Throughput-Optimal Rate Adaptation for Best-M Feedback in OFDM Systems

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***Abstract*—In rate-adaptive orthogonal frequency division multiplexing (OFDM) systems, limited feedback schemes are essential to reduce the number of subchannels for which the channel state information is fed back by the users. For the practically important best-M scheme, in which each user feeds back only its M strongest subchannels and their indices to the base station (BS), we derive a throughput-optimal rate adaptation policy that enables the BS to assign rates to the subchannels of every user. We present it in closed-form for the widely used exponential correlation model. The novelty of the policy lies in its exploitation of the structure of the information fed back by the best-M scheme and the correlation among subchannel gains. We also present a near-optimal, lower computational complexity approach. In effect, our approach facilitates rate adaptation and scheduling by the BS even on subchannels that are not fed back by a user due to feedback constraints. For various schedulers, we show that it improves the downlink throughput compared to several conventional approaches, without requiring any additional feedback.**

I. INTRODUCTION

Rate adaptation is an indispensable technique that enables current and next generation wireless standards such as long term evolution (LTE) and LTE-advanced (LTE-A) to achieve high spectral efficiencies. In it, the transmit rate is adapted based on the channel conditions. To do so, the base station (BS) needs channel state information (CSI). This must be fed back by the users in the uplink when the uplink and downlink channels are either not reciprocal, as is the case in frequency-division-duplex systems, or not symmetric, as is the case in time-division-duplex systems with asymmetric uplink and downlink interferences. For example, in LTE, the BS transmits to a user over one or more groups of twelve contiguous subcarriers, which we shall refer to as subchannels. For some or all subchannels, a user feeds back to the BS the index of one modulation and coding scheme (MCS) among sixteen pre-specified ones that it can reliably receive [1, Chap. 10].

In order to reduce the significant uplink feedback overhead, many *limited feedback schemes* have been studied in the orthogonal frequency division multiplexing (OFDM) literature. These include threshold-based feedback scheme [2], one-bit feedback scheme [3], subcarrier clustering [4], and best-M scheme [5], [6]. In the best-M scheme, which is the focus of this paper, a user feeds back or *reports* the M largest subchannel signal-to-noise-ratios (SNRs) along with the subchannel indices. It is practically important because its variant has been adopted in LTE [1]. Small values of M are preferred to keep the feedback overhead low [7]. However, reducing M lowers the downlink throughput because it increases the odds that few or even zero users report a subchannel, which limits the ability of the scheduler at the BS to exploit multi-user diversity.

A. Contributions

In this paper, we address the relatively less studied problem of transmitting reliably and at higher rates on subchannels regardless of the number of users that report them. To this end, we develop a novel, throughput-optimal rate adaptation policy for the best-M scheme that enables the BS to determine the best possible transmit rates for reported as well as unreported subchannels. Our contributions are as follows:

- We first show that the optimal MCS to be assigned to a subchannel is the one that maximizes the product of its rate and the conditional probability of its successful transmission given the CSI fed back by the best-M scheme. We then derive the optimal policy in closed-form. We do so first for the insightful scenario in which the subchannel gains are mutually independent and thereafter for the widely studied exponential correlation model [8]–[10]. These bring out how the structure of the limited feedback generated by the best-M scheme and the subchannel correlation are both exploited by the optimal policy.
- We then develop two approaches to reduce the complexity in computing the optimal rate. First, we use the Markov property of the exponential correlation model [9], to show that the optimal rate depends on at most three SNRs reported by the user, regardless of the value of M. Our second approach, called *clipping*, is motivated by the decay of the correlation between subchannels as their separation increases. In it, the correlation between subchannels that are separated by more than N_w subchannels is ignored. The parameter N_w trades off between complexity and numerical accuracy of the rate computed.
- We also evaluate the system-level impact of the proposed policy for two different schedulers, which cover a wide range of the trade-off between throughput and user fairness. We see that it achieves a higher throughput than the approaches pursued in [5], [6], [11] for both schedulers. While the approach in [10], [12] turns out to be near-optimal, it requires a careful adjustment of a rate backoff parameter that is not required by the optimal policy.

B. Related Literature on Rate Adaptation

Throughput-optimal rate adaptation when CSI is available at the BS without error is studied in [13, Chap. 9]. In it, the BS

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simply selects the highest rate MCS that can be transmitted successfully given the channel conditions. In [14]–[16], rate adaptation when the channel gains fed back by the user are corrupted by additive Gaussian noise is studied. In [14], the transmit rate is adapted to maintain a target bit error rate. Transmit rate and power are jointly adapted in [15] subject to a constraint on the outage probability. User scheduling is considered in addition in [16]. Rate and power adaptation algorithms to maximize the expected goodput, which is the average rate at which data is delivered without error, in the presence of CSI noise with an arbitrary distribution are developed in [17]. While continuous rate adaptation is assumed in [14]–[16], discrete rate adaptation is considered in [17].

