# Direct Link-Aware Relay Selection for Average Interference-Constrained Underlay Cognitive Radio

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***Abstract*—Cooperative relaying combined with selection exploits spatial diversity to significantly improve the performance of interference-constrained secondary users in an underlay cognitive radio network. We present a novel and optimal relay selection (RS) rule that minimizes the symbol error probability (SEP) of an average interference-constrained underlay secondary system that uses amplify-and-forward relays. A key point that the rule highlights – for the first time – is that, for the average interference constraint, the signal-to-interference-plus-noise-ratio (SINR) of the direct source-to-destination (SD) link affects the choice of the optimal relay. Furthermore, as the SINR increases, the odds that no relay transmits increase. We also propose a simpler, more practical, and near-optimal variant of the optimal rule that requires just one bit of feedback about the state of the SD link to the relays. Compared to the SD-unaware ad hoc RS rules proposed in the literature, the proposed rules markedly reduce the SEP by up to two orders of magnitude.**

## I. Introduction

Cognitive radio (CR) promises to improve the utilization of scarce wireless spectrum [1]. Different modes have been proposed to access licensed or primary users' (PUs) spectrum by the secondary users (SUs), such as interweave, overlay, and underlay [1]. In the underlay mode of CR, which is the focus of this paper, an SU can simultaneously transmit on the same band as a higher priority PU as long as the interference it causes to the PU is tightly controlled. However, this interference constraint also limits the data rate and reliability of communications by the SU.

Cooperative relaying combined with selection enhances the performance of the SUs by exploiting spatial diversity. In it, a single "best" relay is selected to forward a message from a secondary source $S$ to a destination $D$ based on the instantaneous channel conditions. It is practically appealing because it avoids the challenging timing synchronization problems that arise in distributed networks when multiple geographically separated relays have to transmit simultaneously. In conventional cooperative relay networks, the relay with the maximum signal-to-noise-ratio (SNR) at $D$ is selected [2], [3]. However, in underlay CR, this is no longer the case due to the interference constraint. It may not be preferable to select a relay with the maximum SNR if it causes excessive interference to a primary receiver $P_{\text{Rx}}$. Therefore, the relay selection rule is now also a function of the links between the relays and the $P_{\text{Rx}}$.

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### A. Literature on Relay Selection (RS) Rules in Underlay CR

Interference-aware RS rules have been extensively investigated in the CR literature. In [4], [5], the transmit power of the relay is inversely proportional to the power gain of the channel from the relay to the $P_{\text{Rx}}$ so as to satisfy the peak interference constraint. In [4], [5], the relay that maximizes the minimum of the SNRs of the source-to-relay (SR) and relay-to-destination (RD) links is selected. We shall refer to the RS rules in [4], [5] as *variable-power max-min rules*.

Pruning based RS rules are instead considered in [6]–[8], where fixed-power relays that do not satisfy the peak interference constraint are first excluded. Among the remaining relays, the one that maximizes the minimum of the SNRs of SR and RD links is selected in [6] and the one that maximizes the difference between the SNR at $D$ and the interference caused by the relay to the $P_{\text{Rx}}$ is selected in [7]. Instead, in [8], relays for whom the minimum of the SR and RD link SNRs is below a threshold or that do not satisfy the peak interference constraint are excluded. Among the remaining relays, the one that maximizes the ratio of the minimum of the SNRs of the SR and RD links and the interference caused by the relay to the $P_{\text{Rx}}$ is selected. We shall refer to the RS rules in [6]–[8] as *max-min*, *low-interference*, and *quotient rules*, respectively.

### B. Contributions

The key idea that we develop in this paper is that for the average interference constraint, the choice of the optimal relay is affected not just by the SR, RD, and relay-to-$P_{\text{Rx}}$ (RP) links, which are local to the relays, but also by the state of the direct source-to-destination (SD) link, despite it not being local to any relay. This results in significant performance gains of the secondary system over the SD-unaware RS rules that do not consider the state of the SD link for RS. To the best of our knowledge, this important aspect, which is unique to underlay CR, has not been studied in the extensive literature on RS.[1]

We make the following specific contributions:

- For an underlay CR system that uses fixed-power AF relays, we present a fully SD-aware optimal rule for RS that yields the lowest symbol error probability (SEP) among the class of all RS rules that satisfy an average interference constraint. The functional form of the optimal

[1]We note that this idea is different from incremental relaying, which has been proposed for conventional cooperative systems [9]. Incremental relaying favors a direct transmission to a two-hop relay-aided transmission because of the spectral inefficiency of the latter option. On the other hand, in our problem, it is the interference constraint that fundamentally drives the dependence of the RS rule on the state of the SD link.

