Throughput Analysis of Best-m Feedback in OFDM Systems with Uniformly Correlated Subchannels

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***Abstract*—Practical orthogonal frequency division multiplexing (OFDM) systems, such as Long Term Evolution (LTE), exploit multi-user diversity using very limited feedback. The best-m feedback scheme is one such limited feedback scheme, in which users report only the gains of their m best subchannels (SCs) and their indices. While the scheme has been extensively studied and adopted in standards such as LTE, an analysis of its throughput for the practically important case in which the SCs are correlated has received less attention. We derive new closed-form expressions for the throughput when the SC gains of a user are uniformly correlated. We analyze the performance of the greedy but unfair frequency-domain scheduler and the fair round-robin scheduler for the general case in which the users see statistically non-identical SCs. An asymptotic analysis is then developed to gain further insights. The analysis and extensive numerical results bring out how correlation reduces throughput.**

***Index Terms*—Orthogonal frequency division multiplexing (OFDM), Correlation, Channel quality feedback, Best-m, Frequency-domain scheduling, Adaptive modulation and coding.**

I. Introduction

Next generation wireless systems like Long Term Evolution (LTE) achieve high data rates and support a large number of users. In these systems, orthogonal frequency division multiplexing (OFDM) is the preferred downlink radio access scheme. In OFDM, the available bandwidth is divided into numerous orthogonal subcarriers. A group of contiguous subcarriers, in turn, forms a subchannel (SC). For example, in LTE, a group of twelve subcarriers forms a physical resource block (PRB), which is the smallest unit of allocation for transmission and has a bandwidth of 180 kHz.

These systems achieve high data rates by employing frequency-domain scheduling and rate adaptation [1]. In frequency-domain scheduling, an SC is opportunistically allocated by the base station (BS) to a user, based on the SC gains of all the users. Further, the BS chooses a suitable modulation and coding scheme (MCS) for transmission on that SC. For example, in LTE there are 32 possible MCSs [2].

In order for the BS to schedule in this way, it needs to have access to the downlink SC gains of all the users for all the SCs. However, this information is not available a priori to the BS in frequency division duplexing (FDD) systems, since the downlink and uplink channels are not reciprocal. A similar problem arises even in time division duplexing (TDD) systems when the uplink and downlink interferences are not reciprocal or due to calibration errors [3]. Hence, each user ideally needs to feed back to the BS all of its SC gains on the uplink. Such excessive feedback clogs the uplink and is spectrally inefficient.

Many feedback schemes have been proposed in the literature in order to reduce the amount of feedback. In [4], a thresholding scheme is proposed, in which users report only those SCs whose gains exceed a chosen threshold. The BS, then, assigns an SC, among all users that reported it, to the user with the maximum SC gain, which is akin to greedy scheduling. Instead of reporting the SC gain, a one bit feedback, which indicates if the SC gain exceeds a threshold or not, is proposed in [5], [6]. In [7], the SCs are grouped into disjoint clusters, and a user reports a cluster only if all the SC gains in the cluster exceed a threshold.

A different feedback scheme called the best-m scheme is proposed in [8]. In it, each user feeds back the m highest SC gains and the corresponding indices of the SCs. The BS employs a greedy scheduler in that it assigns an SC to the user that reported the highest SC gain. The best-m scheme is analyzed in [9] for a round-robin (RR) scheduler. Its performance with quantized signal-to-noise-ratio (SNR) feedback and a greedy scheduler is analyzed in [10]. In [11], the throughput of the best-m and threshold schemes, and a hybrid scheme are analyzed. A practical standard such as LTE uses a combination of these techniques to reduce the feedback. In it, a 4-bit channel quality information (CQI) value is used to indicate the MCS that can be supported. The feedback is further reduced by reporting the average CQI value over a cluster of PRBs [2].

All the above papers assume that the SCs are identical and independently distributed (i.i.d.). However, in practice, the SCs are highly correlated. For example, for the typical urban (TU) and rural area (RA) channel models [12], the correlation coefficient between two SCs that are 180 kHz apart is 0.90 and 0.95, respectively. An analysis of the performance of the best-m scheme with correlated SCs is not available in the literature, and is the problem that we address in this paper.

