Performance Analysis of Subband-Level Channel Quality Indicator Feedback Scheme of LTE

Sushruth N. Donthi and Neelesh B. Mehta, *Senior Member, IEEE*

***Abstract*—Frequency-domain scheduling and rate adaptation enable next generation wireless cellular systems such as Long Term Evolution (LTE) to achieve significantly higher downlink throughput. LTE assigns subcarriers in chunks, called physical resource blocks (PRBs), to users to reduce control signaling overhead. To reduce the enormous feedback overhead, the channel quality indicator (CQI) report that is used to feed back channel state information is averaged over a subband, which, in turn, is a group of multiple PRBs. In this paper, we develop closed-form expressions for the throughput achieved by the subband-level CQI feedback mechanism of LTE. We show that the coarse frequency resolution of the CQI incurs a significant loss in throughput and limits the multi-user gains achievable by the system. We then show that the performance can be improved by means of an offset mechanism that effectively makes the users more conservative in reporting their CQI.**

I. Introduction

Orthogonal frequency division multiple access (OFDMA) is the downlink access technique of choice in fourth generation wireless cellular systems. It has been adopted in the third generation partnership project (3GPP) Long Term Evolution (LTE) standard and is also being standardized in the IEEE 802.16m Advanced WiMAX standard. It helps LTE deliver peak data rates as high as 100 Mbps on the downlink and increases spectral efficiencies by a factor of 3 to 4 compared to Release 6 High Speed Downlink Packet Access (HSDPA) [1].

A key feature of the LTE downlink is frequency-domain scheduling, which exploits multiuser diversity. The bandwidth is divided into several hundreds of subcarriers, and subcarriers are assigned to users with higher channels gains to improve system throughput. With proportional fair schedulers, fairness can also be ensured among users [2] [3, Sec. 6.7.1]. In order to perform frequency-domain scheduling, the base station (BS), which is also called the eNodeB in LTE parlance, ideally needs to know the instantaneous channel state information (CSI) for all subcarriers for all users (UEs) in the cell.

In the popular frequency division duplex (FDD) mode of operation in LTE, the uplink and downlink channels are not reciprocal. Therefore, this channel information needs to be fed back to the BS by each user. Such extensive subcarrier level feedback is practically infeasible as it consumes an extremely large amount of uplink resources. Hence, a balance needs to be struck between gains due to multiuser diversity and the amount of feedback required. Several partial feedback techniques have been studied in the literature. In [4], every user sends CSI for a subcarrier only if the subcarrier's channel gain is above a certain threshold. In [2], each user only indicates which n subcarriers have the best gains, and what their gains are. In [5], a one bit feedback scheme is shown to be asymptotically optimal in terms of capacity. Even more drastic feedback reduction techniques are resorted to in a practical system such as LTE, where CSI is quantized into a 4-bit value called channel quality indicator (CQI). This CQI is reported at quite a low frequency resolution, as we shall see later.

In this paper, we develop an analysis for the performance of the subband-level feedback mechanism used in LTE downlink. Such an analysis is relevant because most of the LTE-specific literature that deals with either scheduling algorithms or limited feedback is simulation based [6]–[10] given the analytical complexity of the problem. We develop expressions for the average throughput of the system for the general case in which the channels seen by different users are statistically non-identical. To build intuition, we also present simplified expressions for the special case in which the channels are statistically identical. In order to assess the impact of the coarse feedback used by LTE, we also analyze the performance of a hypothetical benchmark scheme in which each user sends its CQI report at a higher frequency resolution. We show that the subband-level CQI feedback of LTE incurs a significant performance loss due to its coarse frequency granularity. We then propose an offset technique to change the CQI reporting and show that it improves performance.

The paper is organized as follows. We first provide a brief overview of the LTE frame structure and its feedback reporting schemes in Sec. II. This motivates the system model developed in Sec. III, and its analysis in Sec. IV. Numerical results follow in Sec. V. We conclude in Sec. VI.

II. LTE Frame Structure and CQI Feedback

Downlink Frame Structure: In LTE, each downlink *frame* is of 10 ms duration, and consists of 10 subframes. Each subframe of duration 1 ms, which is called a transmission time interval (TTI), consists of two 0.5 ms slots. Each *slot*, in turn, consists of 7 OFDM symbols. In the frequency domain, the system bandwidth, B, is divided into several subcarriers, each of bandwidth of 15 kHz. For example, when $B = 10$ MHz, 600 subcarriers that are obtained using a 1024-point Discrete Fourier Transform are used for data and control information. A set of 12 consecutive subcarriers for a duration of one slot is called as *Physical Resource Block (PRB)*.