However, the above works do not consider subchannel correlation or the best-M scheme. Unlike the best-M scheme, in which the BS has the CSI of only M subchannels, the CSI of all the subchannels, albeit imperfect, is available at the BS in the above works. This makes their modeling, analyses, and results very different from ours. In [10], [12], a minimum mean square error (MMSE) estimate of the subchannel SNR is generated for the unreported subchannels. However, this approach is sub-optimal for maximizing the throughput because the mean square error criterion does not capture the fact that if the SNR is overestimated and leads to the choice of a rate that exceeds the capacity of the subchannel, then the packet cannot be decoded. To overcome this shortcoming, an ad hoc rate backoff technique is employed. In it, an MCS whose rate is lower than that indicated by the subchannel SNR estimate is assigned in order to improve the odds that it can be decoded by the receiver. However, determining the optimal rate backoff is difficult because it depends on M, subchannel correlation, scheduler, and the number of users served by the BS.

C. Organization and Notations

The paper is organized as follows. The system model is discussed in Section II. The throughput-optimal rate adaptation policy is developed in Section III. The simulation results are presented in Section IV. Our conclusions follow in Section V.

Notations: The probability of an event A is denoted by $\mathbb{P}(A)$. The conditional probability of A given B is denoted by $\mathbb{P}(A|B)$. The probability density function (PDF) of a random variable (RV) X is denoted by $f_X(\cdot)$. Expectation over RV X is denoted by $\mathbb{E}_X[\cdot]$ and the expectation conditioned on an event A by $\mathbb{E}[\cdot|A]$. Let $\mathbf{1}(\cdot)$ denote the indicator function. Let $|c|$ and $\overline{c}$ denote the absolute value and complex conjugate of c, respectively. The complement of a set $\mathcal{A}$ is denoted by $\mathcal{A}^c$.

II. System Model

We consider an OFDM cellular system with one BS and K users, each equipped with a single antenna. The system bandwidth is divided into N orthogonal subchannels. We focus on the single cell model, as has been done in [11], [14]–[17].

Channel Model: Let $H_{k,n}$ denote the complex baseband channel gain from the BS to user k for subchannel n. We assume Rayleigh fading. Thus, $H_{k,n}$ is a circularly symmetric complex Gaussian RV with zero mean and variance Ω_k.

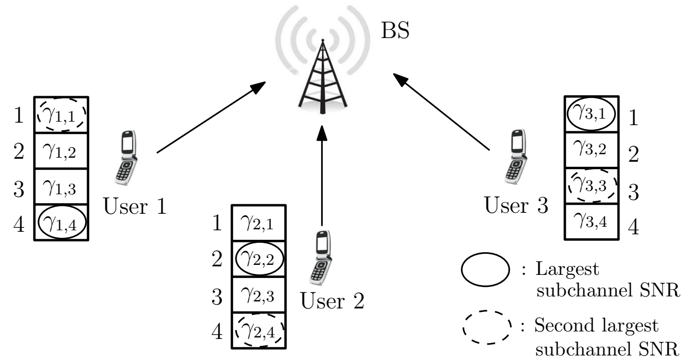

Fig. 1. Illustration of Best-M scheme for $M = 2$, $K = 3$ users, and $N = 4$ subchannels. Users report the circled SNRs and their indices to the BS.

Therefore, for subchannel n of user k, the SNR $\gamma_{k,n} = |H_{k,n}|^2$ is an exponentially distributed RV with mean Ω_k. The subchannel SNRs of a user are statistically identical but correlated. This follows from the uncorrelated scatterers assumption [13, Chap. 3]. The subchannel SNRs of different users are mutually independent because the users are located sufficiently far apart from each other relative to the wavelength.

The correlation across subchannels of any user is given by the exponential correlation model. Here, the covariance of $H_{k,n}$ and $H_{k,m}$ is $\mathbb{E}\left[H_{k,n}\overline{H}_{k,m}\right] = \Omega_k \rho^{|n-m|}$, where ρ is the correlation coefficient. This model is widely used because it is tractable and it captures the decrease in correlation between the subchannels as their separation increases [8]–[10], [12]. We note that the joint PDF of the subchannel SNRs for an arbitrary power delay profile is intractable as it involves multiple integrals [8]. The joint PDF of the subchannel SNRs $\gamma_{k,1}, \ldots, \gamma_{k,N}$ of a user k is then given by [9]

$$f_{\gamma_{k,1},\ldots,\gamma_{k,N}}(x_1,\ldots,x_N) = \frac{\exp\left(-\frac{x_1 + x_N + (1+\rho^2)\sum_{m=2}^{N-1} x_m}{\Omega_k(1-\rho^2)}\right)}{\Omega_k^N (1-\rho^2)^{N-1}} \times \sum_{i=0}^{\infty} \delta^i \sum_{\substack{0 \le l_1 \le \cdots \le l_{N-1} \le i \\ l_1 + l_2 + \cdots + l_{N-1} = i}} \frac{x_1^{l_1} x_2^{l_1+l_2} \ldots x_{N-1}^{l_{N-2}+l_{N-1}} x_N^{l_{N-1}}}{\left(l_1! l_2! \ldots l_{N-1}!\right)^2}, \quad \text{for } x_j \ge 0,\ j = 1,\ldots,N, \tag{1}$$

where $\delta = \rho^2 \Omega_k^{-2}\left(1-\rho^2\right)^{-2}$. The subchannel SNRs are mutually independent for $\rho = 0$.

