EESM-based Link Adaptation in OFDM: Modeling and Analysis

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***Abstract*—In orthogonal frequency division multiplexing systems, such as Long Term Evolution (LTE) and WiMAX, the different subcarriers over which a codeword is transmitted may see different signal-to-noise-ratios (SNRs). Thus, adaptive modulation and coding (AMC) in these systems must be based on a vector of subcarrier SNRs seen by the codeword, and is considerably more involved. Exponential effective SNR mapping (EESM) simplifies the problem by mapping the vector of SNRs into a single equivalent flat-fading SNR. However, the analysis of AMC using EESM is challenging owing to its non-linear nature and because it uses an SNR scaling parameter that depends on the modulation and coding scheme. We first propose a novel statistical model for EESM based on the Beta distribution, which is motivated by the central limit approximation for the sum of random variables with finite support. Unlike several ad hoc statistical models, which require three or more parameters to be computed numerically, the proposed model requires only two parameters, for which closed-form expressions are derived for both correlated and uncorrelated subcarrier SNRs. Despite its simplicity, it is as accurate as the ad hoc models. We then present a novel, tight upper bound and an accurate approximation in closed-form for the throughput of a frequency-selective system that uses EESM for AMC.**

I. Introduction

High data rate requirements coupled with scarcity of the spectrum have driven the quest for techniques that improve spectral efficiency. One important technique for doing so is adaptive modulation and coding (AMC). In it, the transmitter chooses its modulation and coding scheme (MCS) from a finite set of MCSs depending on the channel conditions to maximize the throughput, subject to a constraint on the probability of error. AMC is an integral part of current and next generation systems such as Long Term Evolution (LTE) and IEEE 802.16e/m WiMAX, which are wideband in nature and use orthogonal frequency division multiplexing (OFDM).

In OFDM, the available bandwidth is divided into orthogonal subcarriers over which the data is transmitted. In a practical OFDM system, AMC is not done on a per-subcarrier basis because of its significant feedback and control signal overhead. Instead, the same MCS is used on all the subcarriers assigned to a user. Due to the frequency-selective nature of the wideband channel, the channel gains of these subcarriers can be different. Thus, the MCS must be chosen as a function of a vector of subcarrier gains. In principle, this requires an unwieldy multi-dimensional look-up-table, which is cumbersome to generate and store, and is seldom used. This is much more involved than classical rate adaptation in narrowband fading channels [1, Chap. 9]. Therefore, link quality metrics (LQMs) have been proposed to simplify this problem and make it similar to AMC over narrowband channels. An LQM maps the vector of signal-to-noise-ratios (SNRs) to a scalar, which is then easily mapped to block error rate (BLER) using a 1-dimensional look-up-table.

A popular and accurate LQM is exponential effective SNR mapping (EESM) [2]. It maps the vector of channel SNRs into an effective SNR, which is interpreted as the equivalent SNR in an additive white Gaussian noise (AWGN) channel that results in the same BLER. If γ_i denotes the SNR of the i^{th} subcarrier, for $1 \leq i \leq N$, then the effective SNR using EESM, $\gamma_{\text{eff}}^{(m)}$, for MCS m is given by

$$\gamma_{\text{eff}}^{(m)} = -\beta_m \ln\left(\frac{1}{N}\sum_{i=1}^{N}\exp\left(-\frac{\gamma_i}{\beta_m}\right)\right), \quad (1)$$

where $\beta_m > 0$ is an MCS-dependent SNR scaling parameter. Besides its use in AMC, EESM is widely used as a physical layer abstraction tool in system-level simulations [3], [4]. EESM can be easily extended to handle hybrid automatic repeat request (HARQ) and multiple antennas [5]. Furthermore, EESM can be used to generate channel quality indicators (CQIs) [6], which are fed back to the base station for enabling link adaptation and scheduling.

However, as can be seen from (1), EESM is a highly non-linear function of the subcarrier SNRs. No exact closed-form expression for its statistics is known. Thus, ad hoc distributions have been used in the literature to approximately characterize its statistics [6], [7]. Another critical issue is that the parameter β_m is different for different MCSs [8]. Thus, AMC using EESM involves computing several effective SNRs, one for each MCS, and then selecting an MCS using all of them. Consequently, EESM-based AMC is more involved than AMC in a narrowband system, and is the focus of this paper.

A. Literature on LQM-based AMC in OFDM

An optimal AMC algorithm using LQMs is detailed in [9]. It proceeds as follows: A sequential search is initiated, starting with the highest rate MCS. An LQM-based BLER estimate is obtained for this MCS, and if it is less than the target BLER, then this MCS is selected. Else, the next lower rate MCS is considered, and so on. Thus, this scheme chooses the MCS with the highest rate that satisfies the BLER constraint. Its throughput has thus far been characterized using extensive numerical simulations [9]–[12]. The approaches pursued

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in [10]–[12] can also be considered as providing bounds on the throughput. However, they do not lead to analytically insightful expressions for it.