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RS rule brings out how the choice of the selected relay is affected by the instantaneous signal-to-interference-plus-noise-ratio (SINR) of the SD link.
- However, one practical challenge with this rule is that the SINR of the SD link now needs to broadcast to all the relays since it affects how suitable a relay is for selection and is not known a priori by any relay. We then develop a simpler, novel 1-*bit rule*, which requires just one bit of feedback about the state of the SD link to all the relays, and enables scalable distributed relay selection. The bit informs them whether or not the SINR of the SD link exceeds a threshold $\gamma_{\text{th}}$, which is optimized. Numerical results show that the SEP of the 1-bit rule is close to the fully SD-aware optimal rule over a wide range of SINRs.
- When the interference constraint is inactive, we show that both proposed rules reduce to the optimal RS rule for conventional cooperative systems [2], [3]. Thus, they are generalizations of the conventional optimal RS rule. Furthermore, in the absence of the SD link, the fully SD-aware optimal rule reduces to the SD-unaware optimal rule proposed in [10], and, thus, generalizes it as well.
- Extensive benchmarking shows that both proposed rules reduce the SEP by up to two orders of magnitude compared to the many aforementioned ad hoc rules that do not take the state of the SD link into account [4]–[8], [10]. This also translates into significant power savings at the source and relays for the same SEP.

*Comments:* We focus on classical fixed-power AF relaying because it has attracted considerable interest in the literature on conventional cooperation [3] and underlay CR [6], [8]. Further, it enables the use of energy-efficient transmit power amplifiers at the relays. Although each relay is assumed to have the instantaneous channel power gain from itself to the $P_{\text{Rx}}$, we study the average interference constraint because it is less restrictive than the conservative peak interference constraint and provides better secondary performance. It is well motivated when the packet duration spans multiple channel coherence times [11].

The paper is organized as follows. Section II develops the system model and the problem statement. The fully SD-aware optimal rule is derived in Section III. The low feedback variant of the optimal rule is developed in Section IV. Numerical results and benchmarking are presented in Section V. Our conclusions follow in Section VI.

We shall use the following notation henceforth. The absolute value of $x$ is denoted by $|x|$. The probability of an event $A$ and the conditional probability of $A$ given $B$ are denoted by $\Pr(A)$ and $\Pr(A|B)$, respectively. For a random variable (RV) $X$, $\mathbb{E}_X[.]$ denotes expectation with respect to $X$. Scalar and vector variables are written in normal and bold fonts, respectively. The notation $X \sim CN(0, \sigma^2)$ means that $X$ is a circularly symmetric zero-mean complex Gaussian RV with variance $\sigma^2$.

## II. System Model and Problem Statement

Fig. 1 illustrates our system. It comprises of a primary network, in which a primary transmitter $P_{\text{Tx}}$ sends data to a primary receiver $P_{\text{Rx}}$, and an underlay secondary network, in which a source $S$ transmits to a destination $D$ using $L$ relays $1, \ldots, L$. Each node has one antenna. As we shall see, our problem formulation and solution are novel even for the single antenna model, which has been widely studied in the literature [4]–[8]. It leads to low complexity source, relays, and destination, which is one of the motivations for cooperative relaying. The complex baseband channel gain from $S$ to $P_{\text{Rx}}$ is denoted by $h_{SP}$, from $S$ to $D$ by $h_{SD}$, from $S$ to relay $i$ by $h_{Si}$, from relay $i$ to $D$ by $h_{iD}$, and from relay $i$ to $P_{\text{Rx}}$ by $h_{iP}$. Let $\mathbf{h}_S \triangleq [h_{S1}, h_{S2}, \ldots, h_{SL}]$, $\mathbf{h}_D \triangleq [h_{1D}, h_{2D}, \ldots, h_{LD}]$, $\mathbf{h}_P \triangleq [h_{1P}, h_{2P}, \ldots, h_{LP}]$, and $\mathbf{h} \triangleq [\mathbf{h}_S, \mathbf{h}_D, \mathbf{h}_P]$. All channels are frequency-flat, block fading channels that undergo Rayleigh fading and remain constant over the duration of at least two symbol transmissions. Therefore, for $i = 1, 2, \ldots, L$, $h_{Si} \sim CN(0, \mu_{Si})$, $h_{iD} \sim CN(0, \mu_{iD})$, $h_{iP} \sim CN(0, \mu_{iP})$, $h_{SD} \sim CN(0, \mu_{SD})$, and $h_{SP} \sim CN(0, \mu_{SP})$.

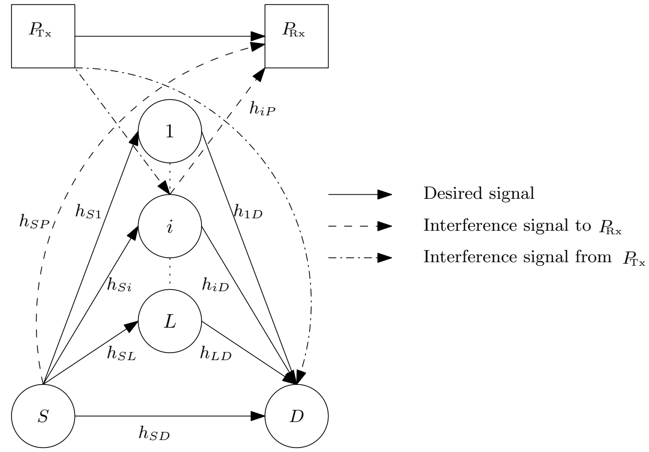


Fig. 1. An underlay CR with a primary transmitter $P_{\text{Tx}}$, a primary receiver $P_{\text{Rx}}$, a secondary source $S$, a secondary destination $D$, and $L$ secondary relays.