Feedback: The feedback information sent by the (UE) is called the Channel Quality Indicator (CQI). The 4-bit CQI

The authors are with the Dept. of Electrical Communication Eng. in the Indian Institute of Science (IISc), Bangalore, India.

Emails: sushruth@ece.iisc.ernet.in, nbmehta@ece.iisc.ernet.in

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value indicates an estimate of the modulation and coding scheme (MCS) that the UE can receive reliably. It is typically based on the measured received signal quality on the downlink. The list of 16 MCSs is tabulated in [11, Tbl. 7.2.3-1].

The BS controls how often and when the UE feeds back CQI. LTE allows for two types of feedback: *Aperiodic feedback* and *Periodic feedback*. In aperiodic feedback, the UE sends CQI only when it is asked to by the BS. On the other hand, in periodic feedback, the UE sends CQI periodically to the BS; the period between 2 consecutive CQI reports is communicated by the BS to the UE at the start of the CQI reporting process. In both these types of feedback, the finest possible frequency resolution for CQI reporting is a subband, which consists of q contiguous PRBs. Depending on the system bandwidth and the type of feedback, q ranges from 2 to 8.

The UE can report CQI at different frequency granularities in aperiodic CQI feedback. Specifically, in *Wideband feedback*, the UE reports one wideband CQI value for the whole system bandwidth. In *Subband-level feedback*, the UE reports CQI for each subband. In *UE selected subband feedback,* the UE reports the position of M preferred subbands that have the highest subband CQIs and a single CQI value for these subbands.[1] In periodic CQI feedback, only wideband and UE selected subband feedback are possible. Even in the latter, the CSI feedback is very limited; the subbands are further clustered into bandwidth parts, and the UE reports the CQI of only one subband from each bandwidth part.

PRB Allocation and Signaling: Based on the CQI reports from all the UEs, the BS decides which PRB to allocate to which UE. It then uses one of three *Resource Allocation Types* to signal to each user on the downlink control channel the specific PRBs that are allocated to it. The three allocation types trade off the control signaling overheads in slightly different ways. *A key point to note is that the PRB is the smallest block of frequency that can be allocated to a UE. However, this allocation is based on a coarser subband frequency resolution.*

III. System Model

In this paper, we will focus on the subband-level CQI reporting scheme. The wideband scheme is not of interest to us since it does not enable frequency-domain scheduling. An analysis of the UE selected subband feedback scheme is beyond the scope of this paper.

We consider a BS that serves K users. Let N denote the total number of PRBs available. The total number of subbands is $S = \lceil N/q \rceil$, where $\lceil . \rceil$ denotes the ceil function. The channel of each user is assumed to undergo block Rayleigh fading, and is assumed to be constant over a 1 ms subframe. For a given user, all the subcarriers within a PRB have the same channel gain, and the channel gains across different PRBs are assumed to be independent and identically distributed (i.i.d.). This is a valid and common assumption [12] in LTE because the 180 kHz bandwidth of a PRB is close to the coherence bandwidth of the channel for the typical delay spreads of 5 ms encountered in LTE [13, Sec. 5.3.2].

The received signal in a TTI for the k^{th} user in the n^{th} PRB is

$$y_{n,k} = h_{n,k} x_{n,k} + w_{n,k}, \quad \forall\, 1 \le k \le K,\ \forall\, 1 \le n \le N, \tag{1}$$

where $x_{n,k}$ is the signal transmitted by the BS, $h_{n,k}$ is the channel gain for the k^{th} user on the n^{th} PRB, and $w_{n,k}$ is circular symmetric complex Gaussian noise with unit variance. Since the channels are Rayleigh, $h_{n,k}$ is a circular symmetric Gaussian random variable whose variance depends upon the distance of the UE from the BS and shadowing. The signal to noise ratio (SNR) of the k^{th} user on the n^{th} PRB is then $\gamma_{n,k} = |h_{n,k}|^2$, where $\gamma_{n,k}$ is an exponential RV with mean σ_k^2.