Focus and contributions: In this paper, we derive closed-form expressions for the throughput of the best-m scheme when the SC gains are correlated. We focus on the best-m scheme because it is effective and has been adopted in contemporary OFDM systems such as LTE. In order to make the analysis tractable and gain insights, we focus on the

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This research has been partially supported by a research grant from the Indo-UK Advanced Technology Consortium (IUATC).

978-1-4799-2361-8/14/$31.00

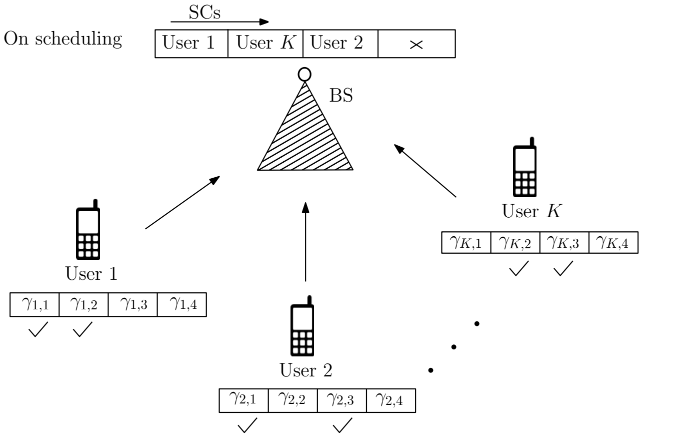

Fig. 1. Illustration of best-m feedback by users and scheduling at BS ($N = 4$ and $m = 2$). Tick mark indicates SC is fed back.

uniformly correlated SC model. The analysis for a general correlation model is extremely involved and intractable since it requires dealing with order statistics of correlated random variables (RVs). For example, this can be seen in [13], which derives an expression for the joint cumulative distribution function (CDF) of correlated Nakagami RVs. It contains as many integrations as the number of RVs being ordered. The uniform correlation model also gives the worst case scenario for the correlation [14]. For the above reasons, it has also been used in [14]–[16].

To gain intuition about the problem, we first analyze the case where the SC gains of different users are i.i.d. The analysis is then extended to the general non-i.i.d. case, in which the SCs of different users are not statistically identical. This arises when the users are at different distances from the BS. We analyze the performance of the greedy and RR schedulers. The performance analysis of the proportional-fair scheduler is beyond the scope of this paper. We focus on discrete rate adaptation, since this is invariably used in practice. In order to gain further insights, we then present considerably simpler expressions for the asymptotic scenario of a large number of users and low correlation. These effectively bring out how the correlation reduces throughput. Expressions for the outage probability of an SC, which is the probability that no user reports it to the BS, are also derived.

The paper is organized as follows. The system model is discussed in Sec. II. The throughput analysis is presented in Sec. III. The simulation results are given in Sec. IV, which are followed by conclusions in Sec. V.

We shall use the following notation. The probability of an event is denoted by $P[.]$. The CDF of an RV is denoted by $F(.)$ and the expectation by $E[.]$. The multinomial coefficient $\binom{n}{l_1,\ldots,l_p}$ is equal to $\frac{n!}{l_1!\ldots l_p!}$ and $L(x,k)$ is the incomplete Gamma function, which is given by $\frac{1}{(k-1)!}\int_0^x \exp(-t)t^{k-1}\,dt$ [17].

II. System model

We consider the downlink of a single cell OFDM system with K users. Each user is equipped with one antenna. The system bandwidth is divided into N flat-fading SCs. Let $H_{k,n}$ denote the SC gain from the BS to user k for SC n. The SC gains $H_{k,n}$, for $n = 1, 2, \ldots, N$, of user k are identically distributed circular symmetric complex normal RVs with zero mean and variance σ_k^2, which models Rayleigh fading [1]. The SC power gains $\gamma_{k,n} = |H_{k,n}|^2$ are exponential RVs each with mean σ_k^2. The SC gains of different users are independent of each other and, hence, so are the SC power gains.

