Revisiting Incremental Relaying and Relay Selection for Underlay Cognitive Radio

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***Abstract*—Cooperative relaying combined with selection exploits spatial diversity to improve the performance of interference-constrained secondary users in an underlay cognitive radio network. While a relay improves the signal-to-interference-plus-noise ratio, it requires two hops and also generates interference to the primary. Therefore, in underlay cognitive radio, new criteria are needed to determine which relay to select. We present an optimal relay selection rule that maximizes the fading-averaged transmission rate of an average interference-constrained underlay secondary network. It differs from the many rules proposed in the literature. We then analyze its fading-averaged channel capacity. Numerical results show that the proposed rule outperforms direct transmission and several other rules, such as incremental relaying, proposed in the literature.**

I. Introduction

Cognitive radio (CR) promises to significantly improve the utilization of scarce wireless spectrum [1]. In the underlay mode of CR, which is the focus of our paper, a secondary user can simultaneously transmit on the same band as a higher priority primary user so long as the interference it causes to the primary receiver is tightly constrained [1].

However, the interference constraint can severely limit the data rates achievable by the cognitive users. In order to mitigate their impact, cooperative relaying with selection has been extensively studied [2]–[4]. In it, a single "best" relay is selected to forward a message from a secondary source (S) to a destination (D) based on the instantaneous channel conditions. Selection avoids the need for tight time synchronization among spatially separated relays that transmit simultaneously.

While relaying exploits spatial diversity and improves the signal-to-noise-ratio (SNR) at D, it requires two time slots instead of the one time slot required by a direct transmission. Solutions such as incremental relaying (IR) attempt to improve the spectral efficiency of relaying by allowing it only when direct transmission fails [5]–[8]. In CR, in addition to this aspect, the use of a relay is also governed by the interference its transmissions cause to the primary receiver. Therefore, new criteria need to be used for determining which relay to select and whether any relay should even be selected.

A. Literature on IR and Relay Selection (RS)

Several variants of IR have been proposed in the literature for conventional (interference-unconstrained) cooperative systems [6], [9] and underlay CR [7], [8]. In [7], [8], the transmit powers of the source and relays are adapted to meet the peak interference power constraints. In [6]–[9], relaying is used only if the SNR of the direct source-to-destination (SD) link falls below a threshold. In [7], the decode-and-forward (DF) relay that maximizes the minimum of the SNRs of the source-to-relay (SR) and relay-to-destination (RD) links is selected. Instead, in [6], [8], the DF relay that successfully decodes the received signal from S and maximizes the RD link SNR is selected. The outage capacity for the rate-optimal RS rule is studied in [10] for conventional cooperative systems with DF relays. IR for amplify-and-forward (AF) RS is studied in [6], [9] for conventional cooperative systems, where the relay that maximizes the end-to-end SNR at D is selected. In [11], incremental RS is studied for AF relays, but for the interweave mode of CR.

B. Focus and Contributions

In this paper, we focus on rate-optimal RS in underlay CR in which the relays are subject to an average interference constraint. We make the following specific contributions:

- We develop an optimal RS rule that yields the highest fading-averaged rate between S and D among the class of all RS rules that use AF relays and satisfy an average interference constraint. It differs from the incremental RS rules in [7], [8], [11], and the symbol error probability (SEP)-optimal RS rules in [2], [3] for underlay CR. This is because minimizing the average SEP does not imply maximizing the average rate due to the non-linear relationship between the two.
- We then derive the fading-averaged capacity of the optimal rule in the interference-unconstrained and interference-constrained regimes. We note that the average capacity analysis is more involved since the optimal rule turns out to be a non-linear function of the signal-to-interference–plus-noise ratio (SINR) of the SD link and is governed by the interference constraint.
- We present extensive numerical results and benchmark the performance of the proposed rule. We show that it improves the rate compared to the best-relay incremental rule [6] and direct transmission over a wide SINR range.

Comments: We study the fixed-power AF relay model for three reasons. First, it is extensively studied in the literature [6], [9], [12]. Second, it enables the use of energy-efficient transmit power amplifier at the relay. Third, the relay does not need to decode the source message. We consider the average interference constraint because it is less restrictive than the conservative peak interference constraint and is well-motivated

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This work was partially supported by a research grant from the Aerospace Networking Research Consortium (ANRC).

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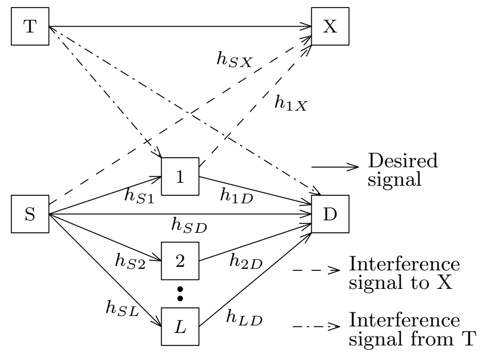

Fig. 1. An underlay CR system with a primary transmitter T, a primary receiver X, a secondary source S, a secondary destination D, and L secondary relays $1, 2, \ldots, L$.

when the packet transmission duration of the primary network spans multiple channel coherence times. Even for larger coherence times, it can be justified depending on the quality-of-service requirements of the primary network [13].

C. Outline and Notation

Section II presents the system model. The optimal RS rule is developed in Section III. Its average capacity is derived in Section IV. Numerical results and benchmarking are presented in Section V, followed by our conclusions in Section VI.