A. Subband-Level CQI Feedback

To enable analytical tractability, we assume that the CQI value for a subband is the MCS that corresponds to the average SNR over its constituent PRBs. Alternate models such as effective exponential SNR [14] also exist. However, these are analytically intractable and are beyond the scope of this paper. In effect, the use of average SNR leads to an overestimation of achievable rate. We shall revisit this overestimation in the offset scheme considered in IV-C.

Let $\overline{\gamma}_{s,k}$ be the average SNR of the k^{th} user for the subband s. Since the PRBs are i.i.d., $\overline{\gamma}_{s,k}$ is a χ^2 RV with $2q$ degrees of freedom. Its probability density function (PDF) is

$$f_{\overline{\gamma}_{s,k}}(x) = \frac{q^q x^{q-1} e^{-\frac{qx}{\sigma_k^2}}}{(q-1)!(\sigma_k^2)^q}. \tag{2}$$

For a subband s of user k, a set of link adaptation thresholds, $T_0, \ldots, T_L$ determine how $\overline{\gamma}_{s,k}$ gets mapped into a CQI value, $\overline{C}_{s,k}$, which can take one of L possible values. Let r_i denote the rate in bits/symbol achieved by using the MCS corresponding to the i^{th} CQI value. These thresholds ensure that a target block error rate of 10% is met [13, Fig. 10.1], should the BS transmit over the entire subband. Formally, the CQI reporting rule is $\overline{C}_{s,k} = i$ if $\overline{\gamma}_{s,k} \in [T_{i-1}, T_i)$.

The BS assigns a PRB, n, to the UE, denoted by $k^*(n)$, that reported the highest CQI for its subband, $s(n)$. Let this highest CQI be denoted by $\overline{C}^*_{s(n)}$. Then,

$$\overline{C}^*_{s(n)} = \max_{1 \le k \le K} \overline{C}_{s(n),k} \text{ and } k^*(n) = \arg \max_{1 \le k \le K} \overline{C}_{s(n),k}. \tag{3}$$

In case multiple UEs report the same highest CQI, one of them is chosen randomly with uniform probability. The BS then transmits data to UE $k^*(n)$ on the n^{th} PRB using the MCS corresponding to $\overline{C}^*_{s(n)}$. Since the CQI value corresponds to the average SNR for the entire subband $s(n)$, the actual SNR for the n^{th} PRB may be below the lower threshold of the MCS

[1]The CQI overhead in both subband-level and UE selected subband feedback is further reduced in LTE as follows. The UE reports a wideband CQI value for the whole system bandwidth, and a 2-bit differential CQI value for each subband. We shall ignore the minor impact of this differential feedback in our analysis.

being used. In such a case, we say that an *outage* has occurred and the throughput is 0 in that TTI.[2]

B. Benchmarking Scheme: PRB-Level CQI Feedback

To understand the impact of subband averaging in reporting CQI, we also analyze a scheme in which each UE reports an L-valued CQI for *each* PRB. Let $C_{n,k}$ be the CQI value reported by the k^{th} UE for the n^{th} PRB. Then $C_{n,k} = i$ if $\gamma_{n,k} \in [T_{i-1}, T_i)$. The BS assigns the n^{th} PRB to the UE $k^*(n)$ that reports the highest CQI value, C_n^*, for it. Then,

$$C_n^* = \max_{1 \le k \le K} C_{n,k} \text{ and } k^*(n) = \arg \max_{1 \le k \le K} C_{n,k}. \quad (4)$$

As before, if multiple UEs report the highest CQI value, then one of them is chosen randomly with uniform probability. The BS sends data to the user $k^*(n)$ using the MCS corresponding to C_n^*.

IV. Analysis

A. PRB-Level Feedback Scheme

Let R_n be the average throughput for the n^{th} PRB. A throughput of r_i is achieved if the CQI value reported by the selected UE is i. The following result provides a closed-form expression for R_n.

Result *1:* The average throughput, R_n, of PRB n of the PRB-level CQI feedback scheme is

$$R_n = \sum_{i=1}^{L} r_i \left[\prod_{j=1}^{K} \left(1 - e^{-\frac{T_i}{\sigma_j^2}} \right) - \prod_{j=1}^{K} \left(1 - e^{-\frac{T_{i-1}}{\sigma_j^2}} \right) \right]. \quad (5)$$

Proof: The proof is relegated to Appendix A. ■

For the symmetric case, in which $\sigma_k^2 = \sigma^2$, for all $1 \le k \le K$, the above expression simplifies to

$$R_n = \sum_{i=1}^{L} r_i \left[\left(1 - e^{-\frac{T_i}{\sigma^2}} \right)^K - \left(1 - e^{-\frac{T_{i-1}}{\sigma^2}} \right)^K \right]. \quad (6)$$

B. Subband-Level Feedback Scheme

As mentioned, since we have only average CQI information about the entire subband, an outage occurs if the i^{th} MCS is used but the SNR is less than T_{i-1}. Let $p_{\text{out}}(i)$ be the probability of outage given that the BS uses the i^{th} MCS. The following 2 lemmas help find the probability of outage.

