Joint Evaluation of Reduced Feedback Scheme, Scheduling, and Rate Adaptation in OFDMA Systems with Feedback Delays

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***Abstract*—Orthogonal frequency division multiple access (OFDMA) systems exploit multiuser diversity and frequency-selectivity to achieve high spectral efficiencies. However, they require considerable feedback for scheduling and rate adaptation, and are sensitive to feedback delays. We develop a comprehensive analysis of the OFDMA system throughput as a function of the feedback scheme, frequency-domain scheduler, and discrete rate adaptation rule in the presence of feedback delays. We analyze the popular best-n and threshold-based feedback schemes. We show that for both the greedy and round-robin schedulers, the throughput degradation, given a feedback delay, depends primarily on the fraction of feedback reduced by the feedback scheme and not the feedback scheme itself. Even small feedback delays at low vehicular speeds are shown to significantly degrade the throughput. We also show that optimizing the link adaptation thresholds as a function of the feedback delay can effectively counteract the detrimental effect of delays.**

I. Introduction

In orthogonal frequency division multiple access (OFDMA), the system bandwidth is divided into several orthogonal subchannels. In an OFDMA system, the base station (BS) achieves high spectral efficiency by exploiting the frequency-selective nature of the wireless channel and using rate adaptation. This involves assigning users (UEs) to subchannels on the basis of their instantaneous subchannel gains. For effective scheduling and rate adaptation, the frequency-domain scheduler at the BS ideally needs to know the downlink channel gains of each subchannel for each user served by it. However, in the frequency division duplex (FDD) mode of operation, the uplink and downlink channels are not reciprocal. Therefore, each user needs to feed its channel information back to the BS. This also holds for the time division duplex (TDD) mode when the uplink and downlink interferences are asymmetric. Such extensive feedback is practically infeasible and inefficient.

In order to strike a balance between multiuser diversity gains and feedback overhead, various feedback reduction schemes have been studied in the literature. In [1], a user feeds back channel state information only for the subchannels whose channel power gains exceed a certain threshold. In [2], at most one bit per subchannel is fed back. Thresholding is combined with a random access protocol in [3] and with subcarrier grouping in [4]. In [5], [6], for every subchannel, users with higher subchannel gains send their feedback earlier. On the other hand, in [7], [8], each user only feeds back the indices and subchannel gains of a subset of its subchannels that have the highest gains among all subchannels. Reducing the subset size reduces the feedback, but also the system throughput.

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In addition to the feedback reduction scheme employed, a key issue that affects the performance of the scheduler is feedback delay as it leads to outdated channel estimates. If the BS underestimates the subchannel gain, then the selected user is served at a rate lower than the rate its assigned subchannel can support. On the other hand, if the BS overestimates the rate, then the data may not be decoded correctly resulting in an *outage* in that subchannel. Outdated estimates can also lead to a sub-optimal assignment of subchannels to users.

Related Literature: In [9], [10], closed-form expressions were derived for the throughputs of the threshold-based and the best-n feedback schemes and their variants, but without feedback delays. In [11], the best-n scheme with quantized feedback was analyzed, but only for time-invariant channels. The impact of feedback delay on multiuser diversity was studied in [12], but without considering feedback reduction schemes. In [13], adaptive modulation with outdated channel knowledge was studied, but frequency-selective channels and reduced feedback schemes were not considered. The throughput of a multiuser OFDMA system with imperfect channel state was analyzed in [14]. However, feedback reduction schemes were not considered. [9]–[13] assumed that the users know subchannel gain without estimation error. Further, perfect channel estimates and single cell environment are assumed in most papers [9]–[12], [14]. Thus, while considerable work has been done on feedback reduction for OFDMA systems, a thorough analysis and performance optimization that takes into account the frequency-domain scheduler, rate adaptation scheme, and feedback delays is missing for several popular feedback schemes.

Contributions: In this work, we develop a comprehensive analysis of the impact of feedback delays on OFDMA system throughput. Our analysis subsumes several inter-dependent mechanisms such as the feedback scheme, scheduler, and practical discrete rate adaptation [15]. We analyze the threshold-based and best-n channel feedback schemes with outdated channel information. The threshold-based scheme is relevant because it has been extensively studied in the literature [1], [9] and because it maximizes the sum rate in the asymptotic limit of a large number of users [2]. Similarly, the best-n feedback

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scheme is relevant since its variants are integral components of next generation OFDMA systems such as Long Term Evolution (LTE) [16]. Our analysis leads to novel expressions for the throughput of the above feedback schemes under different schedulers. We also show how the link adaptation thresholds, which drive discrete rate adaptation, should be fine-tuned as a function of the feedback delay. Otherwise, the throughput degradation can be unacceptably large.

The paper is organized as follows. Section II presents the system model. The analysis is developed in Section III. Section IV provides simulation results and is followed by our conclusions in Section V. Proofs are given in the Appendix.