B. Contributions

We first present a novel statistical model for EESM, in which the quantity inside the logarithm in (1), is modeled as a Beta random variable (RV). Moment-matching is used to compute its parameters. The choice of the Beta distribution, which has a compact support, is motivated by the popular central limit approximation for RVs with finite support [13, Chap. 11]. We show that its accuracy is better than that of the lognormal model considered in [6] and is comparable to the Pearson and generalized extreme value (GEV) distributions [7], which are analytically very involved and require evaluating more parameters. Secondly, we derive simple closed-form expressions for the moments for both independent and correlated subcarriers, which makes it easy to evaluate the parameters of the Beta distribution. This is unlike the EESM moments that are required by the lognormal, GEV, and Pearson models, the expressions for which are extremely involved or intractable [7].

Another key advantage is the analytical tractability of the proposed model, as it yields closed-form expressions for the throughput of AMC in an OFDM link. We present a novel, tight upper bound and an accurate approximation for it. The analysis covers different multiple antenna diversity modes and easily accounts for the correlation among subcarriers.

We focus on EESM because it has a relatively simpler definition than mutual-information based LQMs [2], [14], [15] and is only marginally less accurate. Thus, EESM strikes a good balance between the complexity required to compute it and the accuracy of the BLER estimate obtained from it.

The paper is organized as follows. The point-to-point link system model is described in Section II. A novel statistical model for EESM is developed in Section III. Throughput analysis and simulations are given in Section IV, and are followed by our conclusions in Section V.

II. System Model: Point-to-point Link

We consider a point-to-point OFDM link where the transmitted codeword is encoded across N subcarriers. The receiver selects an MCS based on the N subcarrier SNRs and feeds it back to the transmitter using a feedback channel. The feedback is assumed to be error-free and the feedback delay is assumed to be negligible. The transmitter then uses the reported MCS for transmission.

Let $h_i(k,l)$ denote the complex baseband channel gain between the l^{th} transmit antenna and the k^{th} receive antenna of the i^{th} subcarrier. The complex channel gains are identically distributed and are circularly symmetric complex Gaussian RVs with variance σ^2 [1]. The channel gains between different transmit-receive (Tx-Rx) antenna pairs are assumed to be independent, i.e., $h_i(k,l)$ is independent of $h_j(m,n)$ if $k \neq m$ or $l \neq n$, for any i and j. This assumption is valid when the antennas are sufficiently spaced apart in a rich scattering environment. Let N_r and N_t denote the number of receive and transmit antennas, respectively. Let γ_i denote the i^{th} subcarrier SNR, for $1 \leq i \leq N$. It depends on the antenna diversity mode and is a function of $h_i(k,l)$ [6]. Let $\mathbf{\Gamma}$ denote the vector of subcarrier SNRs: $\mathbf{\Gamma} = [\gamma_1, \gamma_2, \dots, \gamma_N]$.

Let Ω denote the set of L MCSs used for rate adaptation. The information rate of MCS m is denoted by r_m and $\hat{m}$ denotes the selected MCS. BLER_t denotes the maximum allowable block error rate. The MCSs are indexed in the increasing order of their rates, i.e., $r_1 \leq r_2 \leq \cdots \leq r_L$.

A. Optimal AMC Using EESM

In order to maximize the average throughput, the MCS should be chosen as a function of $\mathbf{\Gamma}$ as follows:

$$\hat{m} = \underset{m\in\Omega}{\arg\max}\ r_m, \tag{2}$$

$$\text{s. t. } \text{BLER}(\mathbf{\Gamma}, m) \leq \text{BLER}_t, \tag{3}$$

where $\text{BLER}(\mathbf{\Gamma}, m)$ is the BLER of MCS m with $\mathbf{\Gamma}$ as the vector of subcarrier SNRs.

Using EESM, the BLER constraint in (3) can be mapped to a constraint on the effective SNR as follows: Let T_m denote the lowest SNR at which the BLER in the AWGN channel using MCS m, $\text{BLER}_{\text{AWGN}}(T_m, m)$, is BLER_t, i.e.,

$$\text{BLER}_{\text{AWGN}}(T_m, m) = \text{BLER}_t, \quad m = 1, \dots, L. \tag{4}$$

Thus, the BLER constraint is equivalent to having the effective SNR of MCS m greater than or equal to T_m.

The optimal AMC scheme is now driven by the thresholds $T_1, T_2, \dots, T_N$, and proceeds as follows: It starts with the highest MCS $m = L$. It is chosen if its effective SNR is greater than or equal to T_L, i.e., $\gamma_{\text{eff}}^{(L)} \geq T_L$. Else, the scheme moves to the next highest MCS, and so on. If $\gamma_{\text{eff}}^{(i)} < T_i$, for all $i = 1, \dots, L$, then no data transmission takes place.