### A. Relay Selection Rule

A RS rule selects one out of $L$ relays or decides that no relay is selected so as to avoid interfering with the $P_{\text{Rx}}$ depending on the instantaneous channel conditions. When no relay is selected, for notational convenience, we denote it by a virtual relay 0 with $h_{S0} = h_{0D} = h_{0P} = 0$. A fully SD-aware RS rule $\phi$ is a mapping:

$$\phi : \mathbb{R}^+ \times (\mathbb{R}^+)^L \times (\mathbb{R}^+)^L \times (\mathbb{R}^+)^L \to \{0, 1, \ldots, L\}, \quad (1)$$

that selects one out of the $L+1$ relays for every realization of $|h_{SD}|^2$, $\left\{|h_{Si}|^2\right\}_{i=1}^{L}$, $\left\{|h_{iD}|^2\right\}_{i=1}^{L}$, and $\left\{|h_{iP}|^2\right\}_{i=1}^{L}$.

### B. Data Transmission

The data transmission occurs over two slots. In the first time slot, $S$ transmits a data symbol $x$ that is drawn with equal probability from a constellation of size $M$. The received signals $y_{Si}$ at the relay $i$ and $y_{SD}$ at $D$ are given by

$$y_{Si} = \sqrt{P_t} h_{Si} x + n_i + w_i, \ \ 1 \le i \le L, \quad (2)$$

$$y_{SD} = \sqrt{P_t} h_{SD} x + n_D + w_D, \quad (3)$$

where $P_t$ is the source transmit power and $\mathbb{E}\left[|x|^2\right] = 1$. The noises at the relay $i$ and $D$ are $n_i \sim CN(0, \sigma_0^2)$ and $n_D \sim CN(0, \sigma_0^2)$, respectively. The interferences at the relay $i$ and $D$ due to transmissions by the $P_{\text{Tx}}$ are $w_i$ and $w_D$, respectively. These are assumed to be Gaussian, as has also been assumed in [12], [13]. Therefore, $w_i \sim CN(0, \sigma_1^2)$ and $w_D \sim CN(0, \sigma_2^2)$. The assumption is justified with one $P_{\text{Tx}}$ when it transmits a constant amplitude signal over a Rayleigh fading link [13]. It is also justified when there are many $P_{\text{Tx}}$s due to the central limit theorem. Furthermore, it is justified when the $P_{\text{Tx}}$ is far away from the secondary network, as has been assumed in [4], [6], [8]. In general, the Gaussian assumption corresponds to a worst case model for the interference [12] and ensures mathematical tractability.

In the second time slot, if relay $\beta$ is selected, it amplifies the signal $y_{S\beta}$ by a factor $\alpha_\beta = \sqrt{\frac{P_r}{P_t|h_{S\beta}|^2+\sigma_0^2+\sigma_1^2}}$ [8] so that its transmit power is $P_r$, and forwards it to $D$. Therefore, the received signal $y_{\beta D}$ at $D$ in the second time slot is given by

$$y_{\beta D} = y_{S\beta}\alpha_\beta h_{\beta D} + n_D^{'} + w_D^{'}, \qquad (4)$$

where $n_D^{'} \sim CN(0, \sigma_0^2)$ is the noise at $D$ and $w_D^{'} \sim CN(0, \sigma_2^2)$ is the interference from the $P_{\text{Tx}}$ at $D$ in the second time slot. After maximal ratio combining and coherent demodulation, the end-to-end SINR at $D$ is given by [6]:

$$\gamma_\beta = \frac{\gamma_{S\beta}\gamma_{\beta D}}{\gamma_{S\beta} + \gamma_{\beta D} + 1}, \qquad (5)$$

where $\gamma_{S\beta} = \frac{P_t|h_{S\beta}|^2}{\sigma_0^2+\sigma_1^2}$ and $\gamma_{\beta D} = \frac{P_r|h_{\beta D}|^2}{\sigma_0^2+\sigma_2^2}$ are the SINRs of the first and second hops, respectively.

### *C. Channel State information (CSI) Assumptions*

The selected relay $\beta$ is assumed to know the instantaneous channel power gains of its local links, i.e., $|h_{S\beta}|^2$, $|h_{\beta D}|^2$, and $|h_{\beta P}|^2$. The destination is assumed to know the baseband channel gains $h_{SD}$, $h_{S\beta}$, and $h_{\beta D}$ to enable coherent demodulation [14], [15]. This CSI can be acquired by a training protocol [16] and by exploiting reciprocity, and is assumed in the related literature [4]–[8]. The channel statistics based parameters $\sigma_1^2$ and $\sigma_2^2$, which change over a larger time scale than the instantaneous channel gains, can also be estimated by the selected relay and the destination, respectively [17]. Note that no phase information of the baseband channel gains is required for RS in our model.