II. System Model

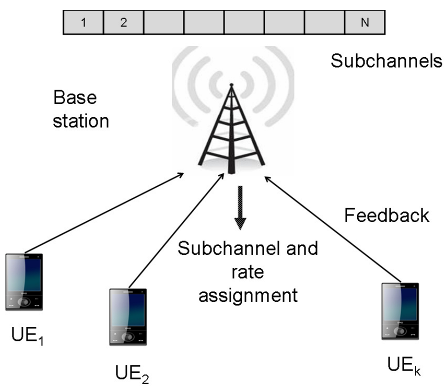

Fig. 1. System model with a base station and k users (UEs). The users feed back channel state information back to the BS, which then assigns subchannels to the users and also determines the transmission rate for each subchannel.

The system bandwidth is divided into N orthogonal subchannels. Let G_{ui} denote the i^{th} subchannel power gain of User u that is used for determining the rate and subchannel assignment. The corresponding subchannel power gain at the time of transmission is G_{ui}^d, where d stands for delay.

We assume a wide sense stationary Rayleigh fading process, which implies that G_{ui} and G_{ui}^d are correlated exponential random variables (RVs), both with mean Ω_u. As per the Jakes' fading model, their correlation coefficient is [17]

$$\rho = J_0^2(2\pi f_d \tau), \tag{1}$$

where f_d is the maximum Doppler spread, τ is the delay between the time instants of rate adaptation and transmission, and $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind [18, Chp. 9]. The joint density $f(G_{ui}, G_{ui}^d)$ of G_{ui} and G_{ui}^d is given by [19, Chp. 6]

$$f_{G_{ui},G_{ui}^d}(x,y)=\frac{e^{-\frac{x+y}{\Omega_u(1-\rho)}}}{\Omega_u^2(1-\rho)} I_0\!\left(\frac{2\sqrt{\rho}\sqrt{xy}}{\Omega_u(1-\rho)}\right), \quad x,y \geq 0, \tag{2}$$

where $I_0(\cdot)$ is the 0^{th} order modified Bessel function of the first kind [18, Chp. 9].

Given the space constraints, the analysis is presented for the case where the users see statistically identical channels. Thus, $\Omega_u = \Omega$, for all users $1 \leq u \leq k$. It can be generalized to handle statistically non-identical users and also handle other schedulers such as the proportional fair (PF) scheduler [20].

A. Assumptions

In order to develop a tractable model that captures the interactions of the scheduler, rate adaptation scheme, and reduced feedback scheme in a wideband system, we make the following assumptions. All of these assumptions are also often employed in the related literature, as explained below.

- We consider a single cell system, as has also been assumed in [4], [9]–[12], [14], [21]–[23].[1]
- The gains across different subchannels are assumed to be independent and identically distributed (i.i.d.). This assumption holds when the coherence bandwidth of the channel is close to the subchannel bandwidth, and has also been assumed in [2], [4], [5], [8], [10], [11], [21].
- The users are assumed to know the subchannel gains without estimation error [9]–[13], [21], and feed the gains back as per the feedback scheme employed. In practice, only the index of the highest rate that the subchannel can support is fed back [11], [16]. The analysis can be generalized to handle the latter scenario, but the details are omitted due to space constraints. Our investigations indicate that subchannel gain feedback overestimates the system throughput by up to 10% for up to 35 users per cell compared to rate index feedback.

B. Schedulers

The BS uses the channel information fed back by the users to decide which user to serve over which subchannel(s). This decision depends on the scheduler used by the BS.

Among the users feeding back, the greedy scheduler selects the user with the highest channel power gain for each subchannel. The round-robin (RR) scheduler instead serves the users in a sequential fashion, thereby ensuring fairness among users. However, it does not exploit multi-user diversity.

C. Discrete Rate Adaptation

We focus on discrete rate adaptation, given that it is used in practice in all rate adaptive systems. The set of available rates is $\{r_1, \ldots, r_{M-1}\}$, where $0 = r_1 < r_2 < \cdots < r_{M-1}$. The channel power gain range is divided into $M-1$ intervals by a set of link adaptation thresholds $T_1, T_2, \ldots, T_M$, where $0 = T_1 < T_2 < \cdots < T_M = \infty$.

For subchannel i, let the selected user be denoted by S_i. If $T_j \leq G_{S_i i} < T_{j+1}$, then S_i is served with rate r_j. Further, if $G_{S_i i}^d < T_j$ then an outage occurs in that subchannel. Thus, the selected user successfully receives at rate r_j if and only if $T_j \leq G_{S_i i} < T_{j+1}$ and $G_{S_i i}^d \geq T_j$ for $2 \leq j \leq M-1$.

D. Reduced Feedback Schemes

Threshold-based feedback: In it, for every subchannel, a user feeds back its subchannel gain only if it exceeds a threshold λ. The threshold λ is chosen so that a fraction f

[1]Our model can be extended to a multi-cell scenario by accounting for co-channel interference through its fading-averaged power [10]. However, with a limited number of co-channel interferers, this extension does have its limitations.

of the users feed back on average. Let the index w be such that $\lambda \in [T_w, T_{w+1})$.

Best-n feedback: In it, each user feeds back the gains and indices of its best n subchannels. For a subchannel i, the BS selects the best user among the subset of users that have reported subchannel i as one of its best n subchannels. Thus, the scheme reduces the feedback overhead of each user to a fraction $f = \frac{n}{N}$ of complete per-subchannel feedback.