Lemma *1:* Let UE j be selected (sel.) for the n^{th} PRB and the CQI value reported by it be i. Then, the probability that $\gamma_{n,j}$ is less than T_{i-1} is

$$\begin{aligned} &\Pr\left(\gamma_{n,j} < T_{i-1} | \overline{C}_{s(n)}^* = i, j \text{ is sel. for } n^{\text{th}} \text{ PRB} \right) \\ &= (q-1)! \sum_{l=0}^{q-2} \frac{(-1)^l T_{i-1}^{l+1}}{(l+1)!(q-2-l)!(\sigma_j^2)^{l+1}} \\ &\times \frac{\left[\gamma\left(q-1-l, \frac{qT_i}{\sigma_j^2}\right) - \gamma\left(q-1-l, \frac{qT_{i-1}}{\sigma_j^2}\right) \right]}{\gamma\left(q, \frac{qT_i}{\sigma_j^2}\right) - \gamma\left(q, \frac{qT_{i-1}}{\sigma_j^2}\right)}, \end{aligned} \quad (7)$$

where $\gamma(k, x)$ is the Incomplete Gamma function [15].

Proof: The proof is relegated to Appendix B. ■

Lemma *2:* The probability that UE j is selected for the n^{th} PRB is

$$\begin{aligned} &\Pr\left(j \text{ is sel. for } n^{\text{th}} \text{ PRB}\right) \\ &= \sum_{i=1}^{L} \Pr\left(\overline{C}_{s(n),j} = i\right) \sum_{l=0}^{K-1} \sum_{r=1}^{\binom{K-1}{l}} \frac{1}{l+1} \times \\ &\left[\prod_{t_1 \epsilon \alpha_{l,j}(r)} \Pr\left(\overline{C}_{s(n),t_1} = i\right) \right] \left[\prod_{t_2 \epsilon \alpha_{l,j}^c(r)} \Pr\left(\overline{C}_{s(n),t_2} \le i-1\right) \right], \end{aligned} \quad (8)$$

where $\Pr\left(\overline{C}_{s(n),j} \le i\right) = \frac{\gamma\left(q, \frac{qT_i}{\sigma_j^2}\right)}{(q-1)!}$ and $\Pr\left(\overline{C}_{s(n),j} = i\right) = \frac{1}{(q-1)!}\left(\gamma\left(q, \frac{qT_i}{\sigma_j^2}\right) - \gamma\left(q, \frac{qT_{i-1}}{\sigma_j^2}\right)\right)$. The r^{th} l-element subset of $\{1, K\} \setminus \{j\}$ is $\alpha_{l,j}(r)$; the number of possible subsets is $\binom{K-1}{l}$.

Proof: Since $\Pr\left(\overline{C}_{s(n),j} \le i\right) = \Pr\left(0 \le \overline{\gamma}_{s(n),j} \le T_i\right)$, integrating the PDF of $\overline{\gamma}_{s,k}$ in (2) from 0 to T_i gives the result for $\Pr\left(\overline{C}_{s(n),j} \le i\right)$ and, thus, $\Pr\left(\overline{C}_{s(n),j} = i\right) = \Pr\left(\overline{C}_{s(n),j} \le i\right) - \Pr\left(\overline{C}_{s(n),j} \le i-1\right)$. A UE j is selected for the n^{th} PRB if $j = \arg\max_{1 \le k \le K} \{\overline{C}_{s(n),k}\}$. If l other UEs also report the same maximum value, then j is selected with probability $1/(l+1)$. Hence, we get the above result. ■