The SC gains for a user are identically distributed [1] but they are not independent. We assume that the SCs are correlated with each other by the same coefficient of correlation ρ:

$$\frac{E[H_{k,n}H_{k,m}^\star]}{\sigma_k^2} = \rho, \quad \text{for } k = 1, 2, \ldots, K, \quad n \neq m, 0 \leq n, m \leq 1, \ldots, N. \quad (1)$$

where * denotes the complex conjugate.

We assume that each user knows its SC gains perfectly, and that there are no errors during feedback. The coherence time of the channel is assumed to be greater than the time required by the BS to acquire CQI, schedule users, and transmit data to the users [5], [6], [8], [9]. This is a valid assumption for LTE for pedestrian and vehicular speeds of up to 30 km/h, since the feedback delays are of the order of 10 milliseconds.

Each user orders its SC power gains. For a user k, the ordered SC power gains are denoted as

$$\gamma_{k,1:N} \leq \gamma_{k,2:N} \leq \cdots \leq \gamma_{k,N:N},$$

where $r : N$ denotes the index of the SC with the r^{th} highest power gain [18]. The user, then, feeds back the m highest SC power gains, $\gamma_{k,N-m+1:N}, \ldots, \gamma_{k,N:N}$ along with their indices to the BS. The BS acquires this information from all the users and schedules as follows. Among the users that report an SC, the one with the highest SC power gain is chosen to transmit on that SC. Let the set of users that report SC n be $\mathcal{S}$. The user scheduled on this SC is given by

$$i_n^\star = \arg\max_{i\in\mathcal{S}} \gamma_{i,n}. \quad (2)$$

If an SC is not reported by any user, then $i_n^\star = 0$, $\gamma_{i_n^\star,n} = 0$. Further, the BS transmits no data to any user on it since it has no channel state information.

The BS assigns one among M rates $0 = R_1 < R_2 < \cdots < R_M$ to the user scheduled on an SC as follows. The range of SC power gains is divided by means of $M + 1$ thresholds $0 = \Gamma_1 < \Gamma_2 < \cdots < \Gamma_{M+1} = \infty$ into M disjoint intervals. If the SC power gain lies in the interval $[\Gamma_r, \Gamma_{r+1})$, an MCS corresponding to the rate R_r is assigned to the selected user.

III. Throughput Analysis

We analyze the throughput of the best-m scheme for the uniform correlation model. Since the SCs are statistically identical, the throughput is the same for all the SCs. We, therefore, focus on a single SC n. The subscript n is dropped in $i_n^\star$, which is hereafter referred to by $i^\star$. The throughput $\bar{R}$ for SC n in the downlink for both schedulers is

$$\begin{aligned}\bar{R} &= \sum_{r=1}^{M} R_r P[\Gamma_r \leq \gamma_{i^\star,n} < \Gamma_{r+1}], \\ &= \sum_{r=1}^{M} R_r \left(P[\gamma_{i^\star,n} < \Gamma_{r+1}] - P[\gamma_{i^\star,n} < \Gamma_r] \right). \end{aligned} \tag{3}$$

We first derive expressions for the CDF, $P[\gamma_{i^\star,n} < x]$, for the relatively simpler scenario with i.i.d. users, in which $\sigma_k^2 = \sigma^2$, $k = 1, 2, \ldots, K$. Thereafter, the more general case of non-i.i.d. users is analyzed.

A. I.I.D. users

We shall denote the CDF of $\gamma_{k,r:N}$ as $F_{k,r:N}(x)$, for $1 \leq k \leq K$.