III. Subchannel Throughput Analysis

Let η_{G} and η_{RR} denote the average throughputs per subchannel for the greedy and RR schedulers, respectively. $\Pr(A)$ denotes the probability of event A and $\Pr(A|B)$ denotes the conditional probability of A given B. Further, $\{X_i\}_{i=a}^{b}$ denotes the sequence $X_a, X_{a+1}, \dots, X_b$. Further, $\{X_i\}_{i=a}^{b} \geq \lambda$ shall mean that $X_a \geq \lambda, X_{a+1} \geq \lambda, \dots, X_b \geq \lambda$.

A. Threshold-based Feedback

1) Greedy Scheduler: For the threshold-based feedback scheme, feedback occurs on a per-subchannel basis. Since the subchannel gains are statistically identical for each user, it is sufficient to focus on a given subchannel, say the i^{th} one.

Result *1:* The average throughput for the threshold-based feedback scheme with the greedy scheduler is given by

$$\eta_{\text{G}} = \sum_{l=1}^{k} l \binom{k}{l} e^{-\frac{\lambda}{\Omega}(l-1)} \left(1 - e^{-\frac{\lambda}{\Omega}}\right)^{k-l}$$
$$\times \left[r_w \int_{\lambda}^{T_{w+1}} \int_{T_w}^{\infty} \frac{\left[1 - e^{-\frac{x-\lambda}{\Omega}}\right]^{l-1} e^{-\frac{x+y}{\Omega(1-\rho)}} I_0\!\left(\frac{2\sqrt{\rho}\sqrt{xy}}{\Omega(1-\rho)}\right)}{\Omega^2(1-\rho)} dydx \right.$$
$$\left. + \sum_{j=w+1}^{M-1} r_j \int_{T_j}^{T_{j+1}} \int_{T_j}^{\infty} \frac{\left[1 - e^{-\frac{x-\lambda}{\Omega}}\right]^{l-1} e^{-\frac{x+y}{\Omega(1-\rho)}} I_0\!\left(\frac{2\sqrt{\rho}\sqrt{xy}}{\Omega(1-\rho)}\right)}{\Omega^2(1-\rho)} dydx \right]. \quad (3)$$

Further, it is lower bounded by

$$\eta_{\text{G}} \geq \sum_{l=1}^{k} l \binom{k}{l} e^{-\frac{\lambda(l-1)}{\Omega}} \left(1 - e^{-\frac{\lambda}{\Omega}}\right)^{k-l} \sum_{q=0}^{l-1} \frac{(-1)^q e^{\frac{q\lambda}{\Omega}}}{\Omega^2(1-\rho)}$$
$$\times \binom{l-1}{q} \left[r_w \xi_{q,\rho}^{[L]}(\lambda, T_{w+1}; T_w) + \sum_{j=w+1}^{M-1} r_j \xi_{q,\rho}^{[L]}(T_j, T_{j+1}; T_j) \right], \quad (4)$$

where

$$\xi_{q,\rho}^{[L]}(a,b;c) \triangleq \sum_{p=0}^{L} \frac{1}{(p!)^2} \frac{\rho^p}{(\Omega(1-\rho))^{p-1}} \Gamma\left(p+1, \frac{c}{\Omega(1-\rho)}\right)$$
$$\times \left[\gamma\left(p+1, \frac{b(q-\rho q+1)}{\Omega(1-\rho)}\right) - \gamma\left(p+1, \frac{a(q-\rho q+1)}{\Omega(1-\rho)}\right) \right]$$
$$\times \left(\frac{\Omega(1-\rho)}{q-\rho q+1}\right)^{p+1}, \quad (5)$$

and $\gamma(a,b)$ and $\Gamma(a,b)$ are the lower and upper incomplete gamma functions, respectively [18, Chp. 6].

Proof: The derivation is relegated to Appendix A. ∎

The lower bound becomes tighter as L increases. Since it is tight, we show it as being approximately equal to the exact throughput expression henceforth.

2) RR Scheduler: Since the RR scheduler serves the users in a sequential fashion, it can be shown that its throughput is the same as that of a greedy scheduler that serves one user. Therefore, substituting $k=1$ in (4) yields

$$\eta_{\text{RR}} \approx \frac{r_w \xi_{0,\rho}^{[L]}(\lambda, T_{w+1}; T_w) + \sum_{j=w+1}^{M-1} r_j \xi_{0,\rho}^{[L]}(T_j, T_{j+1}; T_j)}{\Omega^2(1-\rho)}. \quad (6)$$

B. Best-n Feedback

Let $\gimel_{ui}$ denote the number of subchannels of user u whose gains exceed the gain of its i^{th} subchannel G_{ui}. If User u reports subchannel i, then at most $n-1$ subchannels of User u can have gains that exceed G_{ui}; thus, $\gimel_{ui} \leq n-1$.

1) Greedy Scheduler: Since the subchannels are statistically identical, we focus on the throughput of the i^{th} subchannel.