We shall use the following notation henceforth. The absolute value of a complex number x is denoted by $|x|$. The probability of an event A and the conditional probability of A given B are denoted by $\Pr(A)$ and $\Pr(A|B)$, respectively. $\mathbb{E}_X[.]$ denotes the expectation with respect to a random variable (RV) X; the subscript is dropped if it is obvious from the context. $X \sim CN(0,\sigma^2)$ means that X is a circularly symmetric zero-mean complex Gaussian RV with variance σ^2, and $1_{\{a\}}$ denotes the indicator function; it is 1 if a is true and is 0 otherwise. $X \sim \mathcal{E}\{\mu\}$ means X is an exponential RV with mean μ.

II. System Model and Problem Statement

Fig. 1 illustrates our system. It comprises of a primary network, in which a primary transmitter T sends data to a primary receiver X, and an underlay secondary network, in which S transmits data to D using L relays $1, 2, \ldots, L$. Each node is equipped with a single antenna. The complex baseband channel gain from S to X is h_{SX}, from S to D is h_{SD}, from S to relay i is h_{Si}, from relay i to D is h_{iD}, and from relay i to X is h_{iX}. Let $\mathbf{h}_S \triangleq [h_{S1}, h_{S2}, ..., h_{SL}], \mathbf{h}_D \triangleq [h_{1D}, h_{2D}, ..., h_{LD}]$, $\mathbf{h}_X \triangleq [h_{1X}, h_{2X}, ..., h_{LX}]$, and $\mathbf{h} \triangleq [\mathbf{h}_S, \mathbf{h}_D, \mathbf{h}_X]$. All channels are assumed to be frequency-flat, block fading channels that remain constant over the duration of at least two transmissions by S.

A. Relay Selection

A RS rule selects one out of L relays or decides that no relay should transmit depending on the instantaneous channel power gains. We shall denote the case where no relay transmits by a virtual relay 0 with $h_{S0} = h_{0D} = h_{0X} \triangleq 0$. Therefore, an SD-aware RS rule ϕ is a mapping: $\phi : \mathbb{R}^+ \times (\mathbb{R}^+)^L \times (\mathbb{R}^+)^L \times (\mathbb{R}^+)^L \to \{0, 1, \ldots, L\}$ that selects one out of $L+1$ relays (including relay 0) for every realization of the channel power gains $|h_{SD}|^2$, $\left\{|h_{Si}|^2\right\}_{i=1}^L$, $\left\{|h_{iD}|^2\right\}_{i=1}^L$, and $\left\{|h_{iX}|^2\right\}_{i=1}^L$.

B. Data Transmissions

The transmission occurs over two slots. In the first time slot, S transmits a data symbol x_s. After accounting for the interferences from T at the relays and destination, the received signals y_{Si} at relay i and y_{SD} at D are given by

$$y_{Si} = \sqrt{P_s} h_{Si} x_s + n_i + w_i, \quad 1 \le i \le L, \tag{1}$$
$$y_{SD} = \sqrt{P_s} h_{SD} x_s + n_D + w_D, \tag{2}$$

Here, P_s is the source transmit power and $\mathbb{E}\left[|x_s|^2\right] = 1$. The noises at relay i and D are $n_i \sim CN(0,\sigma_0^2)$ and $n_D \sim CN(0,\sigma_0^2)$, respectively. The interferences at relay i and D from T are w_i and w_D, respectively. These are assumed to be Gaussian, which is a worst case statistical model for the interference [14]. The reader is referred to [2] and the references therein for a discussion of the validity of this assumption. Therefore, $w_i \sim CN(0,\sigma_1^2)$ and $w_D \sim CN(0,\sigma_2^2)$.

In the second time slot, if relay $\beta \in \{1, \ldots, L\}$ is selected, it amplifies the signal $y_{S\beta}$ by a factor $\alpha_\beta = \sqrt{\frac{P_r}{P_s|h_{S\beta}|^2 + \sigma_0^2 + \sigma_1^2}}$ such that it transmits at a fixed power P_r, and forwards it to D. Therefore, the received signal $y_{\beta D}$ at D in this slot is

$$y_{\beta D} = y_{S\beta} \alpha_\beta h_{\beta D} + n_D^{'} + w_D^{'}, \tag{3}$$

where $n_D^{'} \sim CN(0,\sigma_0^2)$ is the noise at D and $w_D^{'} \sim CN(0,\sigma_2^2)$ is the interference from T at D in the second time slot. The instantaneous SINR at D after maximal ratio combining is $\gamma_{SD} + \gamma_\beta$, where $\gamma_{SD} = \frac{P_s|h_{SD}|^2}{\sigma_0^2+\sigma_2^2}$ is the SINR for the direct SD link and γ_β is the end-to-end SINR for the selected relay link, and is given by [6]

$$\gamma_\beta = \frac{\gamma_{S\beta}\gamma_{\beta D}}{\gamma_{S\beta} + \gamma_{\beta D} + 1}, \tag{4}$$

with $\gamma_{S\beta} = \frac{P_s|h_{S\beta}|^2}{\sigma_0^2+\sigma_1^2}$, and $\gamma_{\beta D} = \frac{P_r|h_{\beta D}|^2}{\sigma_0^2+\sigma_2^2}$ are the SINRs of the first and second hops, respectively.