Result 1: The CDF of the SC power gain of the selected user $i^\star$ is given by

$$\begin{aligned} P[\gamma_{i^\star,n} < x] &= \left(1 - \frac{m}{N}\right)^K + \sum_{a=1}^{K} \binom{K}{a} \left(1 - \frac{m}{N}\right)^{(K-a)} \\ &\times \left(\frac{1}{N}\right)^a \left(\sum_{p=N-m+1}^{N} (-1)^{p+N-m+1} \binom{p-2}{N-m-1} \right. \\ &\left. \times \binom{N}{p} F_{1,p:p}(x) \right)^a, \end{aligned} \tag{4}$$

where

$$\begin{aligned} F_{1,p:p}(x) &= \frac{1-\rho}{1+(p-1)\rho} \sum_{n=0}^{\infty} \left(\frac{\rho}{1+(p-1)\rho} \right)^n \\ &\times \sum_{\substack{l_1,\ldots,l_p \\ 0 \leq l_1,\ldots,l_p \leq n \\ l_1+\cdots+l_p = n}} \binom{n}{l_1,\ldots,l_p} \prod_{i=1}^{p} L\left(\frac{x}{\sigma^2(1-\rho)}, l_i + 1 \right). \end{aligned} \tag{5}$$

Proof: The proof is given in Appendix A. ∎

Substituting (4) in (3) yields the final throughput expression.

For the RR scheduler, the throughput is obtained by replacing K with one because the users are statistically identical [19]. The CDF of the selected user simplifies to

$$\begin{aligned} P[\gamma_{i^\star,n} < x] &= \left(1 - \frac{m}{N}\right)^K + \frac{1}{N} \sum_{p=N-m+1}^{N} (-1)^{p+N-m+1} \\ &\times \binom{p-2}{N-m-1} \binom{N}{p} F_{1,p:p}(x). \end{aligned} \tag{6}$$

1) Outage probability: The outage probability for an SC n, $p_{out}(n)$, is equal to the probability that no user feeds back that SC to the BS. It is given by

$$\begin{aligned} p_{out}(n) &= P[\text{SC } n \text{ is at most } (m-1)^{th} \text{ best for } K \text{ users}], \\ &= \left(1 - \frac{m}{N}\right)^K. \end{aligned} \tag{7}$$

The outage probability is a useful performance measure as it has been used to determine m in the literature [11]. Notice that the outage probability does not depend on the correlation coefficient. Also, as m increases, the outage probability decreases.

2) Asymptotic analysis: While the above analysis is exact, the final expressions are quite involved. Therefore, we now analyze the asymptotic regime in which the number of users K tends to ∞. This provides several insights about the effect of the correlation coefficient ρ on the throughput.

Result 2: For large K, the difference Δ between the maximum rate achievable and the throughput scales exponentially in K, and is given by

$$\Delta = R_M - \bar{R} = (R_M - R_{M-1}) \left(1 - \frac{m}{N} + \frac{m\varphi}{N}\right)^K, \tag{8}$$

where

$$\varphi = P[\gamma_{1,n} \leq \Gamma_M | \text{User 1 reports SC } n]. \tag{9}$$

Proof: The proof is omitted due to space constraints, and is available in [20]. ∎

In order to bring out the dependence of the throughput on ρ, a simplified expression for the conditional CDF φ in the form of an upper bound is presented in the following result.

Result 3: For small ρ, φ is upper bounded by

$$\begin{aligned} \varphi \leq & \frac{1}{m} \binom{N}{N-m+1} \left(1 - \exp\left(-\frac{\Gamma_M}{\sigma^2}\right)\right)^{N-m+1} \\ & \times \left(1 + \frac{\rho\Gamma_M}{\sigma^2} \frac{\exp\left(-\frac{\Gamma_M}{\sigma^2}\right)}{1 - \exp\left(-\frac{\Gamma_M}{\sigma^2}\right)}\right)^{N-m} \\ \times & \left(1 - \frac{\rho\Gamma_M \exp\left(-\frac{\Gamma_M}{\sigma^2}\right) \left[(N-m)\left(1 - \frac{\rho\Gamma_M}{\sigma^2}\right) + \rho - 1\right]}{\sigma^2 \left(1 - \exp\left(-\frac{\Gamma_M}{\sigma^2}\right)\right)(1 + (N-m)\rho)}\right). \end{aligned} \tag{10}$$

Proof: The derivation uses the Taylor series expansion of the incomplete Gamma function for small ρ. It is available in [20]. ∎

Substituting (10) in (8) yields the desired upper bound on the difference Δ.