Best-M Feedback Scheme [11], [12], [18]: In it, each user orders its subchannel SNRs. For a user k, they are denoted as $\gamma_{k,i_1^{(k)}} \ge \cdots \ge \gamma_{k,i_N^{(k)}}$, where $i_r^{(k)}$ indexes the subchannel with the r^{th} largest SNR of user k. User k then feeds back its M largest subchannel SNRs, $\mathbf{\Gamma}_{k,M} = \left[\gamma_{k,i_1^{(k)}}, \ldots, \gamma_{k,i_M^{(k)}}\right]$, along with their indices $\mathbf{I}_{k,M} = \left[i_1^{(k)}, \ldots, i_M^{(k)}\right]$ to the BS. In practice, the index of the MCS is fed back [1]. However, modeling this is beyond the scope of the paper. The users are assumed to know their subchannel SNRs without error and the feedback channel is assumed to be error-free [2]–[5].

Discrete Rate Adaptation: We consider discrete rate adaptation, since it is inevitably used in practice [1], [13]. The

BS has available to it L MCSs indexed $1, 2, \ldots, L$ with rates $0 = R_1 < R_2 < \cdots < R_L$. An SNR threshold T_l is associated with MCS l such that the transmission to user k at rate R_l on subchannel n is successful if $\gamma_{k,n} > T_l$; else an outage occurs. We set $T_1 = 0$ and $T_{L+1} = \infty$.

III. Throughput-Optimal Rate Adaptation

The BS uses the best-M feedback it receives from a user to determine the optimal rate with which to transmit to the user on each subchannel, which we derive below. For notational simplicity, we drop the user from the notations in this section.

Let random vectors $\mathbf{\Gamma}_M$ and $\mathbf{I}_M$ denote the vector of reported subchannel SNRs and the subchannel indices, respectively. Let $\Gamma_M = [x_{i_1}, \ldots, x_{i_M}]$ and $I_M = [i_1, \ldots, i_M]$ denote a realization of $\mathbf{\Gamma}_M$ and $\mathbf{I}_M$, respectively. For subchannel n, a rate adaptation policy π_n maps the ordered pair (Γ_M, I_M) to an element of the set of MCS indices $\{1, 2, \ldots, L\}$. Let Δ denote the set of all rate adaptation policies.

Let $\Psi_n(\Gamma_M, I_M)$ denote the throughput on subchannel n conditioned on the fed back CSI (Γ_M, I_M). It is given by

$$\Psi_n(\Gamma_M, I_M) = \mathbb{E}_{\gamma_n}\left[R_{\pi_n} \mathbf{1}\left(\gamma_n \geq T_{\pi_n}\right) | \Gamma_M, I_M\right], \quad (2)$$

where R_{π_n} is the transmit rate on subchannel n and the indicator function specifies whether the transmission is successful or not. The following lemma gives the optimal policy.

Lemma *1:* The throughput-optimal MCS $\pi_n^*(\Gamma_M, I_M)$ for subchannel n given the best-M feedback (Γ_M, I_M) is

$$\pi_n^*(\Gamma_M, I_M) = \underset{1 \leq l \leq L}{\operatorname{argmax}} \left\{R_l \mathbb{P}\left(\gamma_n \geq T_l | \Gamma_M, I_M\right)\right\}. \quad (3)$$

Proof: The proof is relegated to Appendix A. ■

Comments: If subchannel n has been reported, then $\mathbb{P}\left(\gamma_n \geq T_l | \Gamma_M, I_M\right)$ is 1 if $\gamma_n \geq T_l$ and 0, otherwise. Therefore, for a reported subchannel, the optimal rate adaptation reduces to classical rate adaptation in which the BS assigns rate R_l to subchannel n if $\gamma_n \in [T_l, T_{l+1})$ [13, Chap. 9]. For an unreported subchannel, the optimal policy looks similar to that in [19], which selects the MCS that maximizes the product of the rate and the probability of success. However, [19] assumes complete CSI at the BS and, thus, does not condition on the limited CSI available from the best-M scheme.

We now present the optimal policy in an explicit closed-form. We first do so for the analytically insightful and simple case where the subchannels of a user are mutually independent. This arises in practice if the subchannel bandwidth exceeds the coherence bandwidth of the channel. Thereafter, we generalize the results to exponentially correlated subchannels.