Result *2:* The average throughput of the subband-level CQI feedback scheme is

$$R_n = \sum_{i=1}^{L} \frac{r_i(1-p_{\text{out}}(i))}{((q-1)!)^K} \left[\prod_{j=1}^{K} \gamma\left(q, \frac{qT_i}{\sigma_j^2}\right) - \prod_{j=1}^{K} \gamma\left(q, \frac{qT_{i-1}}{\sigma_j^2}\right) \right],$$

where $p_{\text{out}}(i)$ is obtained from Lemmas 1 and 2 as:

$$\begin{aligned} p_{\text{out}}(i) &= \sum_{j=1}^{K} \Pr\left(j \text{ is sel. for } n^{\text{th}} \text{ PRB}\right) \\ &\times \Pr\left(\gamma_{n,j} < T_{i-1} | \overline{C}_{s(n)}^* = i, j \text{ is sel. for } n^{\text{th}} \text{ PRB}\right). \end{aligned} \quad (9)$$

Proof: The proof is relegated to Appendix C. ■

For the symmetric case, $p_{\text{out}}(i)$ simplifies to

$$\begin{aligned} p_{\text{out}}(i) &= (q-1)! \sum_{l=0}^{q-2} \frac{(-1)^l T_{i-1}^{l+1}}{(l+1)!(q-2-l)!(\sigma^2)^{l+1}} \\ &\times \frac{\left[\gamma\left(q-1-l, \frac{qT_i}{\sigma^2}\right) - \gamma\left(q-1-l, \frac{qT_{i-1}}{\sigma^2}\right) \right]}{\gamma\left(q, \frac{qT_i}{\sigma^2}\right) - \gamma\left(q, \frac{qT_{i-1}}{\sigma^2}\right)}, \end{aligned} \quad (10)$$

and the expression for the average throughput simplifies to

$$R_n = \sum_{i=1}^{L} \frac{r_i(1-p_{\text{out}}(i))}{((q-1)!)^K} \left[\gamma^K\left(q, \frac{qT_i}{\sigma^2}\right) - \gamma^K\left(q, \frac{qT_{i-1}}{\sigma^2}\right) \right]. \quad (11)$$

[2] In practice, when the SNR of the PRB is below the MCS threshold, the data might still be received correctly, albeit with a higher error probability. Therefore, the outage model provides a lower bound on the throughput. However, it is accurate when the block error rate declines sharply with SNR.

C. Subband-Level Feedback With Offset

To increase the throughput of the subband-level feedback scheme without increasing the feedback overhead, we make each UE report a CQI that corresponds to a scaled version of its average SNR. Therefore, $\overline{C}_{s(n),k} = i$ if $\overline{\gamma}_{s(n),k} \in \Delta[T_{i-1}, T_i)$ where Δ is the offset. Most of the analysis for this scheme is similar to the *subband-level feedback* scheme of Section IV-B because $\overline{\gamma}_{s(n),j}$ is now a χ^2 distributed RV with $2q$ degrees of freedom scaled by $\frac{\sigma_k^2}{2q\Delta}$. The throughput and probability of outage expressions are similar to Result 2, and are omitted. They are used to find the optimum Δ.

V. Simulation Results And Comparisons

We now verify the analytical results using Monte Carlo simulations that average over 50,000 samples. The set of link adaptation thresholds are generated using the coding gain loss model of [16], [17]. A rate r_i is achievable (with no outage) if the SNR lies above the threshold T_{i-1} and equals

$$r_i = \log_2 (1 + \alpha T_{i-1}). \quad (12)$$

Here, α is the link degradation parameter that models the coding gain loss. We use $\alpha = 0.398$, which corresponds to a deviation of 4 dB from the Shannon limit and is typical of practical systems [16]. The set of rates $\{r_i\}$ for the 16 different MCSs used in LTE lies in the range 0 to 5.6 bits/symbol, and is tabulated in [13, Tbl. 10.1]. A subband consists of $q = 4$ PRBs. As assumed in the system model, fading across PRBs is simulated to be i.i.d. Since we are characterizing throughput as a function of bits/symbol, the results below hold for all bandwidths.

In Figure 1, we plot the simulation and analysis results for the average throughput as a function of number of users for all the feedback reporting schemes. $\sigma^2 = 13$ dB is chosen for all users. Notice that the simulation and analysis results agree very well for all the schemes. We can see that as the number of users in the cell increases, the average throughput per PRB increases because of frequency-domain scheduling. Also, we can see that the frequency granularity of CQI reporting has a significant impact on the average throughput. Unlike the PRB-level feedback scheme, the subband-level feedback scheme's average throughput saturates early and is about 60% less. We can see that subband-level feedback with an offset (Δ) performs better and improves throughput by 10%. The offset is chosen to maximize throughput.