The first term $\frac{1}{m}\binom{N}{N-m+1}\left(1 - \exp\left(-\frac{\Gamma_M}{\sigma^2}\right)\right)^{N-m+1}$ in (10) corresponds to the bound for φ for i.i.d. SCs; we shall refer to it as φ_{iid}. The remaining two terms capture the effect of correlation on φ. The second term $\left(1 + \frac{\rho\Gamma_M}{\sigma^2} \frac{\exp\left(-\frac{\Gamma_M}{\sigma^2}\right)}{1-\exp\left(-\frac{\Gamma_M}{\sigma^2}\right)}\right)^{N-m}$ is greater than one. It increases exponentially with N and makes the product of the second and third terms exceed one for larger N. Hence, the bound of φ is greater than that for the i.i.d. SCs. Consequently, the difference Δ between the maximum rate and the throughput decreases at a much slower rate in the presence of correlation. For $(N-m)\rho \ll 1$, the bound on φ further simplifies to

$$\varphi \leq \varphi_{\text{iid}} (1 + \rho a), \tag{11}$$

where $a = \frac{(N-m)\Gamma_M}{\sigma^2} \frac{\exp\left(-\frac{\Gamma_M}{\sigma^2}\right)}{1-\exp\left(-\frac{\Gamma_M}{\sigma^2}\right)}$.

B. Non-I.I.D. users

When the users are not statistically identical, the expression for the CDF $P[\gamma_{i^\star,j} < x]$ is more involved. We now have to track which subset of users reported the SC and not just the number of users that did so. For this, let A_l^a denote the l^{th} subset of K users with a elements. There are $\binom{K}{a}$ such subsets.

Result 4: The CDF of the SC power gain of the selected user $i^\star$ is given by

$$P[\gamma_{i^\star,n} < x] = \left(1 - \frac{m}{N}\right)^K + \sum_{a=1}^{K} \left(\frac{1}{N}\right)^a \left(1 - \frac{m}{N}\right)^{(K-a)} \times \sum_{l=1}^{\binom{K}{a}} \left(\prod_{u \in A_l^a} \sum_{p=N-m+1}^{N} (-1)^{p+N-m+1} \binom{p-2}{N-m-1} \times \binom{N}{p} F_{u,p:p}(x) \right). \quad (12)$$

Proof: The proof is given in Appendix B. ■

Substituting this result in (3) yields the expression for the throughput.

For the RR scheduler, the CDF of the SC power gain of the selected user can be shown to be equal to

$$P[\gamma_{i^\star,n} < x] = \left(1 - \frac{m}{N}\right)^K + \frac{1}{NK} \sum_{p=N-m+1}^{N} (-1)^{p+N-m+1} \times \binom{p-2}{N-m-1} \binom{N}{p} \sum_{l=1}^{K} F_{l,p:p}(x). \quad (13)$$

The outage probability for non-i.i.d. users turns out to be the same as that for i.i.d. users (cf. (7)) because the SCs of a user are still statistically identical and the users are independent.

IV. Simulation results

We now present Monte Carlo simulation results that are averaged over 10^4 samples to verify the analysis. The $M = 5$ rates are $0, 1, 2, 4$, and 6 bits/symbol. The thresholds for discrete rate adaptation are calculated using the formula [21]

$$R_r = \log_2 (1 + \zeta \Gamma_r), \quad (14)$$

where $\zeta = 0.398$ accounts for the coding loss of a practical code [21]. The number of users K is 10 and number of SCs N is 10. The simulation results are shown by markers and the analytical results by lines. Unless mentioned otherwise the greedy scheduler is assumed. The CDF $F_{1,j:j}(x)$ in (22) is evaluated by truncating the infinite series in (5). The number of terms required for summation increases with ρ for a given accuracy because of the presence of the $1 - \rho$ term in the denominator of the first argument of the incomplete Gamma function in (5).