A. Independent Subchannels

Result *1:* The throughput-optimal MCS $\pi_n^*(\Gamma_M, I_M)$ for an unreported subchannel n given the CSI from the best-M scheme $\Gamma_M = [x_{i_1}, \ldots, x_{i_M}]$ and $I_M = [i_1, \ldots, i_M]$ is

$$\pi_n^*(\Gamma_M, I_M) = \underset{1 \leq l \leq L}{\operatorname{argmax}} \left\{\frac{R_l \left[\exp\left(-\min\left(\frac{T_l}{\Omega}, \frac{x_{i_M}}{\Omega}\right)\right) - \exp\left(-\frac{x_{i_M}}{\Omega}\right)\right]}{1 - \exp\left(-\frac{x_{i_M}}{\Omega}\right)}\right\}. \quad (4)$$

Proof: The proof is relegated to Appendix B. ■

Observations: The optimal MCS is independent of the number of subchannels N and the subchannel index n. Thus, it is the same for all the unreported subchannels. Also, it depends on the fed back CSI only through the least reported SNR x_{i_M}.

It can be shown that $\pi_n^*(\Gamma_M, I_M)$ is a monotonically non-decreasing function of x_{i_M} and it saturates to $l^\infty = \operatorname{argmax}_{1 \leq l \leq L}\left\{R_l \exp\left(-T_l/\Omega\right)\right\} \leq L$ as x_{i_M} tends to infinity. Since the set of MCSs is finite, this implies that there are thresholds T_l', for $l = 1, \ldots, l^\infty$, such that the optimal rate is R_l for $x_{i_M} \in [T_{l-1}', T_l']$. Further, it can be shown that, for $1 \leq l \leq l^\infty$, $T_l' = \Omega \log\left(R_l - R_{l-1}\right)$ $-\Omega \log\left(R_l \exp\left(-T_l/\Omega\right) - R_{l-1} \exp\left(-T_{l-1}/\Omega\right)\right)$.

B. Exponentially Correlated Subchannels

Consider an unreported subchannel n. Let the reported subchannels that are nearest to n and respectively lower and higher than n be denoted by n_l and n_h. The corresponding SNRs are denoted by x_{n_l} and x_{n_h}. If there is no lower reported subchannel, we set $n_l = 0$ and $x_{n_l} = 0$. Similarly, if there is no higher reported subchannel, we set $n_h = N+1$ and $x_{n_h} = 0$. Then, we have the following key result.

Result *2:* The throughput-optimal MCS $\pi_n^*(\Gamma_M, I_M)$ for an unreported subchannel n given the CSI from the best-M scheme $\Gamma_M = [x_{i_1}, \ldots, x_{i_M}]$ and $I_M = [i_1, \ldots, i_M]$ is

$$\pi_n^*(\Gamma_M, I_M) = \underset{1 \leq l \leq L}{\operatorname{argmax}} \left\{R_l \frac{A_{l,n}}{B_{l,n}}\right\}, \quad (5)$$

where

$$A_{l,n} = \sum_{i=0}^{\infty} \delta^i \sum_{\substack{0 \leq q_{n_l}, \ldots, q_{n_h-1} \leq i \\ q_{n_l} + \ldots + q_{n_h-1} = i}} \frac{\left(x_{n_l}\right)^{q_{n_l}} \left(x_{n_h}\right)^{q_{n_h-1}}}{\eta_n \prod_{r=n_l}^{n_h-1} \left(q_r!\right)^2 \eta_r^{q_{r-1}+q_r+1}} \times \prod_{\substack{r=n_l+1, \\ r \neq n}}^{n_h-1} \Gamma_{\text{inc}}\left(\eta_r x_{i_M}, q_{r-1}+q_r+1\right) \left[\Gamma_{\text{inc}}\left(\eta_n x_{i_M}, q_{n-1}+q_n+1\right) - \Gamma_{\text{inc}}\left(\eta_n \min\left(T_l, x_{i_M}\right), q_{n-1}+q_n+1\right)\right], \quad (6)$$

$$B_{l,n} = \sum_{i=0}^{\infty} \delta^i \sum_{\substack{0 \leq q_{n_l}, \ldots, q_{n_h-1} \leq i \\ q_{n_l} + \ldots + q_{n_h-1} = i}} \frac{\left(x_{n_l}\right)^{q_{n_l}} \left(x_{n_h}\right)^{q_{n_h-1}}}{\eta_n \prod_{r=n_l}^{n_h-1} \left(q_r!\right)^2 \eta_r^{q_{r-1}+q_r+1}} \times \prod_{r=n_l+1}^{n_h-1} \Gamma_{\text{inc}}\left(\eta_r x_{i_M}, q_{r-1}+q_r+1\right), \quad (7)$$

$\Gamma_{\text{inc}}(\cdot, \cdot)$ is the incomplete gamma function [20, Tbl. 6.5], $\eta_2 = \cdots = \eta_{N-1} = (1+\rho^2)/(\Omega(1-\rho^2))$, and $\eta_1 = \eta_N = 1/(\Omega(1-\rho^2))$.