B. Notations

Let $\mathbf{I}_n$ and $\mathbf{0}_n$ denote the $n \times n$ identity matrix and zero matrix, respectively. The determinant of a matrix $\mathbf{A}$ is denoted by $\det(\mathbf{A})$. The matrix $\text{diag}(\mathbf{v})$ denotes a diagonal matrix with elements of the vector $\mathbf{v}$ as its diagonal elements. Let $\left[0, \dots, 0, \underset{(i)}{s}, 0, \dots, 0\right]$ denote a vector whose i^{th} element is s and all other elements are 0. $\text{Re}(C)$ and $\text{Im}(C)$ denote the real and imaginary parts of the complex number C, respectively. Let $\mathbb{E}[X]$ denote the expectation of the RV X.

III. Statistical Model for EESM

In order to analyze the performance of EESM-based AMC, a statistical model for EESM is required. Since, no closed-form expression for the distribution of EESM is available owing to its highly non-linear nature, we first present a novel and tractable approximation for it.

Notice that $e^{-\frac{\gamma_i}{\beta_m}} \in [0,1]$. Thus, EESM involves taking sums over N positive RVs with finite support. This motivates a new statistical model for EESM based on the central limit approximation for RVs with finite support, in which the sum of i.i.d. RVs with finite support is approximated by a Beta distribution [13, Chap. 11]. Thus, we model the RV $Y_m = \frac{1}{N}\sum_{i=1}^{N} e^{-\frac{\gamma_i}{\beta_m}}$ as a Beta RV, whose probability density function is given by

$$f_{Y_m}(y) = \frac{y^{(a_m-1)}(1-y)^{(b_m-1)}}{B(a_m,b_m)}, \quad 0 \le y \le 1, \tag{5}$$

where $B(\cdot,\cdot)$ is the Beta function [16]. In terms of the mean μ_m and variance v_m of RV Y_m, the parameters a_m and b_m of the Beta distribution are given by

$$a_m = \frac{\mu_m(\mu_m - \mu_m^2 - v_m)}{v_m}, \tag{6}$$

$$b_m = \frac{(1-\mu_m)(\mu_m - \mu_m^2 - v_m)}{v_m}. \tag{7}$$

Thus, the cumulative distribution function (CDF) of $\gamma_{\text{eff}}^{(m)}$, $F_{\gamma_{\text{eff}}^{(m)}}(x)$, is equal to $P\left(Y_m \ge e^{\frac{-x}{\beta_m}}\right)$ and is given by

$$F_{\gamma_{\text{eff}}^{(m)}}(x) = 1 - B_i\left(e^{-\frac{x}{\beta_m}}, a_m, b_m\right), \quad x \ge 0, \tag{8}$$

where $B_i(\cdot,\cdot,\cdot)$ is the incomplete Beta function [16].

Notice that we approximate the term inside the logarithm in (1) as a Beta RV. A key advantage of this is the easy computation of its moments. It is unlike the lognormal, Pearson, and GEV distributions, which often need numerical simulations to compute the EESM moments for moment-matching [6].

A. Expressions for Mean and Variance of Y_m

We see from (6) and (7) that the Beta distribution is fully specified in terms of its mean and variance. Hence, we now derive closed-form expressions for these for different subcarrier correlations and antenna diversity modes.

We consider the cases where the subcarrier SNRs are independent and where they are correlated. The former scenario is simple and insightful, and occurs when the subcarrier bandwidth is close to the coherence bandwidth of the channel. It also arises if the subcarriers are noncontiguous, as is the case in the full usage of subchannels (FUSC) or partial usage of subchannels (PUSC) modes of WiMAX. Otherwise, the subcarrier SNRs are correlated. This scenario occurs in LTE and in the Band AMC mode of WiMAX, in which contiguous subcarriers are allotted to a user.

We focus on single-input-single-output (SISO), single-input-multiple-output (SIMO), and multiple-input-single-output (MISO) multiple antenna diversity modes. The multiple-input-multiple-output (MIMO) scenario is skipped due to space constraints. Closed-form expressions can be derived for single stream MIMO with independent subcarriers [17], but, in general, one has to use numerical Monte Carlo methods [18].

1) Independent Subcarriers: We first state the following general result.