Result *2:* The average throughput of the best-n feedback scheme with the greedy scheduler is given by

$$\eta_{\text{G}} \approx \sum_{l=1}^{k} \frac{l}{\Omega^2(1-\rho)} \sum_{j=2}^{M-1} r_j \binom{k}{l} \left(1 - \frac{n}{N}\right)^{k-l} \sum_{p=0}^{L} \frac{1}{(p!)^2}$$
$$\times \sum_{m=0}^{n-1} \binom{N-1}{m} \frac{\rho^p}{(\Omega(1-\rho))^{p-1}} \Gamma\left(p+1, \frac{T_j}{\Omega(1-\rho)}\right)$$
$$\times \int_{T_j}^{T_{j+1}} x^p e^{-x\left(\frac{m}{\Omega} + \frac{1}{\Omega(1-\rho)}\right)} \left(1 - e^{\frac{x}{\Omega}}\right)^{N-1-m} \Upsilon_n^{l-1}(0,x) dx, \quad (7)$$

where

$$\Upsilon_n(a,b) \triangleq \sum_{r=0}^{n-1} \binom{N-1}{r} \sum_{q=0}^{N-1-r} \binom{N-1-r}{q} \frac{(-1)^q}{q+r+1}$$
$$\times \left(e^{-\frac{(q+r+1)a}{\Omega}} - e^{-\frac{(q+r+1)b}{\Omega}}\right). \quad (8)$$

Proof: The derivation is relegated to Appendix B. ∎

The single integral in (7) is evaluated numerically.

2) RR Scheduler: As in Section III-A2, substituting $k=1$ in (7) and simplifying yields

$$\eta_{\text{RR}} \approx \sum_{j=2}^{M-1} r_j \sum_{m=0}^{n-1} \binom{N-1}{m} \sum_{q=0}^{N-m-1} \binom{N-m-1}{q}$$
$$\times \frac{(-1)^q}{\Omega^2(1-\rho)} \xi_{q+m,\rho}^{[L]}(T_j, T_{j+1}; T_j). \quad (9)$$

IV. Simulation Results, Comparisons, and Insights

To independently verify our analytical results, we present results from Monte Carlo simulations that average over 10^5 samples. The time-varying channel is simulated using the modified Jakes' simulator [17] with 512 oscillators. We use $\Omega = 6$ (7.78 dB) and $N = 12$ subchannels in our simulations. The number of terms L used to accurately compute the results depends on the delay. For $f_d\tau \geq 0.15$, $L = 10$ suffices, for $0.06 \leq f_d\tau < 0.15$, $L = 25$ suffices, and for $f_d\tau < 0.06$, $L = 50$ suffices.

The link adaptation thresholds are generated as per

$$r_i = \log_2(1 + \zeta T_i), \quad (10)$$

where ζ models the coding gain loss of a practical code [24]. In our simulations, we set $\zeta = 0.398$ [24]. Since these thresholds are not chosen based on the feedback delay, we shall refer to them as *zero-Doppler thresholds*. The $M - 1 = 16$ rates are selected according to the LTE rate table [16, Tbl. 7.2.3-1], and range from $r_2 = 0.15$ bits/symbol to $r_{16} = 5.55$ bits/symbol.

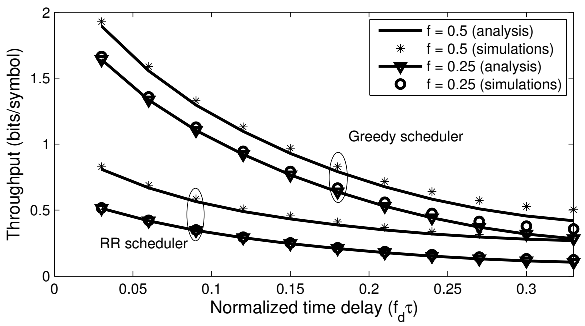

Fig. 2. Average throughput as a function of the normalized feedback delay for the threshold-based feedback scheme ($k = 5$ users).

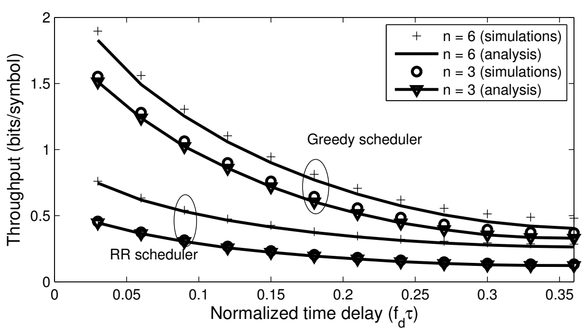

Fig. 3. Average throughput as a function of the normalized feedback delay for the best-n feedback scheme ($k = 5$ users).

The average throughput is plotted as a function of the normalized feedback delay $f_d\tau$ in Fig. 2 for the threshold-based feedback scheme with 5 users for the greedy and RR schedulers for $f = 0.25$ and 0.50. We see that analysis matches simulations well. The minor mismatch occurs because of the limitations of the modified Jakes' simulator in generating multiple independent Rayleigh fading processes. Notice that the throughput degrades markedly as the feedback delay increases. For example, when $f_d\tau = 0.06$, the throughput decreases by 31% compared to the scenario without any delays. This corresponds to a feedback delay of just 1.1 ms when the carrier frequency is 2 GHz and the user speed is 30 kmph.

Figure 3 plots the average throughputs for the best-n scheme of the greedy and RR schedulers as a function of $f_d\tau$ with 5 users for two different values of n. We again observe a good match between analysis and simulations, and the sensitivity of the throughput to $f_d\tau$. Comparing the above two figures, we find that the percentage reduction in the throughput as a function of $f_d\tau$ is almost the same for the two feedback schemes. Thus, the throughput of the two schemes depends primarily on the feedback reduction fraction f. Further, the percentage reduction in throughput for the RR scheduler is lesser than for the greedy scheduler.