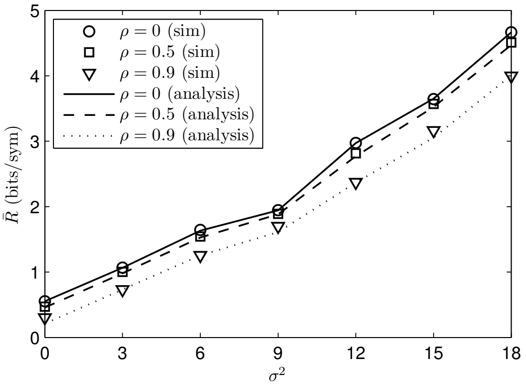

Fig. 2. I.I.D. users: Throughput as a function of mean SC power gain (σ^2) for different ρ ($K = 10, N = 10$, and $m = 2$).

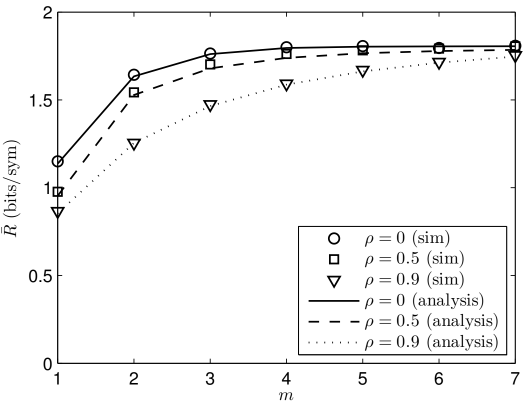

Fig. 3. I.I.D. users: Throughput as a function of number of SCs fed back (m) for different ρ ($K = 10, N = 10$, and $\sigma^2 = 6$ dB).

A. I.I.D. users

Figure 2 plots the throughput as a function of the mean SC power gain σ^2 for the best-m scheme with $m = 2$. The throughput increases with σ^2 for all values of ρ. We see that there is no significant change in the throughput when ρ increases from 0 to 0.5. However, there is a perceptible decrease in the throughput once ρ exceeds 0.5. For example, for $\sigma^2 = 6$ dB, there is a 6% decrease in the throughput when ρ is increased from 0 to 0.5, but a 23% decrease when ρ is increased from 0 to 0.9. The marginal mismatch between the analysis and simulation results at higher values of ρ arises due to the aforementioned truncation of the infinite series in (5).

Figure 3 plots the throughput as a function of the number of SCs fed back, m, for $\sigma^2 = 6$ dB. The throughput increases as m increases. This is because as the feedback increases the probability of an SC being in outage decreases. Most importantly, the throughput is more sensitive to correlation for lower values of m. In fact, for $m = N$, in which all SC gains are fed back, the correlation does not affect the throughput.

Figure 4 plots the expression for $\frac{\log_2\left(\frac{\Delta}{R_M - R_{M-1}}\right)}{K}$ in (8) as a function of ρ for different σ^2. We observe that

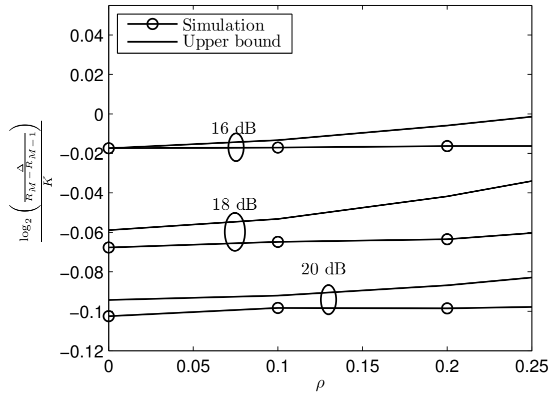

Fig. 4. Asymptotic behavior: $\frac{\log_2\left(\frac{\Delta}{R_M - R_{M-1}}\right)}{K}$ as a function of ρ for different σ^2 ($N = 10$ and $m = 1$).

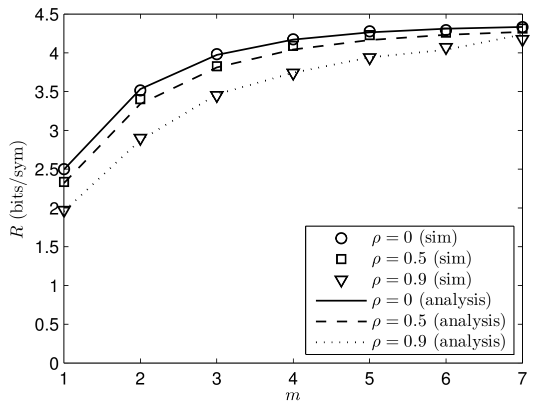

Fig. 5. Non-i.i.d. users: Throughput as a function of number of SCs fed back (m) for different ρ ($K = 10, N = 10$, and $\sigma^2 = 6$ dB).