Proof: The proof is relegated to Appendix C. ■

Notice that the optimal MCS for subchannel n depends only on x_{n_l}, x_{n_h}, and x_{i_M}. Also, the inner summation in (6) and (7) is over $n_h - n_l$ variables unlike the inner summation in (1), which is over $N-1$ variables. This is a consequence of the Markov property of the exponential correlation model [9]. It states that conditioned on $\gamma_{k,j}$, for $1 < j < N$,

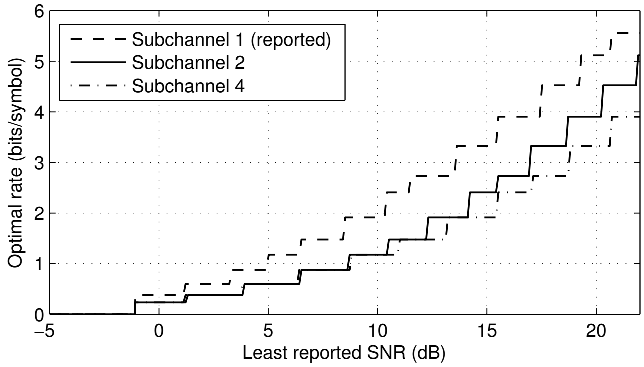

Fig. 2. Correlated subchannels: Optimal rate against the least reported SNR for different subchannels ($N = 4$, $M = 1$, and $\Omega = 14$ dB).

the subchannel SNRs $\gamma_{k,1}, \ldots, \gamma_{k,j-1}$ are independent of $\gamma_{k,j+1}, \ldots, \gamma_{k,N}$. This affords significant savings in numerically computing the optimal MCS as the number of terms in the inner summation drops from $(N+i-2)!/(i!(N-2)!)$ to $(i+n_h-n_l-1)!/(i!(n_h-n_l-1)!)$. Unlike the independent subchannel scenario, the optimal MCS is different for different subchannels.

We now present a visualization of the optimal policy. We plot in Figure 2 the optimal rate as a function of the least reported SNR for $M = 1$ and $N = 4$. We set subchannel 1 as the reported subchannel. The $L = 16$ rates are as specified in LTE [1, Table 10.1]. These range from $R_2 = 0.15$ bits/symbol to $R_{16} = 5.55$ bits/symbol. The threshold T_l is calculated using the formula [12], [18]: $R_l = \log_2(1 + \zeta T_l)$, where $\zeta = 0.398$ accounts for the coding loss of a practical code. The curve for subchannel 3 is skipped to reduce clutter. We see that the optimal rate is monotonically non-decreasing in the least reported SNR for all the subchannels. For a fixed value of the least reported SNR, the optimal rate decreases as the separation from the reported subchannel increases. This shows how the optimal policy takes subchannel correlation into account in estimating the transmit rates.

C. Reducing Computational Complexity

In order to reduce the computational complexity further, we next propose a clipping approach. It is motivated by the decay of the correlation between subchannels as their separation increases. In it, the subchannels that are away by more than N_w are ignored in computing the transmit rate for subchannel n. This reduces the rate adaptation problem with N subchannels to one that involves at the most $2N_w + 1$ subchannels. The parameter N_w trades off between computational complexity and numerical accuracy.

The following four different cases can occur:

1) $n - n_l \leq N_w$ and $n_h - n \leq N_w$. Here, the optimal rate is still given by (5).
2) $n - n_l \leq N_w$ and $n_h - n > N_w$: Here, subchannels with indices higher than $n + N_w$ are ignored. The optimal rate is then computed using (5) with n_h set to $n + N_w + 1$ and x_{n_h} set to 0.
3) $n_h - n \leq N_w$ and $n - n_l > N_w$: This case is similar to the previous one except that the optimal rate is computed with n_l set to $n - N_w - 1$ and x_{n_l} set to 0.
4) $n - n_l > N_w$ and $n_h - n > N_w$: Here, subchannel n is away from all the reported subchannels by more than N_w subchannels. Therefore, the optimal rate is computed using (5) with n_l and n_h set to $n - N_w - 1$ and $n + N_w + 1$, respectively. Also, x_{n_l} and x_{n_h} are set to 0.

We now present a numerical example to illustrate the significant reduction in computational complexity. Consider the case $N = 10$ and $M = 2$ with the first and fifth subchannels being the reported subchannels. Then, for the $i = 10^{\text{th}}$ term in the infinite series, the Markov property reduces the number of terms in the inner summation to 286 from 43758. The clipping approach with $N_w = 1$ reduces it even further to 11.