The instantaneous capacity between S and D when AF relay $\beta \neq 0$ is selected is given by

$$C_\beta\left(\gamma_{SD}, \gamma_\beta\right) = \frac{1}{2}\log_2\left(1 + \gamma_{SD} + \gamma_\beta\right), \tag{5}$$

where the factor $\frac{1}{2}$ is due to the two time slots needed for the transmission by a relay. If no relay is selected, the source sends a new message in the second time slot. Therefore,

$$C_\beta\left(\gamma_{SD}, \gamma_\beta\right) = \log_2\left(1 + \gamma_{SD}\right), \text{ for } \beta = 0. \tag{6}$$

Channel State Information (CSI) Assumptions: A relay i is assumed to know the instantaneous channel power gains of its local links, i.e., $|h_{Si}|^2$ and $|h_{iX}|^2$, and the average interference power σ_1^2 from T to itself. The relay can use a band manager [15] or can exploit channel reciprocity to estimate $|h_{iX}|^2$ by periodically sensing the transmitted signal from X whenever it is engaged in a two-way communication with T [15], [16]. Similarly, the relay i knows h_{Si} using a training protocol [17], [18] and conveys it to D [2], [6], [12]. Using a training protocol [17], [18], D is assumed to know the average interference power σ_2^2 from T to D and the baseband channel gains h_{SD} and $h_{\beta D}$ for coherent demodulation [2].

C. Optimal RS Rule: Problem Statement

Our goal is to find an optimal RS rule ϕ^* that maximizes the fading-averaged rate of the secondary network while ensuring that the average interference caused to X from relay transmissions is below a threshold I_{avg}, i.e., $\mathbb{E}_{h_{SD},\mathbf{h}}\left[P_\beta |h_{\beta X}|^2\right] \leq I_{\text{avg}}$, where $P_\beta = 0$, for $\beta = 0$, and $P_\beta = P_r$, for $1 \leq \beta \leq L$.

Our optimization problem can be stated as follows:

$$\max_{\phi} \quad \mathbb{E}_{h_{SD},\mathbf{h}}[C_\beta\left(\gamma_{SD}, \gamma_\beta\right)], \tag{7}$$

$$\text{s.t.} \quad \mathbb{E}_{h_{SD},\mathbf{h}}\left[P_\beta |h_{\beta X}|^2\right] \leq I_{\text{avg}}, \tag{8}$$

$$\beta = \phi(h_{SD}, \mathbf{h}) \in \{0, 1, \ldots, L\}. \tag{9}$$

Henceforth, we shall refer to any RS rule that satisfies the constraints in (8) and (9) as a *feasible rule*.

Discussion of Constraint in (8)*:* The total average interference caused to X due to transmissions by S and relays can be also constrained in our model as follows. The average interference due to transmissions by S is $P_s \mathbb{E}_{h_{SX}}\left[|h_{SX}|^2\right]$ and due to transmissions by the relays is $\mathbb{E}_{h_{SD},\mathbf{h}}\left[P_\beta |h_{\beta X}|^2\right]$. The total average interference per slot, $\overline{I}$, after accounting for the fact that relay-based and direct transmissions use different numbers of slots, can be shown to be $\overline{I} = \frac{P_s \mathbb{E}_{h_{SX}}\left[|h_{SX}|^2\right] + \mathbb{E}_{h_{SD},\mathbf{h}}\left[P_\beta |h_{\beta X}|^2\right]}{1+\Pr(\beta \neq 0)}$. From (8), it can then be shown that setting $I_{\text{avg}} = I_{\text{th}} - P_s \mathbb{E}_{h_{SX}}\left[|h_{SX}|^2\right]$ ensures that $\overline{I} \leq I_{\text{th}}$, where I_{th} is the total interference threshold.

III. Optimal RS Rule

Let us first consider the case when the average interference constraint in (8) is not active. The optimal rule should select the relay that provides the maximum instantaneous rate. Thus,

$$\beta = \underset{i \in \{0,1,\ldots,L\}}{\operatorname{argmax}} \left\{C_i\left(\gamma_{SD}, \gamma_i\right)\right\}. \tag{10}$$

We shall refer to this as the *unconstrained rule*. From (5) and (6), the above rule is equivalent to

$$\beta = \begin{cases} 0, & \gamma_{SD} + \gamma_{SD}^2 \geq \max_{i \in \{1,2,\ldots,L\}} \{\gamma_i\}, \\ \operatorname{argmax}_{i \in \{1,2,\ldots,L\}} \{\gamma_i\}, & \text{otherwise.} \end{cases} \tag{11}$$

Let I_{un} denote the average interference caused to X from the relays. Clearly, $I_{\text{un}} = \mathbb{E}_{h_{SD},\mathbf{h}}\left[P_\beta |h_{\beta X}|^2\right]$. When $I_{\text{un}} > I_{\text{avg}}$, the unconstrained rule cannot be optimal as it is not even feasible. In general, the optimal RS rule is as follows.

Result *1:* The selected relay $\beta^* = \phi^*(h_{SD}, \mathbf{h})$, where ϕ^* is an optimal rule, is given by

$$\beta^* = \begin{cases} \operatorname{argmax}_{i \in \{0,1,\ldots,L\}} \left\{C_i\left(\gamma_{SD}, \gamma_i\right)\right\}, I_{\text{un}} \leq I_{\text{avg}}, \\ \operatorname{argmax}_{i \in \{0,1,\ldots,L\}} \left\{C_i\left(\gamma_{SD}, \gamma_i\right) - \lambda P_i |h_{iX}|^2\right\}, \\ \qquad\qquad\qquad\qquad I_{\text{un}} > I_{\text{avg}}, \end{cases} \tag{12}$$

where $P_i = 0$, for $i = 0$, and $P_i = P_r$, for $1 \leq i \leq L$. Here, λ is a strictly positive constant that arises only if $I_{\text{un}} > I_{\text{avg}}$. In this case, it is chosen such that the average interference constraint is satisfied with equality, and such a choice exists.