Result *1:* The mean and variance of RV Y_m when subcarrier SNR γ_i is equal to cX_τ, where X_τ is a Chi-square RV with τ degrees of freedom, are given by

$$\mu_m = \left(\frac{\beta_m}{\beta_m + 2c}\right)^{\frac{\tau}{2}}, \tag{9}$$

$$v_m = \frac{1}{N}\left[\left(\frac{\beta_m}{\beta_m+4c}\right)^{\frac{\tau}{2}} - \left(\frac{\beta_m}{\beta_m+2c}\right)^{\tau}\right]. \tag{10}$$

Proof: The proof is relegated to Appendix A. ∎

The mean and variance of RV Y_m for SISO, SIMO, and MISO antenna modes then follow from (9) and (10). For SISO ($N_t = N_r = 1$), $\gamma_i = |h_i(1,1)|^2$. Thus, $c = \frac{\sigma^2}{2}$ and $\tau = 2$. For SIMO ($N_t = 1$, $N_r > 1$) with maximal ratio combining, $\gamma_i = \sum_{j=1}^{N_r} |h_i(j,1)|^2$. Thus, $c = \frac{\sigma^2}{2}$ and $\tau = 2N_r$. Similarly, for MISO ($N_t > 1$, $N_r = 1$) with maximal ratio transmission, $c = \frac{\sigma^2}{2}$ and $\tau = 2N_t$. For MISO with the Alamouti space-time code ($N_t = 1$, $N_r = 2$), $c = \frac{\sigma^2}{4}$ and $\tau = 4$ [1].

2) Correlated Subcarriers: In this scenario, the channel gain between any two subcarriers for the same transmit-receive antenna pair is correlated. Therefore, the vector $\mathbf{h}_{kl} = [h_1(k,l),\ldots,h_N(k,l)]^T$ is a circularly symmetric complex Gaussian random vector with covariance matrix $\mathbf{C}$. The $(i,j)^{\text{th}}$ element C_{ij} of $\mathbf{C}$ is defined as $C_{ij} = \mathbb{E}\left[h_i(k,l)h_j(k,l)^*\right]$.

We state the following general result.

Result *2:* The mean and variance of Y_m with correlated subcarriers, for SISO ($N_t = N_r = 1$), MISO ($N_t > 1, N_r = 1$), and SIMO ($N_t = 1, N_r > 1$), are given by

$$\mu_m = \left(\det\left(\mathbf{I}_{2N} - 2\mathbf{K}\mathbf{P}^{(m)}\right)\right)^{-\frac{\max(N_t,N_r)}{2}}, \tag{11}$$

$$v_m = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\Bigg[\left(\det\left(\mathbf{I}_{2N} - 2\mathbf{K}\mathbf{Q}_{ij}^{(m)}\right)\right)^{-\frac{\max(N_t,N_r)}{2}} - \left(\det\left(\mathbf{I}_{2N} - 2\mathbf{K}\mathbf{P}^{(m)}\right)\right)^{-\max(N_t,N_r)}\Bigg], \tag{12}$$

where $\mathbf{P}^{(m)} = \begin{bmatrix}\mathbf{R}^{(m)} & \mathbf{0}_N \\ \mathbf{0}_N & \mathbf{R}^{(m)}\end{bmatrix}$, $\mathbf{Q}_{ij}^{(m)} = \begin{bmatrix}\mathbf{S}_{ij}^{(m)} & \mathbf{0}_N \\ \mathbf{0}_N & \mathbf{S}_{ij}^{(m)}\end{bmatrix}$, $\mathbf{R}^{(m)} = \text{diag}\left([-\beta_m^{-1},0,\ldots,0]\right)$, $\mathbf{K} = \frac{1}{2}\begin{bmatrix}\text{Re}(\mathbf{C}) & -\text{Im}(\mathbf{C}) \\ \text{Im}(\mathbf{C}) & \text{Re}(\mathbf{C})\end{bmatrix}$,
$\mathbf{S}_{ij}^{(m)} = \text{diag}\left(\left[0,\ldots,0,\underset{(i)}{-\beta_m^{-1}},0,\ldots,0,\underset{(j)}{-\beta_m^{-1}},0,\ldots,0\right]\right)$,
for $i \ne j$, and $\mathbf{S}_{ii}^{(m)} = \text{diag}\left(\left[0,\ldots,0,\underset{(i)}{-2\beta_m^{-1}},0,\ldots,0\right]\right)$.

Proof: The proof is relegated to Appendix B. ∎

MISO with the Alamouti space-time code is equivalent to closed-loop MISO with half the transmit power [1]. Therefore, the mean and variance expressions for the Alamouti space-time code follow from (11) and (12), with $\mathbf{K}$ replaced by $\frac{\mathbf{K}}{2}$.

B. Empirical Verification of the Proposed Beta Distribution

We now compare the CDF and complementary CDF (CCDF) obtained by the proposed model with the empirical CDF and CCDF obtained from Monte Carlo simulations that use 10^5 samples. The CDF captures the accuracy of the fit for low $\gamma_{\text{eff}}^{(m)}$ values. However, it saturates to 1 for large

$\gamma_{\text{eff}}^{(m)}$ values for any distribution. In this regime, comparing the CCDF is more instructive. The average SNR σ^2 of a transmit-receive antenna pair link is 10 dB and β_m is 5.