### *D. Optimal RS Rule: Problem Statement*

Our goal is to find an optimal RS rule $\phi^*$ that minimizes the SEP of the secondary system while ensuring that the average interference caused to the $P_{\text{Rx}}$ by the relays is below a threshold $I_{\text{avg}}$. We note that this model can be generalized to the interference constraint due to transmissions by $S$ as well, as discussed in [10]. We focus on MPSK first. Corresponding SEP-optimal RS rules can be developed for several other constellations such as M-PAM and MQAM [15, (6.1)], M-DPSK and MFSK [14, (8.1)] whose SEP upper bound is an exponentially decaying function of the SINR. The instantaneous SEP for MPSK at $D$ when relay $\beta$ is selected is given by [14, (8.23)]

$$\text{SEP}\left(|h_{SD}|^2, |h_{S\beta}|^2, |h_{\beta D}|^2\right) = \frac{1}{\pi}\int_0^{m\pi} e^{-\frac{q\left(\gamma_{SD}+\gamma_\beta\right)}{\sin^2\theta}}\, d\theta, \qquad (6)$$

where $q = \sin^2(\frac{\pi}{M})$, $m = \frac{M-1}{M}$, $\gamma_{SD} = \frac{P_t|h_{SD}|^2}{\sigma_0^2+\sigma_2^2}$ is the SINR of the SD link, and $\gamma_\beta$ is given in (5).

Therefore, our problem can be stated as the following mixed-integer, stochastic, constrained optimization problem:

$$\min_{\phi} \quad \mathbb{E}_{h_{SD},\mathbf{h}}\left[\text{SEP}\left(|h_{SD}|^2, |h_{S\beta}|^2, |h_{\beta D}|^2\right)\right], \qquad (7)$$

$$\text{s.t.} \quad \mathbb{E}_{h_{SD},\mathbf{h}}\left[P_\beta|h_{\beta P}|^2\right] \leq I_{\text{avg}}, \qquad (8)$$

$$\beta = \phi(h_{SD}, \mathbf{h}) \in \{0, 1, \ldots, L\}, \qquad (9)$$

where $P_\beta = 0$, for $\beta = 0$, and $P_\beta = P_r$, for $1 \leq \beta \leq L$. Henceforth, we shall refer to any RS rule that satisfies the constraints in (8) and (9) as a *feasible rule*. We note that alternate problem formulations that maximize the capacity or minimize the outage probability are also possible, but are beyond the scope of this paper.

## III. Fully SD-aware Optimal Rule

We now develop the SEP-optimal RS rule. Let us first consider the conventional RS rule that minimizes the SEP at $D$ when the average interference constraint in (8) is inactive. From (6), the optimal rule selects the relay with the highest end-to-end SINR at $D$ [2], [3]. Thus,

$$\beta = \underset{i \in \{1,\ldots,L\}}{\text{argmax}}\, \{\gamma_i\}. \qquad (10)$$

We shall refer to this as the *unconstrained rule*. Let $I_{\text{un}}$ denote the average interference caused to the $P_{\text{Rx}}$ due to the selected relay's transmission by the unconstrained rule. It is given by $I_{\text{un}} = P_r\mathbb{E}_{\mathbf{h}}\left[|h_{\beta P}|^2\right]$. Note that $\beta$ also gets averaged over since it is a function of $\mathbf{h}$. However, when $I_{\text{un}} > I_{\text{avg}}$, the unconstrained rule is not feasible, and, thus, cannot be optimal. A general characterization of the optimal RS rule for MPSK is as follows.

**Result** *1:* The selected relay $\beta^* = \phi^*(h_{SD}, \mathbf{h})$ for an optimal rule $\phi^*$ is given as follows:

$$\beta^* = \begin{cases} \text{argmax}_{i\in\{1,\ldots,L\}}\{\gamma_i\}, & I_{\text{un}} \leq I_{\text{avg}}, \\ \text{argmin}_{i\in\{0,\ldots,L\}}\left\{\frac{1}{\pi}\int_0^{m\pi} e^{-\frac{q(\gamma_i+\gamma_{SD})}{\sin^2\theta}} d\theta + \lambda P_i|h_{iP}|^2\right\}, & \\ & I_{\text{un}} > I_{\text{avg}}, \end{cases} \qquad (11)$$

where $P_i = 0$, for $i = 0$, and $P_i = P_r$, for $1 \leq i \leq L$. Here, $\lambda$ is a strictly positive constant that arises only if $I_{\text{un}} > I_{\text{avg}}$. In this case, $\lambda$ is chosen such that the average interference constraint is satisfied with equality, and such a choice of $\lambda$ always exists.

*Proof:* The proof is relegated to Appendix A. ■

As the optimal RS rule in (11) is a function of $\gamma_{SD}$, we shall call it the *fully SD-aware optimal rule*. The constant $\lambda$ is computed numerically, as is typical in several constrained optimization problems in wireless communications, e.g., optimal rate and power adaption and water-filling [15].

It can be shown that when the RS rule is designed to not depend on the instantaneous SINR of the SD link, the SEP-optimal RS rule takes the following simpler form [10]:

$$\beta^* = \begin{cases} \text{argmax}_{i\in\{1,\ldots,L\}}\{\gamma_i\}, & I_{\text{un}} \le I_{\text{avg}}, \\ \text{argmin}_{i\in\{0,\ldots,L\}}\left\{\frac{1}{\pi}\int_0^{m\pi}\frac{e^{-\frac{q\gamma_i}{\sin^2\theta}}}{1+\frac{q\overline{\gamma}_{SD}}{\sin^2\theta}}\,d\theta + \lambda P_i|h_{iP}|^2\right\}, & \\ & I_{\text{un}} > I_{\text{avg}}, \end{cases} \tag{12}$$

where $\overline{\gamma}_{SD} = \frac{P_t\mu_{SD}}{\sigma_0^2+\sigma_2^2}$ is the average SINR of the SD link. We shall refer to this rule as the *SD-unaware optimal rule*.