$\frac{\log_2\left(\frac{\Delta}{R_M - R_{M-1}}\right)}{K}$ is negative and it decreases as σ^2 increases. This means that the throughput approaches its maximum value faster. Also, as ρ increases, $\frac{\log_2\left(\frac{\Delta}{R_M - R_{M-1}}\right)}{K}$ increases, which implies that the throughput approaches its maximum value slower. This brings out the degradation in throughput due to correlation. We see that as ρ increases, the bound becomes loose. This is because of the truncation of the Taylor series expansion that is used in the derivation of the bound [20].

B. Non-I.I.D. users

In order to model non-i.i.d. users, we set $\sigma_k^2 = \sigma^2 \alpha^{k-1}$, where $\alpha > 1$ and $1 \leq k \leq K$. The larger the α, the more asymmetric are the users. For $\alpha = 1.4$ and $\sigma^2 = 6$ dB, the throughput versus m is shown in Figure 5. For $m = 1$, the decrease in throughput for $\rho = 0.5$ is 7% as compared to the uncorrelated case. However, for $\rho = 0.9$, the decrease is 21%. The reduction in the throughput with correlation for non-i.i.d. users is marginally lower than that for i.i.d. users.

V. Conclusion

We analyzed the downlink throughput for an OFDM system that uses the best-m feedback scheme to acquire channel state information for facilitating frequency-domain scheduling and rate adaptation. We derived expressions for the throughput when the SC gains of a user are uniformly correlated, and those of different users are either statistically identical or non-identical. The analysis is applicable to the greedy, but unfair, scheduler and the fair round-robin scheduler, which together span a wide range of schedulers used in such systems. An insightful asymptotic analysis was also developed. We saw that the presence of correlation decreases the throughput, but does not affect the outage probability. Future work involves extending the analysis to other correlation models, and incorporating the effect of feedback delays.

Appendix

A. Proof of Result 1

We now derive the CDF of the SC power gain of SC n of the selected user $i^\star$. From the law of total probability, we have

$$P[\gamma_{i^\star,n} < x] = \sum_{a=0}^{K} P\left[\gamma_{i^\star,n} < x | a \text{ users report SC } n\right] \times P[a \text{ users report SC } n]. \quad (15)$$

A user reports an SC to the BS when the SC power gain is at least the m^{th} best among its N SCs. The probability that an SC is at least m^{th} best is equal to $\frac{m}{N}$ as the SCs are statistically identical. Since the users are i.i.d., the probability that exactly a of them report SC n is given by

$$P[a \text{ users report SC } n] = \binom{K}{a}\left(\frac{m}{N}\right)^a\left(1 - \frac{m}{N}\right)^{(K-a)}. \quad (16)$$

When $a = 0, \gamma_{i^\star} = 0$ by definition. We, therefore, get $P\left[\gamma_{i^\star,n} < x | 0 \text{ users report SC } n\right] = 1$.

We now obtain the conditional CDF of SC power gain of SC n of selected user $i^\star$ for $a \geq 1$. Since the users are i.i.d., the conditional CDF in (15) can be written as

$$\begin{aligned} &P\left[\gamma_{i^\star,n} < x | a \text{ users report SC } n\right] \\ &= P\left[\gamma_{1,n} < x, \ldots, \gamma_{a,n} < x | \text{Users } 1, \ldots, a \text{ report SC } n\right], \\ &= \left(P[\gamma_{1,n} < x | \text{User 1 reports SC } n]\right)^a. \quad (17) \end{aligned}$$