IV. Simulation Results and System-Level Impact

We now present Monte Carlo simulations to evaluate the system-level throughput achieved by the proposed rate adaptation policy. The BS uses the proposed policy to assign rates to the subchannels of different users, which is then followed by user scheduling. We consider the following schedulers, which trade-off between system throughput and fairness differently:

- *Round Robin (RR) Scheduler:* It schedules users in a periodic, predetermined, and channel-agnostic manner. It does not exploit multi-user diversity.
- *Greedy Scheduler:* It selects the user with the highest assigned rate for a subchannel. In case multiple users have the same highest rate, one among them is chosen with equal probability.

Results for other schedulers such as proportional fair (PF) scheduler are not shown due to space constraints.

We benchmark the proposed optimal policy against the following approaches that have been employed in the literature.

- *Outage Approach [4]–[6]:* In this conventional approach, the BS does not transmit on subchannels that were not reported by any user.
- *Data Method [11]:* A subchannel that is not reported by any user is assigned to the user selected for its adjacent subchannel. Its MCS is one level lower than that assigned to the adjacent subchannel. If no adjacent subchannel has been reported by any user, then an outage occurs.
- *MMSE Approach [10], [12]:* As described in Section I-B, the fed back CSI is used to generate an MMSE estimate of an unreported subchannel SNR. It is then used for rate adaptation and scheduling.
- *Full CSI:* The BS is assumed to know all the subchannel SNRs of all the users. While unrealistic, this provides an upper limit on the throughput achievable by any approach.

We employ the clipping approach described in Section III-C with $N_w = 1$. Increasing N_w further makes a negligible difference for $\rho \leq 0.9$. We have found that 25 terms of the infinite series in (5) for $\rho = 0.5$ and 65 terms for $\rho = 0.75$ are sufficient to ensure numerical accuracy. We show results for users with statistically identical channel gains.

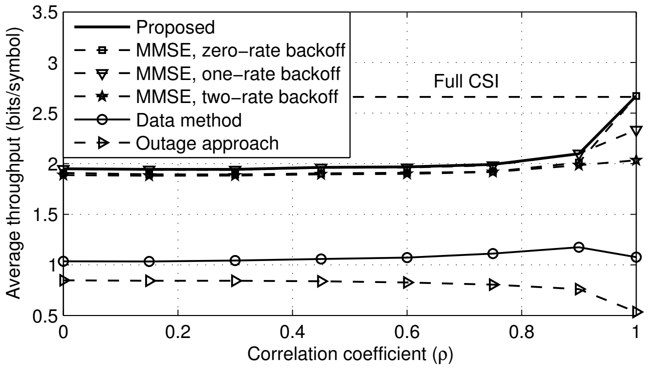

Fig. 3. RR scheduler and $M = 2$: Zoomed-in view of the throughput benchmarking as a function of ρ ($K = 10$, $N = 10$, and $\Omega_k = 14$ dB).

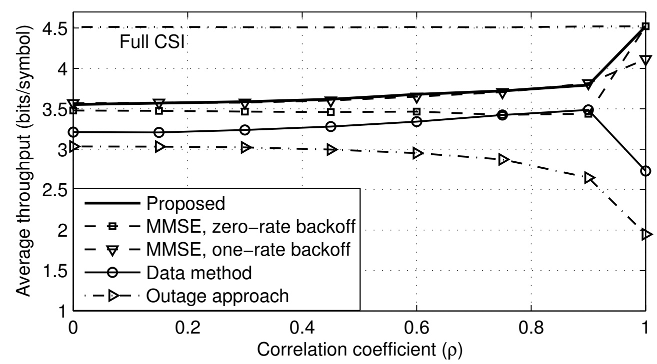

Fig. 4. Greedy scheduler and $M = 1$: Zoomed-in view of the throughput benchmarking as a function of ρ ($K = 10$, $N = 10$, and $\Omega_k = 14$ dB).

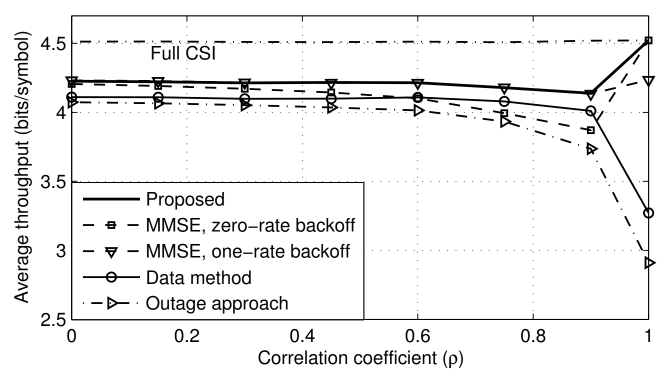

Fig. 5. Greedy scheduler and $M = 2$: Zoomed-in view of the throughput benchmarking as a function of ρ ($K = 10$, $N = 10$, and $\Omega_k = 14$ dB).