1) Independent Subcarriers: Figure 1 compares the empirical CDF and CCDF with the CDF and CCDF obtained from the proposed Beta model, for $\tau = 2$ (SISO). Observe that the Beta model is quite accurate for three orders of magnitude of the CDF and CCDF values even for $N = 4$. Its accuracy improves further as N increases.

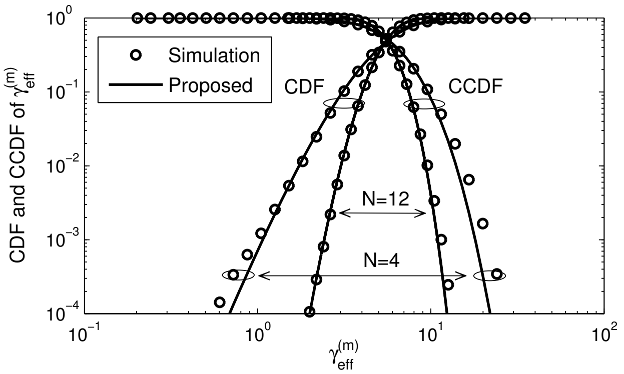

Fig. 1. Comparison of CDF and CCDF of EESM with the proposed Beta model for independent subcarriers ($\tau = 2$ (SISO), $\sigma^2 = 10$ dB, and $\beta_m = 5$).

Figure 2 compares the CDFs of the different approximations. A zoomed-in plot is shown in order to make the curves discernible. GEV, Pearson and lognormal distribution parameters are obtained by moment-matching, and numerical Monte Carlo methods [18] are used to obtain the moments required by them. Observe that the accuracy of the proposed Beta model is comparable to the GEV and Pearson distributions, despite the former's simple form. The proposed model is more accurate than the lognormal model.

2) Correlated Subcarriers: In order to understand the effect of correlation, we set $C_{ij} = \sigma^2 \rho^{|i-j|}$, for $0 < \rho < 1$ [6]. Figure 3 compares the CDFs of the different approximations, with $\rho = 0.4$ and $\tau = 4$ (1×2 SIMO). Note that the proposed model tracks the empirical CDF well, and its accuracy is again comparable to the more involved GEV and Pearson distributions. The results are similar for the typical urban (TU) and rural area (RA) channels [19].

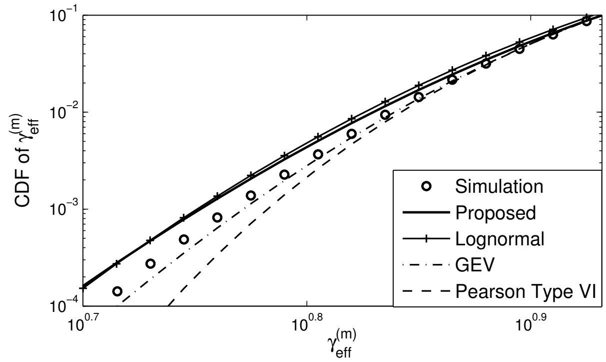

Fig. 3. Comparison of CDFs of different approximations with the empirical CDF for geometrically correlated subcarriers ($\tau = 4$ (1×2 SIMO), $\sigma^2 = 10$ dB, $\beta_m = 5$, $N = 12$, and $\rho = 0.4$).

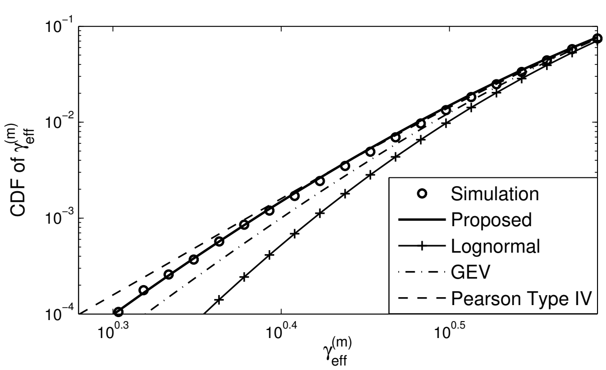

Fig. 2. Comparison of CDFs of different approximations with the empirical CDF for independent subcarriers ($\tau = 2$ (SISO), $\sigma^2 = 10$ dB, $\beta_m = 5$, and $N = 12$).