In Figure 2, we plot the optimum Δ for different numbers of users. Notice that the optimum Δ is always greater than 1, indicating that it is better to be conservative and report a CQI value lesser than the average for the subband. Doing so reduces the probability of outage which increases throughput.

In Figure 3, we plot the throughput vs. the number of users for the asymmetric case with 6 users. In this case, σ_k^2 (in dB) is chosen as 11, 15, 10, 7, 8 and 14 for the 6 users. The behavior is qualitatively similar to the symmetric case.

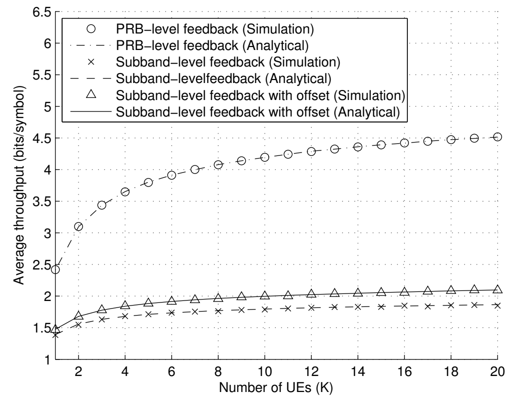

Fig. 1. Symmetric case: Throughput as a function of the number of users for the different feedback schemes.

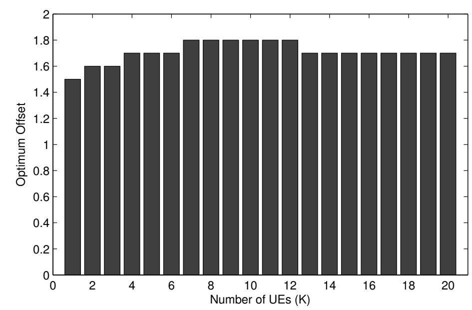

Fig. 2. Symmetric case: Optimum offset as a function of the number of users per cell.

VI. Conclusions

In LTE, the scheduler can assign different PRBs to different UEs. However, the CQI report, on the basis of which the assignment is done, has a coarser frequency granularity of a subband, which consists of multiple PRBs. We showed that the coarse frequency granularity of a subband incurs a significant loss in system throughput. We did this by developing closed-form expressions for the downlink throughput of the subband-

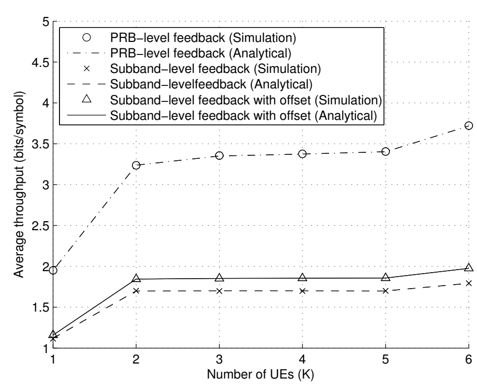

Fig. 3. Asymmetric case: Throughput as a function of the number of users for the different feedback schemes.

level CQI feedback mechanism of LTE and for a benchmark PRB-level feedback scheme. We also showed that the performance of the subband-level feedback scheme can be improved without increasing the feedback overhead by making the users more conservative in reporting their CQIs.

Future work includes analyzing the performance of the UE selected subband feedback mechanism of LTE, which reduces feedback overhead even further, and extending the anlaysis to handle proportional fair schedulers.

APPENDIX

A. Proof of Result 1

From the law of total expectation, we have for PRB n

$$R_n = \sum_{i=1}^{L} r_i \mathrm{Pr}\left(C_n^* = i\right). \tag{13}$$

Since $C_n^* = \max_{1\leq k\leq K}\{C_{n,k}\}$, the PDF of C_n^* is $\mathrm{Pr}\left(C_n^* = i\right) = \prod_{j=1}^{K} \mathrm{Pr}\left(C_{n,j} \leq i\right) - \prod_{j=1}^{K} \mathrm{Pr}\left(C_{n,j} \leq i-1\right)$. But, $\mathrm{Pr}\left(C_{n,j} \leq i\right) = 1 - e^{-\frac{T_i}{\sigma_j^2}}$ since the channels undergo Rayleigh fading. The desired expression then follows.