## IV. Simpler and Low Feedback Variant: 1-bit rule

As mentioned, for the fully SD-aware rule, $D$ needs to broadcast the SINR of the SD link $\gamma_{SD}$ to all the relays, which may be practically challenging. To reduce the feedback burden, we propose a simpler variant of the optimal RS rule, called the 1-bit rule. In it, the selected relay depends on the bit $f$, which is 0 if $\gamma_{SD} \le \gamma_{\text{th}}$ and 1 if $\gamma_{SD} > \gamma_{\text{th}}$. Here, $\gamma_{\text{th}}$ is the SINR threshold and is strictly positive. It is a system parameter that we shall optimize later. Therefore, the 1-bit rule $\phi_{\text{1-bit}}$ is a mapping: $\phi_{\text{1-bit}} : \{0,1\} \times (\mathbb{R}^+)^L \times (\mathbb{R}^+)^L \times (\mathbb{R}^+)^L \to \{0,1,\ldots,L\}$, that selects one out of the $L+1$ relays for every realization of $f \in \{0,1\}$, $\left\{|h_{Si}|^2\right\}_{i=1}^L$, $\left\{|h_{iD}|^2\right\}_{i=1}^L$, and $\left\{|h_{iP}|^2\right\}_{i=1}^L$.

Starting from the optimal RS rule in (11), we develop the 1-bit rule using the following steps:

- Firstly, substituting $\theta = \pi/2$ in the integrand in (6), and using the inequality $e^{-x} \le \frac{1}{1+x}$, for $x \ge 0$, yields the following bound:

$$\text{SEP}\left(|h_{SD}|^2, |h_{S\beta}|^2, |h_{\beta D}|^2\right) \le \frac{me^{-q\gamma_{SD}}}{1+q\gamma_\beta}. \tag{13}$$

- Secondly, we replace the single integral term $\text{SEP}\left(|h_{SD}|^2, |h_{Si}|^2, |h_{iD}|^2\right)$ in (11) with this bound in (13). We also replace $\gamma_{SD}$ with its expected value conditioned on the feedback, i.e., we replace $\gamma_{SD}$ with $\mathbb{E}\left[\gamma_{SD}|\gamma_{SD} \le \gamma_{\text{th}}\right]$ when $f = 0$ and with $\mathbb{E}\left[\gamma_{SD}|\gamma_{SD} > \gamma_{\text{th}}\right]$ when $f = 1$.

Finally, removing the common constant $m$, we get the following simpler 1-bit rule:

For $I_{\text{un}} \le I_{\text{avg}}$,

$$\beta = \underset{i\in\{1,\ldots,L\}}{\text{argmax}}\,\{\gamma_i\}, \tag{14}$$

and for $I_{\text{un}} > I_{\text{avg}}$,

$$\beta = \begin{cases} \text{argmin}_{i\in\{0,\ldots,L\}}\left\{\frac{c_0}{1+q\gamma_i} + \lambda P_i|h_{iP}|^2\right\}, & \gamma_{SD} \le \gamma_{\text{th}}, \\ \text{argmin}_{i\in\{0,\ldots,L\}}\left\{\frac{c_1}{1+q\gamma_i} + \lambda P_i|h_{iP}|^2\right\}, & \gamma_{SD} > \gamma_{\text{th}}, \end{cases} \tag{15}$$

where $c_1 = e^{-q\mathbb{E}[\gamma_{SD}|\gamma_{SD}>\gamma_{\text{th}}]} = e^{-q(\gamma_{\text{th}}+\overline{\gamma}_{SD})}$ and $c_0 = e^{-q\mathbb{E}[\gamma_{SD}|\gamma_{SD}\le\gamma_{\text{th}}]} = \exp\left(\frac{-q\left(\overline{\gamma}_{SD}-(\gamma_{\text{th}}+\overline{\gamma}_{SD})e^{-\frac{\gamma_{\text{th}}}{\overline{\gamma}_{SD}}}\right)}{1-e^{-\frac{\gamma_{\text{th}}}{\overline{\gamma}_{SD}}}}\right)$.

As before, $\lambda$ is strictly positive and is chosen such that the average interference constraint is satisfied with equality. We see that the 1-bit rule in (15) is dependent on the state of the SD link through the bit $f$, which needs to be broadcast by $D$ to all the relays instead of $\gamma_{SD}$. This enables the relays to participate in a distributed selection scheme such as the timer scheme [18] with $D$ serving as the coordinating node. In it, each relay sets a timer whose value is a monotone non-decreasing function of its metric $\left(\frac{c_f}{1+q\gamma_i} + \lambda P_i|h_{iP}|^2\right)$, which depends on $f$. The relay transmits a packet to $D$ containing its identity upon expiry of the timer. The first relay to transmit is the desired best relay. No exchange of information is required between the relays.