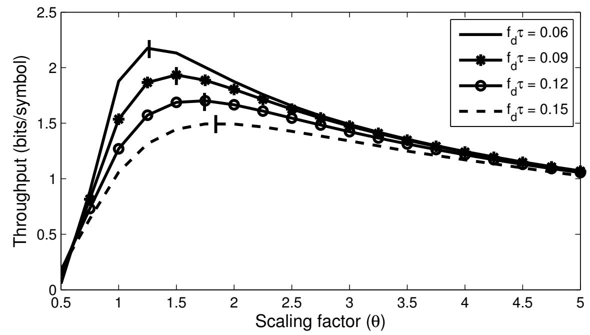

Fig. 4. Average throughput as a function of the link adaptation threshold scaling factor θ for threshold-based feedback scheme ($f = 0.5$) with $k = 10$ users.

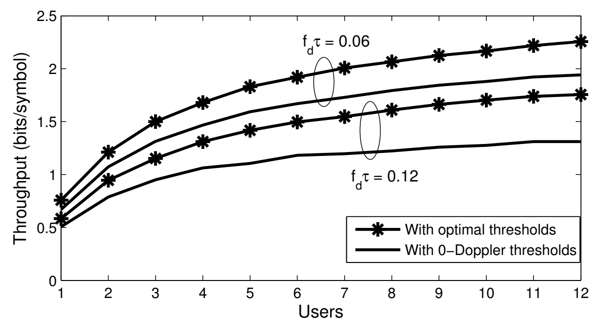

Fig. 5. Average throughput as a function of the number of users for threshold-based feedback scheme ($f = 0.5$) with greedy scheduler using zero-Doppler link adaptation thresholds and optimized link adaptation thresholds.

A. Optimization of Link Adaptation Thresholds

Thus far, we have plotted the throughput when the zero-Doppler thresholds, derived as per (10), are used as link adaptation thresholds. We now investigate how the thresholds should be optimized as a function of the feedback delay. The general problem of optimizing the M adaptation thresholds is analytically intractable and computationally cumbersome.

We, therefore, consider a single-parameter optimization in which the adaptation thresholds are scaled by a factor θ. Thus, the selected user is served with rate r_j if its subchannel gain lies in $[\theta T_j, \theta T_{j+1})$. Recall that an outage occurs only if the subchannel gain at the time of transmission falls below T_j. Figure 4 plots the throughput as a function of θ. It shows that there exists an optimal θ, which is a function of the delay and the number of users in the system, and is indeed different from unity. It increases as the normalized feedback delay increases, which implies a more conservative choice of the thresholds.

Figure 5 compares the average throughput of the threshold-based feedback scheme using the zero-Doppler thresholds and the optimized thresholds. When $f_d\tau = 0.06$ and $k = 5$ users/cell, optimizing the thresholds increases throughput by 15% compared to that with the zero-Doppler thresholds. The gains increase to 34% when $f_d\tau = 0.12$ with 10 users. The behavior is similar for the best-n feedback, and is not shown due to space constraints.

V. Conclusions

In this paper, we analyzed the throughputs of two common feedback reduction schemes that are used in OFDMA, namely, threshold-based and best-n feedback schemes in the presence of feedback delays. The analysis jointly accounted for the frequency-domain scheduler, rate adaptation, feedback reduction scheme, and feedback delay. We saw that the throughput is sensitive to feedback delays and degrades by 17-32% even for small feedback delays at low vehicular speeds. For a given feedback reduction ratio, we saw that the throughput is almost insensitive to the feedback scheme. Further, we saw that adjusting the link adaptation thresholds as a function of the feedback delay can significantly increase throughput. We observed gains as large as 15-35% depending on the number of users and vehicular speed. Future work involves incorporating multiple antenna mode related feedback in the model.

Appendix

A. Proof of Result 1

The probability that the subchannel gains of any l out of k users exceed the threshold λ is $\binom{k}{l}e^{-\frac{\lambda l}{\Omega}}\left(1-e^{-\frac{\lambda}{\Omega}}\right)^{k-l}$. Thus,

$$\eta_{\text{G}} = \sum_{l=1}^{k}\sum_{j=2}^{M-1} r_j \binom{k}{l} e^{-\frac{\lambda l}{\Omega}}\left(1-e^{-\frac{\lambda}{\Omega}}\right)^{k-l} \times \Pr(T_j \le G_{S_i i} < T_{j+1}, G^d_{S_i i} \ge T_j | l \text{ users fed back}). \quad (11)$$