B. Proof of Lemma 1

Given that j is selected for the n^{th} PRB

$$\begin{aligned}&\mathrm{Pr}\left(\gamma_{n,j} < T_{i-1} | \overline{C}^*_{s(n)} = i, j \text{ is sel. for } n^{\text{th}} \text{ PRB}\right)\\ &\quad = \mathrm{Pr}\left(\gamma_{n,j} < T_{i-1} | \overline{C}_{s(n),j} = i, j \text{ is sel. for } n^{\text{th}} \text{ PRB}\right).\end{aligned}$$

Given that $\overline{C}_{s(n),j} = i$, the probability that $\gamma_{n,j}$ is less than T_{i-1} does not depend on whether j is selected for the n^{th} PRB or not. Therefore,

$$\begin{aligned}&\mathrm{Pr}\left(\gamma_{n,j} < T_{i-1} | \overline{C}_{s(n),j} = i, j \text{ is sel. for } n^{\text{th}} \text{ PRB}\right)\\ &\qquad = \frac{\mathrm{Pr}\left(\overline{C}_{s(n),j} = i, \gamma_{n,j} < T_{i-1}\right)}{\mathrm{Pr}\left(\overline{C}_{s(n),j} = i\right)},\end{aligned} \tag{14}$$

since the event $\overline{C}_{s(n),j} = i$ is the same as the event $T_{i-1} \leq \overline{\gamma}_{s(n),j} < T_i$. The numerator is evaluated as follows.

$$\begin{aligned}&\mathrm{Pr}\left(\overline{C}_{s(n),j} = i, \gamma_{n,j} < T_{i-1}\right)\\ &= \mathrm{Pr}\left(T_{i-1} \leq \overline{\gamma}_{s(n),j} < T_i, \gamma_{n,j} < T_{i-1}\right),\\ &= \int_0^{T_{i-1}} f_{\gamma_{n,j}}(y) \mathrm{Pr}\left(T_{i-1} \leq \overline{\gamma}_{s(n),j} < T_i | \gamma_{n,j} = y\right) dy,\end{aligned} \tag{15}$$

where $f_{\gamma_{n,j}}(y)$ is the PDF of $\gamma_{n,j}$. Since $\overline{\gamma}_{s(n),j}$ is the average of q SNRs, given the condition that $\gamma_{n,j} = y$, $\overline{\gamma}_{s(n),j}$ is a χ^2 RV with $2(q-1)$ degrees of freedom that is scaled by $\sigma_j^2/2q$ and shifted by y/q. Thus,

$$\begin{aligned}&\mathrm{Pr}\left(\overline{C}_{s(n),j} = i, \gamma_{n,j} < T_{i-1}\right)\\ &\quad = \int_0^{T_{i-1}} \frac{e^{-\frac{y}{\sigma_j^2}}}{\sigma_j^2} \int_{T_{i-1}-\frac{y}{q}}^{T_i-\frac{y}{q}} \frac{q^{q-1} x^{q-2} e^{-\frac{qx}{\sigma_j^2}}}{(q-2)!(\sigma_j^2)^{q-1}}\, dx\, dy.\end{aligned} \tag{16}$$

Simplifying the integral and substituting in (14) along with results from Lemma 2 yields the desired result.

C. Proof of Result 2

When an outage occurs in a slot, the throughput in that slot is 0. Therefore, the average throughput in bits/symbol is given by

$$R_n = \sum_{i=1}^{L} r_i Pr(\overline{C}^*_{s(n)} = i)(1 - p_{\text{out}}(i)). \tag{17}$$

Since $\overline{C}^*_{s(n)} = \max_{1\leq k\leq K}\{\overline{C}_{s(n),k}\}$, the CDF of $\overline{C}^*_{s(n)}$ equals $\mathrm{Pr}\left(\overline{C}^*_{s(n)} \leq i\right) = \prod_{j=1}^{K} Pr(\overline{C}_{s(n),j} \leq i)$. Hence, $\mathrm{Pr}\left(\overline{C}^*_{s(n)} = i\right) = \prod_{j=1}^{K} Pr(\overline{C}_{s(n),j} \leq i) - \prod_{j=1}^{K} Pr(\overline{C}_{s(n),j} \leq i-1)$. Substituting the expression for $\mathrm{Pr}\left(\overline{C}_{s(n),j} \leq i\right)$ from Lemma 2 in (17), we get the desired result.

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