*Without SD Link* ($\gamma_{SD} = 0$): When no SD link is present, e.g., due to path loss or severe shadowing, the fully SD-aware optimal rule in (11) reduces to $\beta^* = \text{argmin}_{i\in\{0,\ldots,L\}}\left\{\frac{1}{\pi}\int_0^{m\pi} e^{-\frac{q\gamma_i}{\sin^2\theta}}d\theta + \lambda P_i|h_{iP}|^2\right\}$, for $I_{\text{un}} > I_{\text{avg}}$. Note that the SD-unaware optimal rule in (12) also reduces to the same form, which makes intuitive sense. Furthermore, for this scenario, $c_0 = 1$, and the 1-bit rule in (15) simplifies to $\beta = \text{argmin}_{i\in\{0,\ldots,L\}}\left\{\frac{1}{1+q\gamma_i} + \lambda P_i|h_{iP}|^2\right\}$. For $I_{\text{un}} \le I_{\text{avg}}$, all three rules are identical and are given by $\beta = \text{argmax}_{i\in\{1,\ldots,L\}}\{\gamma_i\}$.

## V. Simulation Results and Performance Benchmarking

We now present Monte Carlo simulation results using $10^5$ samples in Matlab to optimize the threshold of the 1-bit rule, benchmark the proposed rules with several ad hoc rules proposed in the literature, and gain quantitative insights into their behavior. For the sake of illustration, we use the following parameters: $P_t = P_r = P = 10$ dB, $I_{\text{avg}} = 15$ dB, $\sigma_0^2 = 0$ dB, and $\sigma_1^2 = \sigma_2^2 = \sigma^2 = 3.36$ dB. Furthermore, we set $\{\mu_{Si}\}_{i=1}^L = \{\mu_{iD}\}_{i=1}^L = \{\mu_{iP}\}_{i=1}^L = \mu_{SD} = \mu$, and we vary $\mu$ from $-5$ dB to 20 dB. Thus, the average SINR $\frac{P\mu}{\sigma_0^2+\sigma^2}$ of the various links varies from 0 to 25 dB.

### A. Optimization of Threshold $\gamma_{th}$ of 1-Bit Rule

We first determine the optimal threshold $\gamma_{\text{th}}^*$, at which the SEP of the 1-bit rule is the minimum. Fig. 2 plots the SEP of the 1-bit rule as a function of $\gamma_{\text{th}}$ for different values of $\mu$. As $\gamma_{\text{th}}$ increases, the SEP initially decreases, reaches a minimum at $\gamma_{\text{th}}^*$ (which is indicated by a vertical line in the plot) and then starts increasing. Further, as $\mu$ increases, the SEP decreases.

### B. Comparison of Proposed Rules and Benchmarking

Fig. 3 compares the SEPs of the fully SD-aware optimal rule and 1-bit rule (with optimal threshold $\gamma_{\text{th}}^*$) as a function of the average SINR $\frac{P\mu}{\sigma_0^2+\sigma^2}$. As a reference, the SEPs of the conventional non-cognitive relay network, which corresponds to $I_{\text{avg}} = \infty$, and a non-cooperative network that uses only the direct SD link, which corresponds to $I_{\text{avg}} = 0$, are shown. All the SEPs lie between these two curves. When $\frac{P\mu}{\sigma_0^2+\sigma^2} \le 10$ dB, the network is not interference-constrained. Hence, the SEPs of the 1-bit and fully SD-aware optimal rules are the same

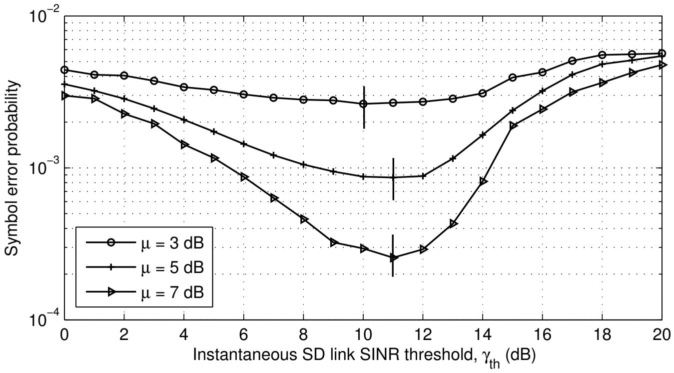


Fig. 2. *Finding the optimum threshold*: SEP of the 1-bit rule as a function of $\gamma_{\text{th}}$ for different average channel power gain $\mu$ ($L = 2$ and QPSK).

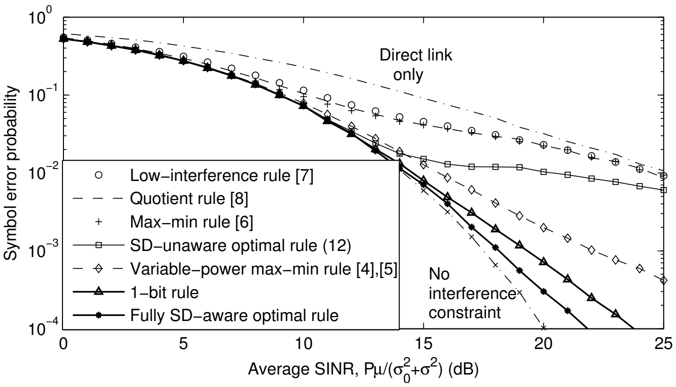


Fig. 3. Comparison of the SEPs of the fully SD-aware optimal and 1-bit rules with several other rules proposed in the literature ($L = 4$ and 8PSK).