IV. Throughput Analysis

We now analyze the throughput of the optimal AMC scheme detailed in Section II-A. It can be mathematically formulated as follows: Let $\mathbb{M}$ denote the set of MCSs whose effective SNRs exceed their corresponding thresholds. Then, the optimal MCS $\hat{m}$ is the maximum element of $\mathbb{M} = \left\{ m \in \{1, \ldots L\} : \gamma_{\text{eff}}^{(m)} \geq T_m \right\}$. Therefore, the average throughput $\overline{R}$ is given by

$$\overline{R} = \sum_{m=1}^{L} r_m \Pr\{\hat{m} = m\}. \tag{13}$$

The probability that MCS m is chosen is

$$\Pr\{\hat{m} = m\} = P\left(\gamma_{\text{eff}}^{(m)} \geq T_m, \gamma_{\text{eff}}^{(m+1)} < T_{m+1}, \ldots, \gamma_{\text{eff}}^{(L)} < T_L \right). \tag{14}$$

Hence, to compute (14), an $(L - m + 1)$-dimensional joint distribution of the effective SNRs, which are correlated, is needed. We tackle this difficult problem by coming up with a novel upper bound and an approximation.

Result *3:* The average throughput is upper bounded as

$$\begin{aligned}\overline{R} \leq \sum_{m=1}^{L-1} r_m \min \Bigg\{ & B_i\left(e^{-\frac{T_{m+1}}{\beta_m}}, a_m, b_m\right) \\ & - B_i\left(e^{-\frac{T_m}{\beta_m}}, a_m, b_m\right),\; B_i\left(e^{-\frac{T_{m+1}}{\beta_{m+1}}}, a_{m+1}, b_{m+1}\right) \\ & - B_i\left(e^{-\frac{T_m}{\beta_{m+1}}}, a_{m+1}, b_{m+1}\right) \Bigg\} + r_L B_i\left(e^{-\frac{T_L}{\beta_L}}, a_L, b_L\right).\end{aligned} \tag{15}$$

Proof: The proof is relegated to Appendix C. ■

The key step in the proof is to upper bound (14) with $P\left(\gamma_{\text{eff}}^{(m)} \geq T_m, \gamma_{\text{eff}}^{(m+1)} < T_{m+1}\right)$. This bound is tight because if reliable transmission with MCS $m+1$ is not possible,

TABLE I
β_m AND SNR THRESHOLDS FOR THE MCSS SPECIFIED IN LTE ($\text{BLER}_t = 0.1$) [8]

Index	Modulation	Code rate	Information rate (bits/symbol)	β_m	SNR threshold (dB)
1	QPSK	0.08	0.15	1.00	-9.48
2	QPSK	0.12	0.23	1.40	-6.66
3	QPSK	0.19	0.38	1.40	-4.10
4	QPSK	0.30	0.60	1.48	-1.80
5	QPSK	0.44	0.88	1.50	0.40
6	QPSK	0.59	1.18	1.62	2.42
7	16-QAM	0.37	1.48	3.10	4.49
8	16-QAM	0.48	1.91	4.32	6.37
9	16-QAM	0.60	2.41	5.37	8.46
10	64-QAM	0.46	2.73	7.71	10.27
11	64-QAM	0.55	3.32	15.50	12.22
12	64-QAM	0.65	3.90	19.60	14.12
13	64-QAM	0.75	4.52	24.70	15.85
14	64-QAM	0.85	5.12	27.60	17.79
15	64-QAM	0.93	5.55	28.90	19.81

then it is unlikely that MCSs with rates higher than r_{m+1} can be transmitted reliably.

Result *4:* The average throughput is approximated as

$$\overline{R} \approx r_L B_i\left(e^{-\frac{T_L}{\beta_L}}, a_L, b_L\right) + \sum_{m=1}^{L-1} r_m \left[B_i\left(e^{-\frac{T_m}{\beta_m}}, a_m, b_m\right) - B_i\left(e^{-\frac{T_{m+1}}{\beta_{m+1}}}, a_{m+1}, b_{m+1}\right)\right]. \quad (16)$$

Proof: The derivation is relegated to Appendix D. ■

Note that both the upper bound and the approximation can be readily computed using the results in Section III.

A. Numerical Results

We now compare the analytical results with numerical simulations. The maximum allowable block error rate, BLER_t, is 0.1 [20] and the number of subcarriers, N, is 24, which is equal to two physical resource blocks (PRBs) in LTE. For SIMO, the number of receive antennas, N_r, is 2. The MCSs used are as per Table I. We consider the TU and RA channels [19]. The TU channel is more frequency-selective than the RA channel. We use 512-point FFT and the bandwidth is 5 MHz. The sampling frequency is 7.68 MHz [3].

The throughput as a function of σ^2 for different antenna modes is plotted in Figures 4 and 5 for the RA and TU channels, respectively. Notice the excellent match between the proposed approximation and the simulation curves. Further, the upper bound becomes tighter as we go from SISO to SIMO, and marginally looser when the correlation across the subcarriers decreases. This is because both the approximation and the upper bound are exactly equal to the throughput in the narrowband regime. The throughput for the RA channel is higher than that for the TU channel. Thus, the throughput increases as correlation across the subcarriers increases.