Proof: The proof is relegated to Appendix A. ■

Comments: The constant λ is computed numerically, as is typical in several constrained optimization problems [19]. The optimal RS rule can be implemented using a centralized polling scheme, in which S sequentially polls each relay about its metric γ_i and $P_r |h_{iX}|^2$ [20]. S also estimates γ_{SD} using a training protocol and exploiting channel reciprocity [17]. It then selects the best relay or directly transmits to D as per (12). Another option is to use a timer-based distributed selection scheme [21], in which each relay sets its timer whose value is a monotone decreasing function of its metric $C_i\left(\gamma_{SD}, \gamma_i\right) - \lambda P_i |h_{iX}|^2$. Upon expiry of its timer, each relay transmits a packet to S containing its identity and the metric. The relay whose timer expires first is the best relay. As above, S then selects β^* as per (12). Note that in order to compute $C_i\left(\gamma_{SD}, \gamma_i\right)$, the relay needs to know γ_{SD}, which needs to be broadcast by D.

IV. Analysis of Average Capacity

We now analyze the fading-averaged capacity $\overline{C}$ when various links undergo Rayleigh fading and are mutually independent. Thus, $h_{SD} \sim CN(0, \mu_{SD})$, $h_{Si} \sim CN(0, \mu_{SR})$, $h_{iD} \sim CN(0, \mu_{RD})$, and $h_{iX} \sim CN(0, \mu_{RX})$, for $i \in \{1, 2, \ldots, L\}$. Therefore, the SINRs of the SD, SR, and RD links are $\gamma_{SD} \sim \mathcal{E}(\overline{\gamma}_{SD})$, $\gamma_{Si} \sim \mathcal{E}(\overline{\gamma}_{SR})$, and $\gamma_{iD} \sim \mathcal{E}(\overline{\gamma}_{RD})$, respectively, where $\overline{\gamma}_{SD} = \frac{P_s \mu_{SD}}{\sigma_0^2 + \sigma_2^2}$, $\overline{\gamma}_{SR} = \frac{P_s \mu_{SR}}{\sigma_0^2 + \sigma_1^2}$, and $\overline{\gamma}_{RD} = \frac{P_r \mu_{RD}}{\sigma_0^2 + \sigma_2^2}$. In general, after exploiting the above assumptions about the channel gains, it can be shown that the the exact expression for the average capacity $\overline{C} = \mathbb{E}_{h_{SD},\mathbf{h}}[C_\beta\left(\gamma_{SD}, \gamma_\beta\right)]$ can at best be reduced to an involved four-dimensional integral form.

In order to addresses this challenge, we approximate γ_i with an *exponential* RV $\gamma_i^{'}$, whose mean $\overline{\gamma} = \mathbb{E}\left[\gamma_i^{'}\right]$ is set equal to $\mathbb{E}\left[\gamma_i\right] \approx \mathbb{E}\left[\frac{\gamma_{Si}\gamma_{iD}}{\gamma_{Si}+\gamma_{iD}}\right]$ [2]. Doing so yields $\overline{\gamma} = \frac{2\sqrt{p}}{15}\, {}_2F_1\left(3, 3; \frac{7}{2}; \frac{1}{2} - \frac{\sigma^{'}}{4\sqrt{p}}\right) + \frac{\sigma^{'}}{10}\, {}_2F_1\left(4, 2; \frac{7}{2}; \frac{1}{2} - \frac{\sigma^{'}}{4\sqrt{p}}\right)$, where $p = \overline{\gamma}_{SR}\overline{\gamma}_{RD}$, $\sigma^{'} = \overline{\gamma}_{SR} + \overline{\gamma}_{RD}$, and ${}_2F_1(a, b; c; z)$ denotes the Gauss hypergeometric function [22, (15.1)]. This is motivated by the observation that in the inequality $\frac{1}{2}\min\{\gamma_{Si}, \gamma_{iD}\} \leq \gamma_i \leq \min\{\gamma_{Si}, \gamma_{iD}\}$, both the upper and lower bounds are exponential RVs.

Result *2:* The average capacity in the interference-constrained regime ($I_{\text{un}} > I_{\text{avg}}$) is given by

$$\begin{aligned} \overline{C} = &\frac{1}{\overline{\gamma}_{SD}} \int_0^\infty \log_2\left(1 + \gamma_{SD}\right)\left(\psi\left(\gamma_{SD}\right)\right)^L e^{-\frac{\gamma_{SD}}{\overline{\gamma}_{SD}}}\, d\gamma_{SD} \\ &+ \frac{L}{\left(2\ln(2)\right)^2 \lambda \overline{\gamma}\overline{\gamma}_{SD}\overline{g}} \int_0^\infty \int_{(1+\gamma_{SD})^2}^\infty \left[\ln(x)\, \mathrm{E}_{\frac{1}{a\overline{g}}}\left(\frac{x}{\overline{\gamma}}\right)\right. \\ &\left. + G_{2,3}^{3,0}\left(\frac{x}{\overline{\gamma}}\middle| \begin{matrix} \frac{1}{a\overline{g}}, \frac{1}{a\overline{g}} \\ 0, \frac{1}{a\overline{g}}-1, \frac{1}{a\overline{g}}-1 \end{matrix}\right)\right] \left[1 - e^{-\frac{[x-(1+\gamma_{SD})]}{\overline{\gamma}}}\right. \\ &\left. + \frac{x}{\overline{\gamma}} e^{\frac{1+\gamma_{SD}}{\overline{\gamma}}} \mathrm{E}_{\frac{1}{a\overline{g}}}\left(\frac{x}{\overline{\gamma}}\right)\right]^{L-1} e^{\left(\frac{1+\gamma_{SD}}{\overline{\gamma}} - \frac{\gamma_{SD}}{\overline{\gamma}_{SD}}\right)}\, dx\, d\gamma_{SD}, \end{aligned} \tag{13}$$

where $a = 2\lambda \ln(2)$, $\overline{g} = P_r \mu_{RX}$, $\psi(\gamma_{SD}) = 1 - e^{-\frac{\gamma_{SD}+\gamma_{SD}^2}{\overline{\gamma}}} + \frac{(1+\gamma_{SD})^2}{\overline{\gamma}} \mathrm{E}_{\frac{1}{a\overline{g}}}\left(\frac{(1+\gamma_{SD})^2}{\overline{\gamma}}\right) e^{\frac{1+\gamma_{SD}}{\overline{\gamma}}}$, $\mathrm{E}_k(x)$ denotes the generalized exponential integral function [22, (5.1.4)], and $G^{m,n}_{p,q}\left(x\middle|\begin{smallmatrix}a_1,\ldots,a_p\\ b_1,\ldots,b_q\end{smallmatrix}\right)$ denotes the Meijer's G-Function [23, (9.301)].