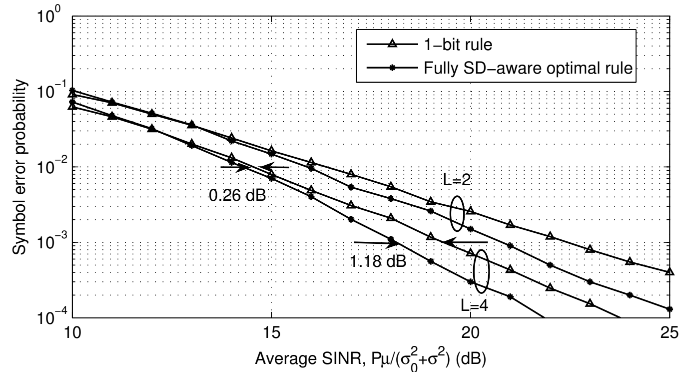


Fig. 4. *Efficacy of* 1*-bit rule for different number of relays* $L$: SEPs of the optimal and 1-bit rules as a function of average SINR and 8PSK.

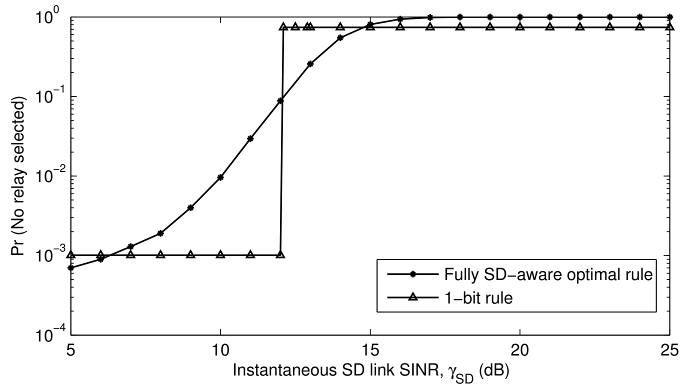


Fig. 5. *Effect of SD link into RS*: Probability that no relay is selected as a function of $\gamma_{SD}$ for different rules (8PSK, $L = 2$, $\mu = 3$ dB, and $\gamma_{\text{th}}^* = 12$ dB).

as that of the unconstrained rule. When $\frac{P\mu}{\sigma_0^2+\sigma^2} > 10$ dB, the network is interference-constrained and the performances of the two rules differ. As expected, the optimal rule has the lowest SEP. The SEP of the 1-bit rule is marginally worse for high average SINRs.

This figure also plots the SEPs of the SD-unaware optimal rule in (12), low-interference rule [7], max-min rule [6], quotient rule [8], and variable-power max-min rule [4], [5] considered in the literature, which have been adapted to our model with the interference threshold set as $I_{\text{avg}}$. We see that the two proposed rules outperform all the benchmark rules over the entire range of average SINRs. For example, at an average SINR of 17 dB, the optimal rule lowers the SEP by a factor of 16.5, 6.0, and 3.0 as compared to the quotient rule, SD-unaware optimal rule, and variable-power max-min rule, respectively. Equivalently, this translates into significant power savings for the same SEP. Also, the performance gap increases as the average SINR increases. This is because the SEPs of the low-interference, quotient, max-min, and SD-unaware optimal rules approach that of a non-cooperative network in order to satisfy the interference constraint. Similar trends occur when different channels are non-identically distributed.

Fig. 4 plots the SEP of the proposed rules as a function of the average SINR in the interference-constrained region for different number of relays $L$. As expected, as $L$ increases, the SEP decreases for both rules. We see that the 1-bit rule is very close to the fully SD-aware optimal rule for low average SINRs, while at high average SINRs, it incurs a marginal degradation in performance. For example, with $L = 4$ relays, the 1-bit rule is within 0.26 dB and 1.18 dB of the fully SD-aware optimal rule at SEPs of $10^{-2}$ and $10^{-3}$, respectively.

In order to understand the effect of the SD link on RS, Fig. 5 plots the probability $\Pr(\beta = 0)$ that no relay gets selected as a function of instantaneous SINR of the SD link $\gamma_{SD}$ for different RS rules. We see that $\Pr(\beta = 0)$ increases monotonically as $\gamma_{SD}$ increases for the fully SD-aware optimal rule. Thus, the relays transmit less often when the SD link is stronger. For the 1-bit rule, from (15), we see that $\Pr(\beta = 0)$ takes only two values, which explains the staircase shape for it. Furthermore, in (15), since $c_1 < c_0$, $\Pr(\beta = 0)$ is greater when $\gamma_{SD} > \gamma_{\text{th}}^*$. This is unlike the SD-unaware rules, for which $\Pr(\beta = 0)$ is a constant for the entire range of $\gamma_{SD}$.

## VI. Conclusions

We proposed two novel SD-aware RS rules for an average interference-constrained underlay CR network. We first derived the fully SD-aware optimal rule, where the choice of the relay depended on the SINR of the SD link. The 1-bit rule was a simpler and low feedback variant of the optimal rule, in which the selected relay depended only on whether the SINR of the SD link exceeded a threshold, which was optimized. We saw that the 1-bit rule was quite close in performance to the optimal rule. In both rules, the odds that no relay got selected increased as the SINR of the SD link increased. Furthermore, both rules reduced the SEP by up to two orders of magnitude compared to several SD-unaware rules proposed in the literature. The significant gains motivate the design of new SD-aware low feedback RS protocols for underlay CR. Analytically characterizing the optimal value of $\gamma_{\text{th}}$ and finding the parameter $\lambda$ is an interesting avenue for future work.