V. CONCLUSIONS

We presented a general mathematical framework for analyzing the average throughput of optimal EESM-based AMC

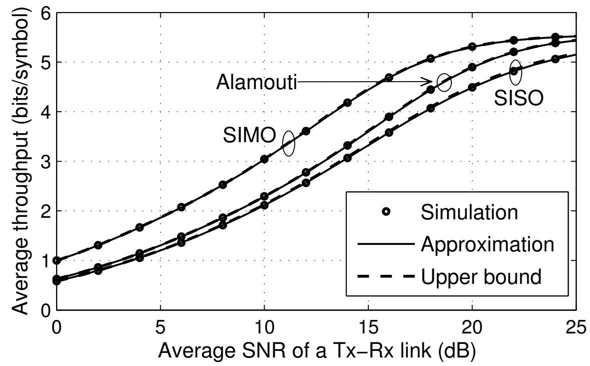

Fig. 4. LTE: Average throughput versus average SNR of a transmit-receive antenna pair link (σ^2) for the RA channel.

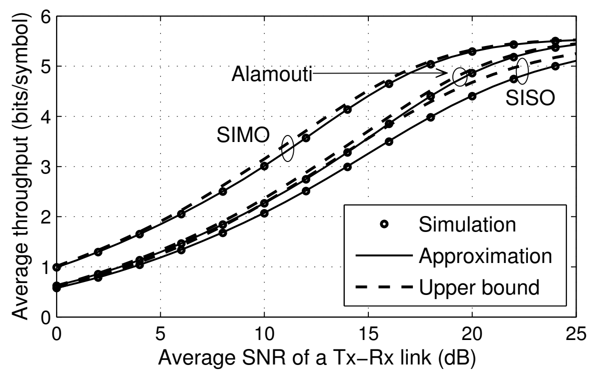

Fig. 5. LTE: Average throughput versus average SNR of a transmit-receive antenna pair link (σ^2) for the TU channel.

for OFDM systems in frequency-selective channels. To this end, we proposed an analytically tractable statistical model for EESM based on the Beta distribution. Its accuracy is comparable to the more involved GEV and the Pearson distributions, which have been proposed in the literature. Another advantage is that the parameters of the Beta distribution can be evaluated in closed-form for different multiple antenna diversity modes. This model then led to a closed-form tight upper bound and an accurate approximation for the average throughput of an EESM-based rate-adaptive system.

The approach presented in this paper can be extended to determine the throughput of systems that use other link quality metrics. Future work involves incorporating correlated antennas, HARQ, and channel imperfections in the framework.

APPENDIX

A. Mean and Variance of Y_m for Independent Subcarriers

The moment generating function (MGF) $\Psi_X(t)$ of an RV X is defined as $\Psi_X(t) = \mathbb{E}\left[e^{tX}\right]$. When the N subcarriers are i.i.d., the mean μ_m and variance v_m of Y_m can be expressed in terms of the MGF of one of the subcarrier SNRs γ_i as

$$\mu_m = \Psi_{\gamma_i}\left(-\beta_m^{-1}\right), \quad (17)$$

$$v_m = \frac{1}{N}\left(\Psi_{\gamma_i}\left(-2\beta_m^{-1}\right) - \left[\Psi_{\gamma_i}\left(-\beta_m^{-1}\right)\right]^2\right). \quad (18)$$

Since $\gamma_i = cX_\tau$, it can be shown that its MGF is given by $\Psi_{\gamma_i}(t) = (1-2ct)^{-\frac{\tau}{2}}$. Substituting this in (17) and (18) yields (9) and (10), respectively.

B. Mean and Variance of Y_m for Correlated Subcarriers

The joint MGF of $\mathbf{\Gamma}$, $\Psi_{\mathbf{\Gamma}}(t_1,\ldots,t_N)$, is defined as $\Psi_{\mathbf{\Gamma}}(t_1,\ldots,t_N) = \mathbb{E}\left[e^{\sum_{i=1}^N t_i\gamma_i}\right]$. The mean μ_m and the variance v_m can be expressed in terms of $\Psi_{\mathbf{\Gamma}}$ as

$$\mu_m = \Psi_{\mathbf{\Gamma}}(t_1,\ldots,t_N)\Big|_{\substack{t_1=-\beta_m^{-1}\\ t_k=0,\text{ else}}}, \tag{19}$$

$$v_m = \frac{1}{N^2}\left[\sum_{i=1}^N \sum_{j=1,j\neq i}^N \Psi_{\mathbf{\Gamma}}(t_1,\ldots,t_N)\Big|_{\substack{t_i,\,t_j=-\beta_m^{-1}\\ t_k=0,\text{ else}}} + \sum_{i=1}^N \Psi_{\mathbf{\Gamma}}(t_1,\ldots,t_N)\Big|_{\substack{t_i=-2\beta_m^{-1}\\ t_k=0,\text{ else}}}\right] - \mu_m^2. \tag{20}$$