Proof: The proof is relegated to Appendix B. ■

While the above expression in (13) is involved, it is simpler than evaluating the aforementioned four-dimensional integral. Furthermore, both $\mathrm{E}_k(x)$ and $G^{m,n}_{p,q}\left(x\middle|\begin{smallmatrix}a_1,\ldots,a_p\\ b_1,\ldots,b_q\end{smallmatrix}\right)$ can be evaluated using computationally-efficient routines available in softwares such as Matlab. In (13), the first term involving the single integral is due to the direct SD link and the second term is due to the L relay links. We see that as L increases, the contribution from the direct link decreases because $(\psi(\gamma_{SD}))^L$ decreases. On the other hand, as λ increases (i.e., a increases) or $\overline{g}$ increases (i.e., interference link becomes stronger), $\mathrm{E}_{\frac{1}{a\overline{g}}}\left(\frac{(1+\gamma_{SD})^2}{\overline{\gamma}}\right)$ increases, which, in turn, increases the contribution from the direct link.

Using Gauss-Laguerre quadrature [22, (25.4.45)], (13) simplifies to the following approximate, single-integral form:

$$\begin{aligned}\overline{C} &\approx \sum_{i=1}^{N} w_i \log_2(1 + x_i \overline{\gamma}_{SD}) (\psi(x_i \overline{\gamma}_{SD}))^L \\ &\quad + \frac{L}{(2\ln(2))^2 \lambda \overline{\gamma} \overline{g}} \sum_{i=1}^{N} w_i \int_{(1+x_i\overline{\gamma}_{SD})^2}^{\infty} e^{\frac{1+x_i\overline{\gamma}_{SD}}{\overline{\gamma}}} \\ &\quad \times \left[\ln(x) \mathrm{E}_{\frac{1}{a\overline{g}}}\left(\frac{x}{\overline{\gamma}}\right) + G^{3,0}_{2,3}\left(\frac{x}{\overline{\gamma}}\middle|\begin{smallmatrix}\frac{1}{a\overline{g}},\frac{1}{a\overline{g}}\\ 0,\frac{1}{a\overline{g}}-1,\frac{1}{a\overline{g}}-1\end{smallmatrix}\right)\right] \\ &\times \left[1 - e^{-\frac{(x-(1+x_i\overline{\gamma}_{SD}))}{\overline{\gamma}}} + \frac{x}{\overline{\gamma}} e^{\frac{1+x_i\overline{\gamma}_{SD}}{\overline{\gamma}}} \mathrm{E}_{\frac{1}{a\overline{g}}}\left(\frac{x}{\overline{\gamma}}\right)\right]^{L-1} dx, \quad (14)\end{aligned}$$

where x_i and w_i, for $1 \leq i \leq N$, are N Gauss-Laguerre abscissas and weights, respectively. Just $N = 3$ terms turn out to be sufficient to compute (14) as accurately as (13).

Result *3:* The average capacity $\overline{C}$ in the interference-unconstrained regime ($I_{\text{un}} \leq I_{\text{avg}}$) is given by

$$\begin{aligned}\overline{C} &= \frac{1}{\overline{\gamma}_{SD}} \int_0^{\infty} \log_2(1+\gamma_{SD}) \left(1 - e^{-\frac{\gamma_{SD}+\gamma_{SD}^2}{\overline{\gamma}}}\right)^L \\ &\quad \times e^{-\frac{\gamma_{SD}}{\overline{\gamma}_{SD}}} d\gamma_{SD} + \frac{L}{2\ln(2)\overline{\gamma}_{SD}} \sum_{k=0}^{L-1} \binom{L-1}{k} \frac{(-1)^k}{(k+1)} \\ &\times \int_0^{\infty} e^{\frac{(k+1)(1+\gamma_{SD})}{\overline{\gamma}}} e^{-\frac{\gamma_{SD}}{\overline{\gamma}_{SD}}} \left[2\ln(1+\gamma_{SD}) e^{-\frac{(k+1)(1+\gamma_{SD})^2}{\overline{\gamma}}}\right. \\ &\qquad \left. + \mathrm{E}_1\left(\frac{(k+1)(1+\gamma_{SD})^2}{\overline{\gamma}}\right)\right] d\gamma_{SD}. \quad (15)\end{aligned}$$

The proof for (15) is similar to that of Appendix B. As before, the first term is due to the direct SD link and the second term is due to the L relay links. As L increases, the contribution from the direct link decreases. Using Gauss-Laguerre quadrature, (15) can be written without any integral.