## APPENDIX

### A. Proof of Result 1

When $I_{\text{un}} \leq I_{\text{avg}}$, the unconstrained rule in (10) is feasible. Now, the SEP-optimal selected relay $\beta^*$ is given from (6) by

$$\beta^* = \underset{i\in\{1,\ldots,L\}}{\operatorname{argmin}} \{e^{-q(\gamma_{SD}+\gamma_i)}\} = \underset{i\in\{1,\ldots,L\}}{\operatorname{argmax}} \{\gamma_i\}. \quad (16)$$

Now consider the case when $I_{\text{un}} > I_{\text{avg}}$. In this case, the unconstrained rule is not feasible because it does not satisfy the interference constraint in (8) and, thus, cannot be optimal. A selection rule in which no relay transmits causes zero relay interference to the $P_{\text{Rx}}$ and is, thus, feasible for any $I_{\text{avg}}$. Therefore, the set of all feasible RS rules, $\mathcal{Z}$, is a non-empty set. Let $\phi \in \mathcal{Z}$ be a feasible rule. For a constant $\lambda > 0$, define an auxiliary function $L_\phi(\lambda)$ associated with $\phi$ as

$$L_\phi(\lambda) \triangleq \mathbb{E}_{h_{SD},\mathbf{h}}\left[\text{SEP}\left(|h_{SD}|^2, |h_{S\beta}|^2, |h_{\beta D}|^2\right) + \lambda P_\beta |h_{\beta P}|^2\right]. \quad (17)$$

It is a function of both $\phi$ and $\lambda$. Further, define a new rule $\phi^* \in \mathcal{Z}$ in terms of the relay $\beta^*$ it selects as follows:

$$\beta^* = \underset{i\in\{0,\ldots,L\}}{\operatorname{argmin}} \{\text{SEP}\left(|h_{SD}|^2, |h_{Si}|^2, |h_{iD}|^2\right) + \lambda P_i |h_{iP}|^2\}. \quad (18)$$

We now prove that $\phi^*$ is the desired optimal RS rule. From (17) and the definition of $\phi^*$ in (18), it follows that $L_{\phi^*}(\lambda) \leq L_\phi(\lambda)$. Therefore,

$$\begin{aligned}&\mathbb{E}_{h_{SD},\mathbf{h}}\left[\text{SEP}\left(|h_{SD}|^2, |h_{S\beta^*}|^2, |h_{\beta^* D}|^2\right) + \lambda P_{\beta^*} |h_{\beta^* P}|^2\right] \\ &\leq \mathbb{E}_{h_{SD},\mathbf{h}}\left[\text{SEP}\left(|h_{SD}|^2, |h_{S\beta}|^2, |h_{\beta D}|^2\right) + \lambda P_\beta |h_{\beta P}|^2\right]. \end{aligned} \quad (19)$$

Choose $\lambda$ such that $\mathbb{E}_{h_{SD},\mathbf{h}}\left[P_{\beta^*}|h_{\beta^* P}|^2\right] = I_{\text{avg}}$.[2] Thus, $\phi^*$ is a feasible rule. Rearranging the terms in (19), we get

$$\begin{aligned}&\mathbb{E}_{h_{SD},\mathbf{h}}\left[\text{SEP}\left(|h_{SD}|^2, |h_{S\beta^*}|^2, |h_{\beta^* D}|^2\right)\right] \\ &\quad\leq \mathbb{E}_{h_{SD},\mathbf{h}}\left[\text{SEP}\left(|h_{SD}|^2, |h_{S\beta}|^2, |h_{\beta D}|^2\right)\right] \\ &\qquad + \lambda\left(\mathbb{E}_{h_{SD},\mathbf{h}}\left[P_\beta |h_{\beta P}|^2\right] - I_{\text{avg}}\right). \end{aligned} \quad (20)$$

[2]That such a unique choice of $\lambda$ exists can be proved using the intermediate value theorem by observing that $0 \leq I_{\text{avg}} < I_{\text{un}}$, and by proving that the average interference is a continuous and monotonically decreasing function of $\lambda$ for $\lambda > 0$. The detailed derivation is not shown due to space constraints.

Since $\phi$ is a feasible rule, $\mathbb{E}_{h_{SD},\mathbf{h}}\left[P_\beta |h_{\beta P}|^2\right] \leq I_{\text{avg}}$. Hence, $\lambda\left(\mathbb{E}_{h_{SD},\mathbf{h}}\left[P_\beta |h_{\beta P}|^2\right] - I_{\text{avg}}\right) \leq 0$. Then, from (20), we get

$$\begin{aligned}&\mathbb{E}_{h_{SD},\mathbf{h}}\left[\text{SEP}\left(|h_{SD}|^2, |h_{S\beta^*}|^2, |h_{\beta^* D}|^2\right)\right] \\ &\quad\leq \mathbb{E}_{h_{SD},\mathbf{h}}\left[\text{SEP}\left(|h_{SD}|^2, |h_{S\beta}|^2, |h_{\beta D}|^2\right)\right]. \end{aligned} \quad (21)$$

Thus, $\phi^*$ yields the lowest average SEP among all feasible rules. It is, therefore, optimal.

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