RR Scheduler: Figure 3 plots the throughputs of the different approaches as a function of ρ for $M = 2$. We see that even at $\rho = 0$, the proposed policy improves the throughput by 88% and 130% compared to the data method and outage approach, respectively. The corresponding gains are 79% and 176% at $\rho = 0.9$. The throughput of the outage approach decreases as ρ increases due to a loss in frequency diversity. The throughput of the data method marginally increases as ρ increases because the estimate of the rate assigned to an unreported subchannel becomes more accurate. However, when ρ is close to 1, its throughput decreases because of the rate backoff it employs.

For the MMSE approach, results with zero, one, and two rate backoffs, in which the rate assigned is zero, one, and two levels lower than the rate that the estimated SNR can support, are shown. We see that two-rate backoff is too conservative and is sub-optimal. While one-rate backoff works best for $\rho \leq 0.9$, zero-rate backoff works best for $\rho > 0.9$. This is because at lower correlations, the one-rate backoff reduces the odds that the BS, on the basis of the MMSE estimate, selects a higher rate than can be supported by the subchannel. At very high correlations, however, no such backoff is needed since the MMSE estimate becomes accurate. While the performance of the MMSE approach is comparable to the optimal policy, it is so only when its rate backoff is carefully adjusted as a function of ρ. In general, this adjustment depends on various factors such as M, ρ, and the scheduler used. No such adjustment is required in the optimal policy.

Greedy Scheduler: Figures 4 and 5 plot the throughputs of the different approaches as a function of ρ for $M = 1$ and 2, respectively. We see that for $M = 1$, the proposed policy improves the throughput by 11% and 18% at $\rho = 0$ compared to the data method and outage approach, respectively. The corresponding gains at $\rho = 0.9$ are 9% and 44%. The gains for $M = 2$ are lower than for $M = 1$ in the previous figure since more CSI is available at the BS, which improves the throughput of the benchmark approaches. Even then, the gains are significant at high correlations. For example, at $\rho = 0.9$, the corresponding gains are 3% and 11%. For the MMSE approach, the curve for two-rate backoff is not shown to reduce clutter. Again, the rate backoff needs to be adjusted in order for its throughput to be close to the optimal policy.

V. Conclusions

We proposed a novel, throughput-optimal rate adaptation policy, which incorporates the structure of best-M feedback and subchannel correlation to determine the transmit rates for all the subchannels of each user for exponentially correlated subchannels. Further, a near-optimal clipping approach was proposed to reduce the complexity of computing the optimal rate. In view of its optimality, the proposed policy serves as a fundamental benchmark for all schemes. We saw that, compared to the outage approach and the data method, the proposed policy improved the throughput of both RR and greedy schedulers without any additional feedback. Further, our results revealed that the MMSE approach of [10], [12] is near-optimal, but required a careful adjustment of the rate backoff as a function of the scheduler used and the correlation.

Future work involves developing sub-optimal policies with lower computational complexity. Deriving optimal policies for quantized feedback from the users, imperfect CSI on the reported subchannels, and incorporating additional constraints on latency or error-rates at the receiver are other interesting avenues for future work.

Appendix

A. Proof of Lemma 1

The throughput-optimal rate adaptation policy π_n^* maximizes the fading- and feedback-averaged downlink throughput. Therefore, $\pi_n^* = \operatorname{argmax}_{\pi_n \in \Delta} \{\mathbb{E}_{\mathbf{\Gamma}_M, \mathbf{I}_M}[\Psi_n]\}$. For a given policy π_n and given (Γ_M, I_M), R_{π_n} is fixed. Hence, $\Psi_n(\Gamma_M, I_M) = R_{\pi_n}\mathbb{P}(\gamma_n \geq T_{\pi_n}|\Gamma_M, I_M)$. Thus,

$$\pi_n^* = \underset{\pi_n \in \Delta}{\operatorname{argmax}} \{\mathbb{E}_{\mathbf{\Gamma}_M, \mathbf{I}_M}[R_{\pi_n}\mathbb{P}(\gamma_n \geq T_{\pi_n}|\Gamma_M, I_M)]\}. \quad (8)$$

From (8), it follows that the optimal policy should maximize the term inside the expectation for each (Γ_M, I_M). Therefore,

$$\pi_n^*(\Gamma_M, I_M) = \underset{1 \leq l \leq L}{\operatorname{argmax}} \{R_l \mathbb{P}(\gamma_n \geq T_l|\Gamma_M, I_M)\}. \quad (9)$$

Hence the result in (3) follows.