Since the users are i.i.d., and a User u feeds back when $G_{ui} \ge \lambda$, the $(l,j)^{\text{th}}$ probability term in the double summation above can be written as

$$\begin{aligned}
&\Pr(T_j \le G_{S_i i} < T_{j+1}, G^d_{S_i i} \ge T_j | l \text{ users fed back}) \\
&= l\Pr(T_j \le G_{1i} < T_{j+1}, G^d_{1i} \ge T_j, \text{User 1 selected} \\
&\qquad\qquad | \text{Users } 1,2,\ldots,l \text{ fed back}), \\
&= l\Pr(T_j \le G_{1i} < T_{j+1}, G^d_{1i} \ge T_j, G_{1i} \ge \{G_{ui}\}_{u=2}^{l} \\
&\qquad\qquad | \{G_{ui}\}_{u=1}^{l} \ge \lambda, \{G_{vi}\}_{v=l+1}^{k} < \lambda), \\
&= l\Pr(T_j \le G_{1i} < T_{j+1}, G^d_{1i} \ge T_j, G_{1i} \ge \{G_{ui}\}_{u=2}^{l} \\
&\qquad\qquad | \{G_{ui}\}_{u=1}^{l} \ge \lambda). \quad (12)
\end{aligned}$$

When $j < w$, the above term equals 0. When $j = w$, using Baye's rule, and conditioning on the values of G_{1i} and G^d_{1i}, it becomes

$$\begin{aligned}
&l\frac{\int_{\lambda}^{T_{w+1}}\int_{T_w}^{\infty}\Pr(\lambda \le \{G_{ui}\}_{u=2}^{l} \le x) f_{G_{1i},G^d_{1i}}(x,y)\,dy\,dx}{e^{-\frac{\lambda l}{\Omega}}} \\
&= le^{\frac{\lambda}{\Omega}}\int_{\lambda}^{T_{w+1}}\int_{T_w}^{\infty}\left(1-e^{-\frac{x-\lambda}{\Omega}}\right)^{l-1} f_{G_{1i},G^d_{1i}}(x,y)\,dy\,dx. \quad (13)
\end{aligned}$$

Here, we have used the fact that the subchannel gains of different users are independent.

Similarly, for $j \ge w+1$, (12) becomes

$$\begin{aligned}
&l\frac{\int_{T_j}^{T_{j+1}}\int_{T_j}^{\infty}\Pr(\lambda \le \{G_{ui}\}_{u=2}^{l} \le x) f_{G_{1i},G^d_{1i}}(x,y)\,dy\,dx}{e^{-\frac{\lambda l}{\Omega}}} \\
&= le^{\frac{\lambda}{\Omega}}\int_{T_j}^{T_{j+1}}\int_{T_j}^{\infty}\left(1-e^{-\frac{x-\lambda}{\Omega}}\right)^{l-1} f_{G_{1i},G^d_{1i}}(x,y)\,dy\,dx. \quad (14)
\end{aligned}$$

Substituting (13) and (14) in (11) yields (3).

The expression in (3) consists of a sum of double integrals. Each double integral in (3) is of the form: $\Delta(a,b,c) = \int_a^b\int_c^{\infty}\left(1-e^{-\frac{x-\lambda}{\Omega}}\right)^{l-1}\frac{e^{-\frac{x+y}{\Omega(1-\rho)}}}{\Omega^2(1-\rho)} I_0\left(\frac{2\sqrt{\rho}\sqrt{xy}}{\Omega(1-\rho)}\right)dy\,dx$. We simplify it below. The modified Bessel function can be expanded as [18]

$$I_0(z) = \sum_{p=0}^{\infty}\frac{\left(\frac{1}{4}z^2\right)^p}{(p!)^2} \ge \sum_{p=0}^{L}\frac{\left(\frac{1}{4}z^2\right)^p}{(p!)^2}. \quad (15)$$

We replace $I_0(\cdot)$ in $\Delta(a,b,c)$ with its truncated expansion that consist of $L+1$ terms and expand $(1-e^{-\frac{x-\lambda}{\Omega}})^{l-1}$ binomially. Then, $\Delta(a,b,c)$ is lower bounded by the following summation of the products of two single integrals:

$$\begin{aligned}
&\Delta(a,b,c) \\
&\ge \sum_{q=0}^{l-1}\binom{l-1}{q}\frac{(-1)^q e^{\frac{q\lambda}{\Omega}}}{\Omega^2(1-\rho)}\sum_{p=0}^{L}\frac{1}{(p!)^2}\left(\frac{\rho}{\Omega^2(1-\rho)^2}\right)^p \\
&\qquad \times\int_c^{\infty} y^p e^{-\frac{y}{\Omega(1-\rho)}}\,dy\int_a^b x^p e^{-x\left(\frac{q}{\Omega}+\frac{1}{\Omega(1-\rho)}\right)}\,dx. \quad (16)
\end{aligned}$$

Writing the single integrals above in terms of incomplete Gamma functions and simplifying further, we get

$$\Delta(a,b,c) \ge \sum_{q=0}^{l-1}\binom{l-1}{q}\frac{(-1)^q e^{\frac{q\lambda}{\Omega}}}{\Omega^2(1-\rho)}\xi^{[L]}_{q,\rho}(a,b;c), \quad (17)$$

where $\xi^{[L]}_{q,\rho}(\cdot,\cdot;\cdot)$ is defined in the result statement. Substituting the lower bound (17) in (3) yields (4).