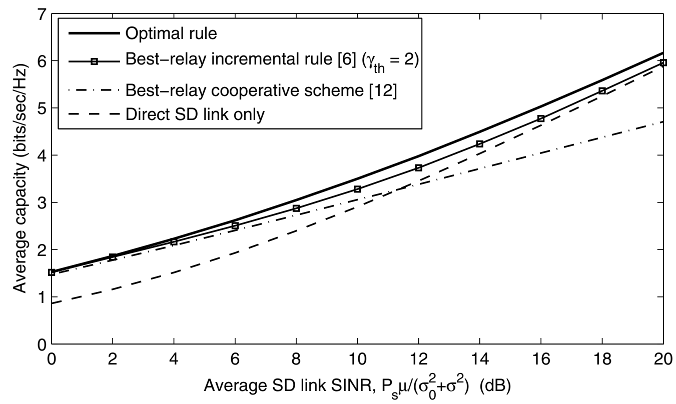

Fig. 2. Comparison of the average capacity of the optimal rule with other rules ($L = 4$, $P_s = P_r = 10$ dB, $\mu_{SD} = \mu$, and $\mu_{SR} = \mu_{RD} = 10\mu$).

V. Numerical Results and Benchmarking

In order to verify our analysis and gain quantitative insights, we now present Monte Carlo simulations averaged over 10^5 samples to compute the average capacity when various links undergo Rayleigh fading. Unless mentioned otherwise, $\sigma_0^2 = 2$ dB, and $\sigma_1^2 = \sigma_2^2 = \sigma^2 = 1.98$ dB. We set $\mu_{SD} = \mu_{RX} = \mu$, $\mu_{SR} = \mu_{RD} = 10\mu$, and vary μ.

A. Comparison and Benchmarking of Proposed Rule

Fig. 2 compares the average capacity of the optimal rule in the interference-unconstrained regime with the aforementioned best-relay incremental rule [6]. These are plotted as a function of the average SD link SINR. In [6], a relay is selected if $\gamma_{SD} \leq \gamma_{\text{th}}$, where γ_{th} (≥ 0) is the SD link SINR threshold. Also shown for reference are the capacities of the best-relay cooperative scheme [12], in which the AF relay that gives the maximum end-to-end SNR at D is always selected, and a non-cooperative scheme that uses only the SD link.

The proposed rule delivers the highest rate compared to all the other rules over the entire range of average SINRs. At low average SINRs, the optimal rule often selects a relay. However, as the average SINR increases, it selects the direct SD link more often so as to avoid the interference caused to X due to relay transmissions and to avoid two slots required by the use of a relay. Consequently, as the average SINR increases, $\overline{C}$ approaches that of direct transmission; so does the capacity of the best-relay incremental rule [6]. We have observed similar performance gains over direct transmission in the interference-constrained regime. We skip this figure to conserve space.

B. Performance Evaluation of Proposed Rule

Fig. 3 plots the average capacity as a function of the average SD link SINR, $P_s\mu/\left(\sigma_0^2 + \sigma^2\right)$, from simulations and analysis for the interference-unconstrained (i.e., $I_{\text{avg}} = \infty$) and interference-constrained regimes. The capacity using direct transmission is also shown as a reference. The analysis and simulation results match each other. This verifies the exponential approximation used in Section IV. For $I_{\text{avg}} = 15$ dB, when $P_s\mu/\left(\sigma_0^2 + \sigma^2\right) < 2$ dB, the system

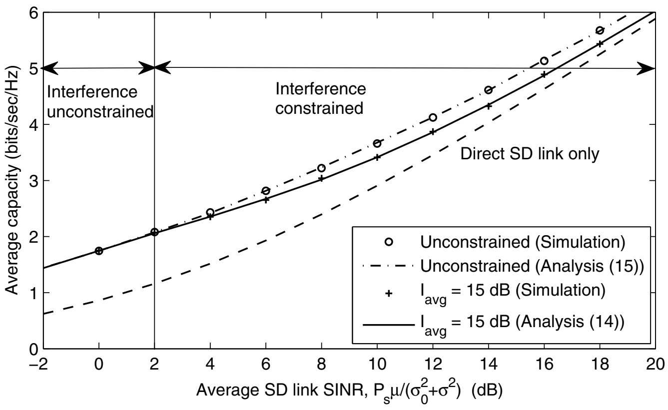

Fig. 3. *Impact of average interference threshold*: Capacity of the optimal rule as a function of average SD link SINR ($L = 2$, $P_s = 10$ dB, $P_r = 10P_s$, $\mu_{SD} = \mu_{RX} = \mu$, and $\mu_{SR} = \mu_{RD} = 10\mu$).

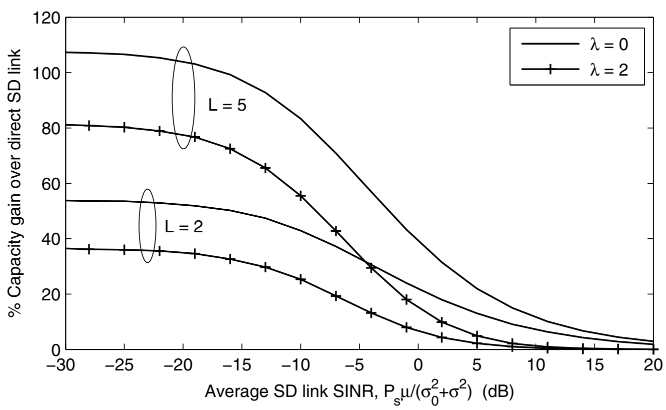

Fig. 4. Percentage capacity gain of the proposed rule over direct transmission as a function of average SD link SINR for different L and λ ($P_s = P_r = 10$ dB, $\mu_{SD} = \mu_{RX} = \mu$, and $\mu_{SR} = \mu_{RD} = \sqrt{\mu\left(\sigma_0^2+\sigma^2\right)/P_r}$).

is in the interference-unconstrained regime and its capacity is equal to the interference-unconstrained capacity. When $P_s\mu/\left(\sigma_0^2+\sigma^2\right) > 2$ dB, the system is in the interference-constrained regime, and the two curves diverge from each other. In the high average SINR regime, the capacity approaches that of direct transmission, as explained in Fig. 2.