B. Derivation of $\pi_n^*(\Gamma_M, I_M)$ for Independent Subchannels

Writing $\mathbb{P}(\gamma_n \geq T_l|\Gamma_M, I_M)$ in (3) in terms of the reported subchannels and their SNRs, we have

$$\begin{aligned}
&\mathbb{P}(\gamma_n \geq T_l|\Gamma_M, I_M) \\
&\quad = \mathbb{P}(\gamma_n \geq T_l|\gamma_p = x_p, p \in I_M; \gamma_q \leq x_{i_M}, q \in I_M^c), \quad (10)\\
&\quad = \mathbb{P}(\gamma_n \geq T_l|\gamma_n \leq x_{i_M}), \quad (11)
\end{aligned}$$

where the last step follows because the subchannel SNRs are independent. Evaluating the conditional probability above for the exponential RV γ_n yields (4).

C. Derivation of $\pi_n^*(\Gamma_M, I_M)$ for Exponential Correlation

Consider the case $n_l \geq 1$ and $n_h \leq N$. Using the Markov property [9], $\mathbb{P}(\gamma_n \geq T_l|\Gamma_M, I_M)$ in (3) can be written in terms of the reported subchannels and their SNRs as

$$\begin{aligned}
\mathbb{P}(\gamma_n \geq T_l|\Gamma_M, I_M) &= \mathbb{P}(\gamma_n \geq T_l|\gamma_{n_l} = x_{n_l}, \gamma_{n_h} = x_{n_h},\\
&\quad \gamma_p \leq x_{i_M}, \text{ for } p = n_l+1, \ldots, n_h - 1). \quad (12)
\end{aligned}$$

The conditional probability in (12) is zero if $T_l > x_{i_M}$. Otherwise, let $\theta_l = \min(T_l, x_{i_M})$ and $\mathbf{z} = [z_{n_l+1}, \ldots, z_{n_h-1}]$. For an MCS l with $T_l \leq x_{i_M}$, using the Bayes' rule, we get

$$\begin{aligned}
&\mathbb{P}(\gamma_n \geq T_l|\Gamma_M, I_M)\\
&= \frac{\int_0^{x_{i_M}} \cdots \int_{z_n=\theta_l}^{x_{i_M}} \cdots \int_0^{x_{i_M}} f_{\gamma_{n_l},\ldots,\gamma_{n_h}}(x_{n_l}, \mathbf{z}, x_{n_h})\, d\mathbf{z}}{\int_0^{x_{i_M}} \cdots \int_0^{x_{i_M}} f_{\gamma_{n_l},\ldots,\gamma_{n_h}}(x_{n_l}, \mathbf{z}, x_{n_h})\, d\mathbf{z}}. \quad (13)
\end{aligned}$$

The joint PDF of $\gamma_{n_l}, \ldots, \gamma_{n_h}$ is given by (1) with $N = n_h - n_l + 1$. Substituting this in (13) and pooling together the terms with the same variable of integration, we eventually get

$$\mathbb{P}(\gamma_n \geq T_l|\Gamma_M, I_M) = \frac{A_{l,n}}{B_{l,n}}, \quad (14)$$

where

$$\begin{aligned}
A_{l,n} &= \sum_{i=0}^{\infty} \delta^i \sum_{\substack{0 \leq q_{n_l}, \ldots, q_{n_h-1} \leq i \\ q_{n_l} + \ldots + q_{n_h-1} = i}} \frac{(x_{n_l})^{q_{n_l}} (x_{n_h})^{q_{n_h-1}}}{\eta_n \prod_{r=n_l}^{n_h-1} (q_r!)^2 \eta_r^{q_{r-1}+q_r+1}}\\
&\times \prod_{\substack{r=n_l+1,\\ r \neq n}}^{n_h-1} \left[\int_0^{x_{i_M}} z_r^{q_{r-1}+q_r} e^{-\eta_r z_r} dz_r\right]\left[\int_{\theta_l}^{x_{i_M}} z_n^{q_{n-1}+q_n+1} e^{-\eta_n z_n} dz_n\right],
\end{aligned} \quad (15)$$

$$\begin{aligned}
B_{l,n} &= \sum_{i=0}^{\infty} \delta^i \sum_{\substack{0 \leq q_{n_l}, \ldots, q_{n_h-1} \leq i \\ q_{n_l} + \ldots + q_{n_h-1} = i}} \frac{(x_{n_l})^{q_{n_l}} (x_{n_h})^{q_{n_h-1}}}{\eta_n \prod_{r=n_l}^{n_h-1} (q_r!)^2 \eta_r^{q_{r-1}+q_r+1}}\\
&\quad \times \prod_{r=n_l+1}^{n_h-1} \int_0^{x_{i_M}} z_r^{q_{r-1}+q_r} e^{-\eta_r z_r} dz_r. \quad (16)
\end{aligned}$$

Writing (15) and (16) in terms of the incomplete gamma function yields (6) and (7), respectively.

The derivation is similar for the cases $n_l = 0, n_h \leq N$ and $n_l \geq 1, n_h = N + 1$.

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