Since the channel gains of different transmit-receive antenna pairs are i.i.d., $\Psi_{\mathbf{\Gamma}}$ simplifies to

$$\begin{aligned}\Psi_{\mathbf{\Gamma}}(t_1,\ldots,t_N) &= \left(\mathbb{E}\left[e^{\sum_{i=1}^N t_i|h_i(1,1)|^2}\right]\right)^{\max(N_t,N_r)},\\ &= \left(\mathbb{E}\left[e^{\mathbf{g}_{11}^{\mathrm{T}}\mathbf{S}\mathbf{g}_{11}}\right]\right)^{\max(N_t,N_r)},\end{aligned} \tag{21}$$

where $\mathbf{S} = \begin{bmatrix}\text{diag}([t_1,\ldots,t_N]) & \mathbf{0}_N\\ \mathbf{0}_N & \text{diag}([t_1,\ldots,t_N])\end{bmatrix}$ and $\mathbf{g}_{11} = \left[\text{Re}(\boldsymbol{h}_{11})^{\mathrm{T}}\ \text{Im}(\boldsymbol{h}_{11})^{\mathrm{T}}\right]^{\mathrm{T}}$. Recall that $\boldsymbol{h}_{11}$ is defined in Section III-A2. Since $\mathbf{g}_{11}$ is a zero-mean Gaussian random vector with covariance matrix $\mathbf{K}$, we can show that $\mathbb{E}\left[e^{\mathbf{g}_{11}^{\mathrm{T}}\mathbf{S}\mathbf{g}_{11}}\right] = \det(\mathbf{I}_{2N} - 2\mathbf{KS})^{-\frac{1}{2}}$. Substituting this in (21), and then in (19) and (20) yields the desired expressions.

C. Upper Bound on Average Throughput

The probability of selecting MCS m is upper bounded by

$$\begin{aligned}&P\left(\gamma_{\text{eff}}^{(m)} \geq T_m, \gamma_{\text{eff}}^{(m+1)} < T_{m+1},\ldots,\gamma_{\text{eff}}^{(L)} < T_L\right)\\ &\qquad\leq P\left(\gamma_{\text{eff}}^{(m)} \geq T_m, \gamma_{\text{eff}}^{(m+1)} < T_{m+1}\right).\end{aligned} \tag{22}$$

It can be shown that $\gamma_{\text{eff}}^{(m)}$ is a strictly increasing function of β_m, except when γ_is are all equal [17]. It then follows that

$$\begin{aligned}&P\left(\gamma_{\text{eff}}^{(m)} \geq T_m, \gamma_{\text{eff}}^{(m+1)} < T_{m+1}\right)\\ &\quad= P\left(T_m \leq \gamma_{\text{eff}}^{(m)} < \gamma_{\text{eff}}^{(m+1)} < T_{m+1}\right),\\ &\quad\leq \min\Big\{P\left(T_m \leq \gamma_{\text{eff}}^{(m)} < T_{m+1}\right),\\ &\qquad\qquad P\left(T_m \leq \gamma_{\text{eff}}^{(m+1)} < T_{m+1}\right)\Big\}.\end{aligned} \tag{23}$$

Equation (23) follows because both $P\left(T_m \leq \gamma_{\text{eff}}^{(m)} < T_{m+1}\right)$ and $P\left(T_m \leq \gamma_{\text{eff}}^{(m+1)} < T_{m+1}\right)$ are upper bounds. Substituting (23) in (13), and expressing the probabilities in terms of the incomplete Beta function using (8) yields (15).

D. Averge Throughput Approximation

We lower bound the expression in (22) to obtain an approximation. This is an approximation because it lower bounds an upper bound. The lower bound is obtained as follows:

$$\begin{aligned}&P\left(\gamma_{\text{eff}}^{(m)} \geq T_m, \gamma_{\text{eff}}^{(m+1)} < T_{m+1}\right)\\ &\quad= P\left(\gamma_{\text{eff}}^{(m)} \geq T_m\right) - P\left(\gamma_{\text{eff}}^{(m)} \geq T_m, \gamma_{\text{eff}}^{(m+1)} \geq T_{m+1}\right),\\ &\quad\geq P\left(\gamma_{\text{eff}}^{(m)} \geq T_m\right) - P\left(\gamma_{\text{eff}}^{(m+1)} \geq T_{m+1}\right).\end{aligned} \tag{24}$$

Substituting (24) in (13), and expressing the probabilities in terms of the incomplete Beta function using (8) yields (16).

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