B. Proof of Result 2

The probability that l users report subchannel (subch.) i is $\binom{k}{l}\left(\frac{n}{N}\right)^l\left(1-\frac{n}{N}\right)^{k-l}$. Therefore, η_{G} can be written as

$$\eta_{\text{G}} = \sum_{l=1}^{k}\sum_{j=2}^{M-1} r_j\binom{k}{l}\left(\frac{n}{N}\right)^l\left(1-\frac{n}{N}\right)^{k-l} \times\Pr(T_j \le G_{S_i i} < T_{j+1}, G^d_{S_i i} \ge T_j | l \text{ users report } i^{\text{th}} \text{ subch.}). \quad (18)$$

Since the subchannel gains of different users are i.i.d., the $(l, j)^{\text{th}}$ probability summation term in (18) can be written as

$$\begin{aligned} &\Pr(T_j \leq G_{S_i i} < T_{j+1}, G^d_{S_i i} \geq T_j | l \text{ users report } i^{\text{th}} \text{ subch.}) \\ &= l\Pr(T_j \leq G_{1i} < T_{j+1}, G^d_{1i} \geq T_j, \text{User 1 selected} \\ &\qquad |\text{Users } 1, \ldots, l \text{ report } i^{\text{th}} \text{ subch.}), \\ &= l\Pr(T_j \leq G_{1i} < T_{j+1}, G^d_{1i} \geq T_j, G_{1i} \geq \{G_{ui}\}_{u=2}^{l} \\ &\qquad |\{\gimel_{ui}\}_{u=1}^{l} \leq n-1, \{\gimel_{vi}\}_{v=l+1}^{k} > n-1). \quad (19) \end{aligned}$$

Again using the independence of the subchannel gains of different users, (19) simplifies to

$$l\Pr(T_j \leq G_{1i} < T_{j+1}, G^d_{1i} \geq T_j, G_{1i} \geq \{G_{ui}\}_{u=2}^{l} \\ |\{\gimel_{ui}\}_{u=1}^{l} \leq n-1). \quad (20)$$

Using Baye's rule, conditioning on G_{1i} and G^d_{1i}, and using the fact that $\Pr\left(\{\gimel_{ui}\}_{u=1}^{l} \leq n-1\right) = \left(\frac{n}{N}\right)^l$, (20) simplifies to

$$l\left(\frac{N}{n}\right)^l \int_{T_j}^{T_{j+1}} \int_{T_j}^{\infty} \Pr(\gimel_{1i} \leq n-1 | G_{1i} = x, G^d_{1i} = y) \\ \times \left(\Pr(G_{ui} \leq x, \gimel_{ui} \leq n-1)\right)^{l-1} f_{G_{1i}G^d_{1i}}(x, y)\, dy\, dx. \quad (21)$$

Since the subchannel gains of different users are independent, it can be shown that

$$\Pr(\gimel_{1i} \leq n-1 | G_{1i} = x, G^d_{1i} = y) = \Pr(\gimel_{1i} \leq n-1 | G_{1i} = x). \quad (22)$$

We now evaluate the terms $\Pr(\gimel_{1i} \leq n-1 | G_{1i} = x)$ and $\Pr(G_{ui} \leq x, \gimel_{ui} \leq n-1)$. Given that $G_{1i} = x$, we know that

$$\Pr(\gimel_{1i} = m | G_{1i} = x) = \binom{N-1}{m}\left(1 - e^{-\frac{x}{\Omega}}\right)^{N-1-m} e^{-\frac{xm}{\Omega}}. \quad (23)$$

Hence,

$$\Pr\left(\gimel_{1i} \leq n-1 | G_{1i} = x\right) \\ = \sum_{m=0}^{n-1} \binom{N-1}{m} \left(1 - e^{-\frac{x}{\Omega}}\right)^{N-1-m} e^{-\frac{xm}{\Omega}}. \quad (24)$$

Since the users are statistically identical, it also follows from from (24) that for any User u,

$$\begin{aligned} &\Pr\left(G_{ui} \leq x, \gimel_{ui} \leq n-1\right) \\ &= \int_0^x \sum_{z=0}^{n-1} \binom{N-1}{z} \left(1 - e^{-\frac{\omega}{\Omega}}\right)^{N-1-z} \left(e^{-\frac{\omega}{\Omega}}\right)^z \frac{e^{-\frac{\omega}{\Omega}}}{\Omega} d\omega, \\ &= \sum_{z=0}^{n-1} \binom{N-1}{z} \sum_{q=0}^{N-1-z} \frac{\binom{N-z-1}{q}(-1)^q \left(1 - e^{-\frac{(q+z+1)x}{\Omega}}\right)}{q+z+1}, \\ &= \Upsilon_n(0, x). \end{aligned} \quad (25)$$

Using (24) and (25), (21) reduces to

$$l\left(\frac{N}{n}\right)^l \int_{T_j}^{T_{j+1}} \int_{T_j}^{\infty} \sum_{m=0}^{n-1} \binom{N-1}{m} e^{-\frac{xm}{\Omega}} \left(1 - e^{\frac{x}{\Omega}}\right)^{N-1-m} \\ \times \frac{e^{-\frac{x+y}{\Omega(1-\rho)}}}{\Omega^2(1-\rho)} I_0\left(\frac{2\sqrt{\rho}\sqrt{xy}}{\Omega(1-\rho)}\right) \left(\Upsilon_n(0, x)\right)^{l-1} dy\, dx. \quad (26)$$

As in Appendix A, replacing $I_0(\cdot)$ in the integrand in (26) with its truncated series and simplifying yields (7).

