device.

2. Analysis

The analytical model considers each period of the RFW-SWS (Fig. 1) to be consisting of two parts: a straight rectangular waveguide section accommodating the beam-holes as apertures on the broad-wall, and rectangular E-plane waveguide bends. The structure has been considered to be loss free and the equivalent circuits of straight waveguide section, waveguide bends and beam-holes (Fig. 1) are arrived at following existing approaches [7–10], and are subsequently cascaded following Curnow’s approach [11] to arrive at the dispersion relation of the structure.

The equivalent circuit parameters for the effective waveguide length per periodicity (Fig. 1) are expressed following Carter and Liu [8] as

\[
L_W = \frac{\mu_0 L_{\text{eff}}}{\pi^2 \left(1 + \frac{L_{\text{eff}}}{b}\right)}
\]

\[
C_W = \varepsilon_0 L_{\text{eff}} \left(1 + \frac{L_{\text{eff}}}{b}\right)
\]

(1)

Here, \(\varepsilon_0\) and \(\mu_0\) are permittivity and permeability of free space, respectively, \(L_{\text{eff}} = 2h + p - b/2\) is the effective length of the waveguide per period, \(2h\) is the straight waveguide length, \(b\) is...
narrow dimension of the waveguide and p is the periodicity of the RFW-SWS.

The discontinuity due to the E-plane corner-bend also introduces additional inductance and capacitance that are expressed following Marcuvitz [9] as

\[
L_R = Y_0 \frac{\omega a}{2\pi b c_0} \left( 1 - 0.114 \left( \frac{2b}{\omega a} \right)^2 \right)
\]

\[
C_R = Y_0 \frac{4b}{\omega a c_0} \left( 0.878 + 0.498 \left( \frac{2b}{\omega a} \right)^2 \right)
\]  

(2)

Here, \( \omega \) is the radian frequency, \( Y_0 (= \lambda_0/(120\pi\lambda_g)) \) is the wave-admittance of the rectangular waveguide, \( \lambda_0 \) is the free space wavelength, and \( \lambda_g \) is the guided-wavelength of the rectangular waveguide.

The equivalent circuit parameters of the beam-hole are expressed as \[9,10\]

\[
L_H = \frac{1280\pi^3 r_L^5}{\omega a o a b}
\]

\[
C_H = \frac{r_L^3}{45\lambda_0 o a b}
\]  

(4)

Following Curnow [11], one can now cascade the equivalent circuits of the waveguide per period and the beam-holes using the equivalent circuit parameters obtained in (1)–(4) and express the dispersion relation for the cascaded network as

\[
\beta = \frac{1}{p} \left[ 2\pi \cos^{-1} \left( 1 - \frac{1}{k} \left( 1 - \alpha^2 L_W C_W \right) L_H \right) \right]
\]

with

\[
F_H = 2 + \alpha_k \alpha_e \alpha^2 L_H C_H
\]

\[
q_k = \frac{L_H}{kL_W}
\]

and

\[
k = 0.3 \exp \left( 20r_c/a \right)
\]  

(5)

For the case of the structure without beam hole, \( q_k \) vanishes and the factor incorporating the effects of the beam-holes (\( F_H \)) assumes the value \( F_H = 2 \). These formulas work within the parametric regime of \( 0 \leq r_c/a \leq 10\% \). Subsequently, using the dispersion relation (5), one can express the on-axis forward space-harmonic interaction impedance (\( K_c \)) of the slow-wave structure following Sumathy et al. [7] and Booske et al. [12] as

\[
K_c = 120\pi \left( \frac{b}{a} \right) \left( \frac{\omega}{\lambda_0} \right) \left( \frac{1}{pb} \right)^2 \left( \frac{\sin pb}{pb^2} \right)^2
\]  

(6)

Eqs. (5) and (6) can be now used for computing the dispersion and interaction impedance characteristics, respectively, with the knowledge of the dimensions of the structure under consideration.

Having the interaction impedance characteristics available, we now consider a figure of merit (Pierce’s normalized circuit gain CN) that will demonstrate the device performance, expressed as [1]

\[
CN = \left( \frac{K_c I_0}{4V_0} \right) \left( \frac{\omega L_{\text{ext}}}{2\pi u_0} \right)
\]

Here, \( C (= (K_c I_0/(4V_0))^{1/3}) \) is the Pierce’s gain parameter [1], \( I_0 \) is the beam current, \( V_0 \) is the beam voltage, \( L_{\text{ext}} \) is the interaction length and \( u_0 \) is the electronic velocity.

3. Results and discussion

For the purpose of benchmarking and numerical appreciation of the analysis, we considered two typical millimeter-wave RFW-SWSs operating in forward space-harmonic mode: one
operating in Ka-band and the other in Q-band for which the dimensional details and slow-wave characteristics are published in the literature [6]. The dispersion and interaction impedance characteristics of these structures were evaluated using present approach and compared to those from 3D electromagnetic modeling in MAFIA for both the structures and against measurement for the Ka-band structure alone. The benchmarking results are presented in Figs. 2 and 3 that show the efficacy of the present approach.

An increase in the beam-hole radius causes a decrease in both phase velocity and interaction impedance which is demonstrated for a typical Ka-band structure in Figs. 4 and 5, respectively. The normalized circuit gain \(\frac{CN}{\Omega}\) has been computed for the Ka-band structure for the beam voltage of 12 kV and beam current of 120 mA. The value of \(CN\) was found to be reduced significantly with the increase in the beam-hole radius (Fig. 6). Thus, one would need to design the electron beam diameter and the corresponding beam-hole diameter suitably for achieving desired RF efficiency of the structure.

4. Conclusion

A complete equivalent circuit model has been developed for the analysis of slow-wave characteristics of a rectangular folded-waveguide slow-wave structure and thoroughly benchmarked against measurement and 3D electromagnetic modeling. Harmful effects of increasing the beam-hole radius are also demonstrated. With the simplicity and the demonstrated reasonable accuracy, the approach is expected to be of ample use for the travelling-wave tube community.

Acknowledgments

The authors are thankful to Prof. B.N. Basu, Dr. Lalit Kumar and Dr. K.S. Bhat for the many valuable suggestions to improve the quality of the manuscript.

References

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