Fig. 4 plots the capacity gain over direct transmission for different L and λ. Here, we set $\mu_{SD} = \mu_{RX} = \mu$ and $\mu_{SR} = \mu_{RD} = \sqrt{\mu\left(\sigma_0^2+\sigma^2\right)/P_r}$, and μ is varied. Consequently, the average SINRs of the SR and RD links scale as $\sqrt{P_r\mu/\left(\sigma_0^2+\sigma^2\right)}$, where we set $P_s = P_r$. This ensures that, for small μ, the average end-to-end SINR of a relay link, which is proportional to the product of the average SINRs of the SR and RD links, remains comparable to the average SD link SINR $P_s\mu/\left(\sigma_0^2+\sigma^2\right)$. Note that the lower the average SD link SINR, the more the capacity gain. Given L, the capacity gain increases as λ decreases, i.e., as the interference constraint becomes more relaxed. Given λ, the capacity gain increases as L increases due to an increase in spatial diversity.

VI. Conclusions

We proposed a novel RS rule that maximizes the fading-averaged rate between S and D when the secondary network is subject to an average interference constraint. It differed from the incremental AF relaying approaches pursued in the literature and outperformed them all. It traded-off between the SINR improvement afforded by the use of a relay and the additional hop required and the interference caused by its use. It also achieved capacity gains over direct transmission. The gains increased as the number of relays increased or as the interference constraint became more relaxed. These results motivate the use of relays to improve the achievable data rate of underlay CR. Investigating the scenario with multiple primary transmitters and receivers is an interesting avenue for future work.

Appendix

A. Brief Proof of Result 1

When $I_{\text{un}} \leq I_{\text{avg}}$: The unconstrained rule in (10) is feasible. It must, therefore, be optimal.

When $I_{\text{un}} > I_{\text{avg}}$: The set of all feasible RS rules, $\mathcal{Z}$, is a non-empty set because a selection rule in which no relay transmits is feasible. Let ϕ be a feasible rule. For a constant $\lambda > 0$, define an auxiliary function $L_\phi(\lambda)$ associated with ϕ as follows:

$$L_\phi(\lambda) \triangleq \mathbb{E}_{h_{SD},\mathbf{h}}\left[C_\beta\left(\gamma_{SD},\gamma_\beta\right) - \lambda P_\beta |h_{\beta X}|^2\right]. \quad (16)$$

Note that $L_\phi(\lambda)$ is a function of both ϕ and λ. Further, define a new rule ϕ^* in terms of the relay β^* it selects as follows:

$$\beta^* = \underset{i\in\{0,1,\ldots,L\}}{\operatorname{argmax}} \left\{C_i\left(\gamma_{SD},\gamma_i\right) - \lambda P_i |h_{iX}|^2\right\}, \quad (17)$$

where λ is chosen such that $\mathbb{E}_{h_{SD},\mathbf{h}}\left[P_{\beta^*}|h_{\beta^* X}|^2\right] = I_{\text{avg}}$.[1] Thus, ϕ^* is a feasible rule.

We now prove that ϕ^* is the desired optimal RS rule. From (17), it follows that $L_{\phi^*}(\lambda) \geq L_\phi(\lambda)$. Therefore,

$$\mathbb{E}_{h_{SD},\mathbf{h}}[C_{\beta^*}\left(\gamma_{SD},\gamma_{\beta^*}\right)] \geq \mathbb{E}_{h_{SD},\mathbf{h}}[C_\beta\left(\gamma_{SD},\gamma_\beta\right)] - \lambda\left(\mathbb{E}_{h_{SD},\mathbf{h}}\left[P_\beta|h_{\beta X}|^2\right] - I_{\text{avg}}\right). \quad (18)$$

Since ϕ is a feasible rule, $\mathbb{E}_{h_{SD},\mathbf{h}}\left[P_\beta|h_{\beta X}|^2\right] \leq I_{\text{avg}}$. Thus,

$$\mathbb{E}_{h_{SD},\mathbf{h}}[C_{\beta^*}\left(\gamma_{SD},\gamma_{\beta^*}\right)] \geq \mathbb{E}_{h_{SD},\mathbf{h}}[C_\beta\left(\gamma_{SD},\gamma_\beta\right)]. \quad (19)$$

Hence, ϕ^* yields highest average rate among all feasible rules.

B. Proof of Result 2

Let β denote the selected relay in (12). Using (5), (6), and $\gamma_i \approx \gamma_i^{'}$, the average capacity $\overline{C}$ simplifies to

$$\overline{C} = \mathbb{E}_{h_{SD},\mathbf{h}}\left[C_\beta\left(\gamma_{SD},\gamma_\beta^{'}\right)\right] = T_1 + LT_2, \quad (20)$$

where $T_1 = \mathbb{E}_{h_{SD}}[\log_2\left(1+\gamma_{SD}\right)\Pr(\beta=0|\gamma_{SD})]$, and $T_2 = \mathbb{E}_{h_{SD},\mathbf{h}}\left[\frac{1}{2}\log_2\left(1+\gamma_{SD}+\gamma_1^{'}\right)\Pr\left(\beta=1|\gamma_{SD},\gamma_1^{'},g_1\right)\right]$, with $g_i \triangleq P_r|h_{iX}|^2$, $1 \leq i \leq L$. The factor L in (20) arises because the L relays are statistically identical.