Using the law of total probability and

$$P[\gamma_{1,n} \text{ is } p^{\text{th}} \text{ best SC power gain}] = \frac{1}{N}, \quad \text{for } 1 \leq p \leq N,$$

we get

$$\begin{aligned} &P[\gamma_{1,n} < x | \text{User 1 reports SC } n] \\ &= \frac{N}{m} \sum_{p=N-m+1}^{N} P[\gamma_{1,j} < x,\ \gamma_{1,n} \text{ is } p^{\text{th}} \text{ best SC power gain}], \\ &= \frac{N}{m} \sum_{p=N-m+1}^{N} P[\gamma_{1,p:N} < x,\ \gamma_{1,n} \text{ is } p^{\text{th}} \text{ best SC power gain}]. \end{aligned}$$

Since the N SCs are statistically identical, it follows that

$$NP[\gamma_{1,p:N} < x,\ \gamma_{1,n} \text{ is } p^{\text{th}} \text{ best SC power gain}] = P[\gamma_{1,p:N} < x] = F_{1,p:N}(x). \quad (18)$$

Therefore,

$$P[\gamma_{1,n} < x|\text{User } 1 \text{ reports SC } n] = \frac{1}{m} \sum_{p=N-m+1}^{N} F_{1,p:N}(x). \quad (19)$$

From [18], the CDF $F_{1,p:N}(x)$ is given by

$$F_{1,p:N}(x) = \sum_{j=p}^{N} (-1)^{(j-p)} \binom{j-1}{p-1} \binom{N}{j} F_{1,j:j}(x). \quad (20)$$

Therefore,

$$P[\gamma_{1,n} < x|\text{User } 1 \text{ reports SC } n] = \frac{1}{m} \sum_{j=p}^{N} \sum_{p=N-m+1}^{N} (-1)^{(j-p)} \binom{j-1}{p-1} \binom{N}{j} F_{1,j:j}(x). \quad (21)$$

The inner summation $\sum_{p=N-m+1}^{j} (-1)^{(j-p)} \binom{j-1}{p-1}$ in (21) can be shown to simplify to $(-1)^{(j+N-m-1)} \binom{j-2}{N-m-1}$. Hence, we get

$$P[\gamma_{1,n} < x|\text{User } 1 \text{ reports SC } n] = \frac{1}{m} \sum_{j=N-m+1}^{N} (-1)^{(j+N-m-1)} \binom{j-2}{N-m-1} \times \binom{N}{j} F_{1,j:j}(x). \quad (22)$$

The CDF $F_{1,j:j}(x)$ is the CDF of the maximum of j uniformly correlated RVs. It is obtained using the expression for the joint probability distribution in [22, (91)]. Finally, substituting the above expression in (17) and then in (15), gives the result.

B. Proof of Result 4

Using the law of total probability, we have

$$P[\gamma_{i^\star,n} < x] = \sum_{a=0}^{K} \sum_{l=1}^{\binom{K}{a}} P[a \text{ users from } A_l^a \text{ report SC } n] \times P[\gamma_{i^\star,n} < x|a \text{ users from } A_l^a \text{ report SC } n], \quad (23)$$

where, as defined earlier, $i^\star$ denotes the user scheduled on the SC n.

The probability that the users from the subset A_l^a report SC n to the BS is given by

$$P[a \text{ users from } A_l^a \text{ report SC } n] = \left(\frac{m}{N}\right)^a \left(1 - \frac{m}{N}\right)^{(K-a)}.$$

We now have

$$P[\gamma_{i^\star,n} < x] = \left(1 - \frac{m}{N}\right)^K + \sum_{a=1}^{K} \sum_{l=1}^{\binom{K}{a}} \prod_{u \in A_l^a} \left(1 - \frac{m}{N}\right)^{(K-a)} \times \left(\frac{m}{N}\right)^a P[\gamma_{u,n} < x|\text{User } u \text{ from subset } A_l^a \text{ reported SC } n].$$

The CDF $P[\gamma_{u,n} < x|\text{User } u \text{ from subset } A_l^a \text{ reported SC } n]$ is the same as that in (22) except that $F_{1,j:j}(x)$ is replaced with $F_{u,j:j}(x)$. Substituting it in the above equation yields the desired result.

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