References

[1] D. Gesbert and M. S. Alouini, "How much feedback is multiuser diversity really worth?," in *Proc. ICC*, pp. 234-238, Jul. 2004.

[2] S. Sanayei and A. Nosratinia, "Opportunistic downlink transmission with limited feedback," *IEEE Trans. Inf. Theory*, vol. 53 pp. 4363–4372, Nov. 2007.

[3] T. Tang and R. Heath, "Opportunistic feedback for downlink multiuser diversity," *IEEE Commun. Lett.*, vol. 9, pp. 948–950, Oct. 2005.

[4] J. Chen, R. A. Berry, and M. L. Honig, "Limited feedback schemes for downlink OFDMA based on subchannel groups," *IEEE J. Sel. Areas Commun.*, vol. 26, pp. 1451–1461, Oct. 2008.

[5] R. Agarwal, V. Majjigi, Z. Han, R. Vannithamby, and J. Cioffi, "Low complexity resource allocation with opportunistic feedback over downlink OFDMA networks," *IEEE J. Sel. Areas Commun.*, vol. 26, pp. 1462–1472, Oct. 2008.

[6] J. So and J. M. Cioffi, "Feedback reduction scheme for downlink multiuser diversity," *IEEE Trans. Wireless Commun.*, vol. 8, pp. 668–672, Feb. 2009.

[7] P. Svedman, S. K. Wilson, L. J. Cimini, and B. Ottersten, "Opportunistic beamforming and scheduling for OFDMA systems," *IEEE Trans. Commun.*, vol. 55, pp. 941–952, May 2007.

[8] Y. J. Choi and S. Bahk, "Selective channel feedback mechanisms for wireless multichannel scheduling," in *Proc. IEEE Intl. Symp. World of Wireless Mobile and Multimedia Networks (WoWMoM)*, pp. 289–300, Jun. 2006.

[9] Y. J. Choi and S. Bahk, "Partial channel feedback schemes maximizing overall efficiency in wireless networks," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 1306–1314, Apr. 2008.

[10] S. N. Donthi and N. B. Mehta, "Joint performance analysis of channel quality indicator feedback schemes and frequency-domain scheduling for LTE," *IEEE Trans. Veh. Technol.*, vol. 60, pp. 3096–3109, Sep. 2011.

[11] Y. J. Choi and S. Rangarajan, "Analysis of best channel feedback and its adaptive algorithms for multicarrier wireless data systems," *IEEE Trans. Mob. Comput.*, vol. 10, pp. 1071–1082, Aug. 2011.

[12] Q. Ma and C. Tepedelenlioglu, "Practical multiuser diversity with outdated channel feedback," *IEEE Trans. Veh. Technol.*, vol. 54, pp. 1334–1345, Jul. 2005.

[13] S. Falahati, A. Svensson, T. Ekman, and M. Sternad, "Adaptive modulation systems for predicted wireless channels," *IEEE Trans. Commun.*, vol. 52, pp. 307–316, Feb. 2004.

[14] A. Kuhne and A. Klein, "Throughput analysis of multi-user OFDMA systems using imperfect CQI feedback and diversity techniques," *IEEE J. Sel. Areas Commun.*, vol. 26, pp. 1440–1450, Oct. 2008.

[15] A. J. Goldsmith, *Wireless Communications*. Cambridge Univ. Press, 2005.

[16] "Evolved universal terrestrial radio access (E-UTRA); physical layer procedures (release 8)," Tech. Rep. 36.213 (v8.3.0), 3rd Generation Partnership Project (3GPP), 2008.

[17] P. Dent, G. E. Bottomley, and T. Croft, "Jakes fading model revisited," *Electron. Lett.*, vol. 29, pp. 1162–1163, Jun. 1993.

[18] M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables*, Dover, 9 ed., 1972

[19] M. K. Simon and M. S. Alouini, *Digital Communication over Fading Channels*, Wiley Interscience 2008.

[20] S. Guharoy and N. B. Mehta, "Joint evaluation of channel feedback schemes, rate adaptation, and scheduling in OFDMA downlinks with feedback delays," *Submitted to IEEE Trans. Veh. Technol.*, 2012.

[21] J. Leinonen, J. Hamalainen, and M. Juntti, "Performance analysis of downlink OFDMA resource allocation with limited feedback," *IEEE Trans. Veh. Technol.*, vol. 8, pp. 2927–2937, Jun. 2009.

[22] J. G. Choi and S. Bahk, "Cell-throughput analysis of the proportional fair scheduler in the single-cell environment," *IEEE Trans. Veh. Technol.*, vol. 56, pp. 766–778, Mar. 2007.

[23] M. Torabi, D. Haccoun, and W. Ajib, "Performance analysis of scheduling schemes for rate-adaptive MIMO OSFBC-OFDM systems," *IEEE Trans. Veh. Technol.*, vol. 59, pp. 2363–2379, Jun. 2010.

[24] K. L. Baum, T. A. Kostas, P. J. Sartori, and B. K. Classon, "Performance characteristics of cellular sysems with different link adaptation strategies," *IEEE Trans. Veh. Technol.*, vol. 52, pp. 1497–1507, Nov. 2003.