[1]That such a unique choice of λ exists can be proved using the intermediate value theorem by observing that $0 \leq I_{\text{avg}} < I_{\text{un}}$, and showing that the average interference is a continuous, monotonically decreasing function of $\lambda \geq 0$.

1) Evaluating T_1: From (12), we get

$$\Pr(\beta=0|\gamma_{SD}) = \Pr\left(\log_2\left(1+\gamma_{SD}\right) > \max_{i\in\{1,2,\ldots,L\}}\left\{\frac{1}{2}\log_2\left(1+\gamma_{SD}+\gamma_i^{'}\right)-\lambda g_i\right\}\,\Big|\,\gamma_{SD}\right). \quad (21)$$

Since $\{g_i\}_{i=1}^{L}$ are independent and identically distributed (i.i.d.) RVs and so are $\{\gamma_i^{'}\}_{i=1}^{L}$, we get

$$\Pr(\beta=0|\gamma_{SD}) = \left[\Pr\left(g_1 > \frac{1}{a}\ln\left(\frac{1+\gamma_{SD}+\gamma_1^{'}}{(1+\gamma_{SD})^2}\right)\,\Big|\,\gamma_{SD}\right)\right]^L,$$

where $a \triangleq 2\lambda\ln(2)$. Substituting the probability density functions (PDFs) of $\gamma_1^{'} \sim \mathcal{E}\{\overline{\gamma}\}$ and $g_1 \sim \mathcal{E}\{\overline{g}\}$, where $\overline{g} = P_r\mu_{RX}$, we get

$$\Pr(\beta=0|\gamma_{SD}) = \left[1-e^{-\frac{\gamma_{SD}+\gamma_{SD}^2}{\overline{\gamma}}} + \frac{1}{\overline{\gamma}}\int_{\gamma_{SD}+\gamma_{SD}^2}^{\infty}\left(\frac{1+\gamma_{SD}+\gamma_1^{'}}{(1+\gamma_{SD})^2}\right)^{-\frac{1}{a\overline{g}}} e^{-\frac{\gamma_1^{'}}{\overline{\gamma}}}\,d\gamma_1^{'}\right]^L. \quad (22)$$

Employing the variable substitution $\frac{1+\gamma_{SD}+\gamma_1^{'}}{(1+\gamma_{SD})^2} = t$, substituting (22) into the expression for T_1, and unconditioning over γ_{SD} yields the first term in (13).

2) Evaluating T_2: As in (21), the conditional probability that relay 1 is selected is given by

$$\begin{aligned}&\Pr\left(\beta=1|\gamma_{SD},\gamma_1^{'},g_1\right)\\ &= \Pr\Big(\frac{1}{2}\log_2\left(1+\gamma_{SD}+\gamma_1^{'}\right)-\lambda g_1 > \log_2\left(1+\gamma_{SD}\right),\\ &\frac{1}{2}\log_2\left(1+\gamma_{SD}+\gamma_1^{'}\right)-\lambda g_1 > \frac{1}{2}\log_2\left(1+\gamma_{SD}+\gamma_2^{'}\right)-\lambda g_2,\\ &\ldots,\frac{1}{2}\log_2\left(1+\gamma_{SD}+\gamma_1^{'}\right)-\lambda g_1 > \frac{1}{2}\log_2\left(1+\gamma_{SD}+\gamma_L^{'}\right)\\ &\qquad -\lambda g_L\,\Big|\,\gamma_{SD},\gamma_1^{'},g_1\Big). \quad (23)\end{aligned}$$

Now, $\{g_i\}_{i=1}^{L}$ are i.i.d. and so are $\{\gamma_i^{'}\}_{i=1}^{L}$. Therefore, we get $\Pr\left(\beta=1|\gamma_{SD},\gamma_1^{'},g_1\right) = \left[\Pr\left(g_2 > \frac{1}{a}\ln\left(\frac{1+\gamma_{SD}+\gamma_2^{'}}{1+\gamma_{SD}+\gamma_1^{'}}\right)+g_1 \,\Big|\,\gamma_{SD},\gamma_1^{'},g_1\right)\right]^{L-1} 1_{\left\{g_1<\frac{1}{a}\ln\left(\frac{1+\gamma_{SD}+\gamma_1^{'}}{(1+\gamma_{SD})^2}\right)\right\}}$. Substituting the PDFs of the exponential RVs $\gamma_2^{'}$ and g_2, we get

$$\begin{aligned}\Pr\left(\beta=1|\gamma_{SD},\gamma_1^{'},g_1\right) = \Bigg[&1-e^{-\frac{\left[\left(1+\gamma_{SD}+\gamma_1^{'}\right)e^{-ag_1}-(1+\gamma_{SD})\right]}{\overline{\gamma}}}\\ &+\frac{\left(1+\gamma_{SD}+\gamma_1^{'}\right)}{\overline{\gamma}}\mathrm{E}_{\frac{1}{a\overline{g}}}\left(\frac{\left(1+\gamma_{SD}+\gamma_1^{'}\right)e^{-ag_1}}{\overline{\gamma}}\right)\\ &\times e^{\left(\frac{1+\gamma_{SD}}{\overline{\gamma}}-ag_1\right)}\Bigg]^{L-1} 1_{\left\{g_1<\frac{1}{a}\ln\left(\frac{1+\gamma_{SD}+\gamma_1^{'}}{(1+\gamma_{SD})^2}\right)\right\}}. \quad (24)\end{aligned}$$

Substituting (24) into the expression for T_2, unconditioning over γ_{SD}, $\gamma_1^{'}$, and g_1, using the variable substitutions $\left(1+\gamma_{SD}+\gamma_1^{'}\right)e^{-ag_1} = x$ and $e^{ag_1} = t$, and simplifying further yields the second term in (13).

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