Cognitive Relay Selection with Incomplete Channel State Information of Interference Links

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***Abstract*—The availability of channel state information (CSI) about the interference links from the secondary transmitters to the primary receivers is widely assumed in the literature on underlay cognitive radio (CR) in order to control the interference caused to the primary network. However, when multiple primary receivers are present, acquiring such CSI about all the interference links in a timely and scalable manner is practically challenging. We study an underlay cooperative relay system, in which the channel gains of only a subset of the interference links are available at the source and relays. Based on such incomplete CSI, the source and relays back-off their transmit powers in order to satisfy an interference outage constraint. We derive a tight upper bound on the outage probability of the secondary system for the rate-optimal relay selection rule. Our numerical results show the effect of incomplete CSI on the secondary system performance and how its impact can be ameliorated.**

***Index Terms*—Cognitive radio, underlay, relays, selection.**

I. Introduction

Cognitive radio (CR) promises to improve the utilization of scarce wireless spectrum by allowing secondary users (SUs) to access the spectrum allocated to primary users (PUs). In the underlay mode of CR, which is the focus of this paper, an SU can simultaneously transmit on the same band as a higher priority PU as long as the interference it causes to the PU is tightly controlled [1]–[3]. However, this interference constraint also limits the data rate and coverage area of the SUs.

Cooperative relaying, which is considered to be a key technology for next generation wireless networks [3], [4], in combination with relay selection, is an attractive solution to address this shortcoming. In it, a single "best" relay that satisfies the interference constraint is selected to forward a message from a secondary source (S) to a destination (D). Selection is practically appealing because it avoids the timing synchronization problems that arise when multiple geographically separated relays have to transmit simultaneously.

A fundamental difference that arises in underlay CR, when compared to conventional cooperative systems, is that the source transmit power, selected relay, and relay transmit power also depend on the interference caused to the PUs. Several factors control this dependence. The first factor is the nature of the interference constraint. Several constraints, such as peak interference constraint [2], [5], average interference constraint [6], and interference outage constraint [1], have been considered in the literature. The second one is the number of primary receivers since the interference constraint needs to be satisfied at all of them. The third one is the channel state information (CSI) available at the secondary transmitter about the interference link to the primary receiver [7]. We discuss the various models considered in the CR literature with multiple primary receivers that capture this dependence.

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A. Literature on Relaying with Multiple Primary Receivers

Without Direct Source-to-destination (SD) Link: A single relay underlay CR network that operates under the peak interference constraint is considered in [2], [5], [8]. Multiple relays that are subject to a peak interference constraint are instead considered in [9]–[12]. The amplify-and-forward (AF) relay that maximizes the source-to-relay (SR) link signal-to-noise-ratio (SNR) is selected in [9], while the same is done in [10], [11] for a decode-and-forward (DF) relay. Instead, in [12], the DF relay that maximizes the minimum of the SR and relay-to-destination (RD) link SNRs is selected.

With Direct SD Link: A peak interference-constrained underlay CR network with a direct SD link is considered in [3], [13], [14]. The outage probability for DF and AF relays is analyzed in [13], in which either the SD link or the best relay link with the maximum end-to-end SNR is selected. In [3], the DF relay that maximizes the minimum of the SR link SNR and the sum of the RD and SD link SNRs is selected. Instead, in [14], the DF relay that maximizes the RD link SNR and has decoded the source signal is selected if the SD link SNR falls below a threshold; otherwise only the SD link is selected.

B. Focus and Contributions

In order to meet the interference constraint, it is assumed in [2], [3], [5], [8]–[14] that the channel power gains of *all* the interference links to the primary receivers are available at the secondary transmitters. However, as the number of primary receivers increases, it becomes difficult to acquire all this information in a timely manner. While the channel power gain of an interference link can be estimated by exploiting channel reciprocity and sensing the transmitted signals from a primary receiver whenever it communicates with a primary transmitter, such transmissions are not under the control of the secondary system [1], [13]. The alternate approach, in which a third party such as a band manager helps exchange the CSI between the PUs and SUs [14], also faces similar challenges.

In order to capture the above challenges and evaluate their impact on the secondary system performance, we study the

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following *incomplete CSI* model for M primary receivers. The source and the relays know the channel power gains of N out of M interference links, where $1 \leq N \leq M$. This can be acquired by the aforementioned methods. Furthermore, the subset of primary receivers, whose interference links are known, can be different for S and the relays. When $N = M$, we shall say that the secondary system has *complete* CSI, which is the model assumed in [3], [8]–[14]. The transmit powers of S and the relays are controlled as a function of the CSI available about the N interference links in order to satisfy an interference outage constraint for each primary receiver.

For this power control scheme, we derive a tight upper bound on the outage probability of the relay selection rule that maximizes the rate of data transmission from S to D. The outage probability is an important performance measure of the secondary system [3], [8]–[14]. The average rate, which is a measure of spectral efficiency of the system, can also be computed from it. From a high signal-to-interference-plus-noise ratio (SINR) asymptotic analysis, we show that full diversity order is achieved even with incomplete CSI. To the best of our knowledge, this analysis is novel even for the special case of complete CSI. We present extensive simulation results to characterize the effect of various system parameters.

C. Outline and Notation

The paper is organized as follows. Section II develops the system model. The outage probability is analyzed in Section III. Numerical results are presented in Section IV, and are followed by our conclusions in Section V.

We shall use the following notation henceforth. The absolute value of x is denoted by $|x|$. The probability of an event A and the conditional probability of A given event B are denoted by $\Pr(A)$ and $\Pr(A|B)$, respectively. $\mathbb{E}_X[.]$ denotes the expectation with respect to a random variable (RV) X. The indicator function $1_{\{a\}}$ is 1 if a is true and is 0 otherwise. And, $X \sim CN(0, \sigma^2)$ means that X is a circularly symmetric zero-mean complex Gaussian RV with variance σ^2.

II. System Model

Consider a primary network, in which a primary transmitter T communicates with M primary receivers $X_1, X_2, \ldots, X_M$. An underlay secondary system co-exists with it. In it, a source S transmits data to a destination D using L DF relays $R_1, R_2, \ldots, R_L$, as shown in Fig. 1. Each node is equipped with a single antenna. The complex baseband channel gain from S to X_m is g_{Sm}, from S to D is h_{SD}, from S to R_i is h_{Si}, from R_i to D is h_{iD}, and from R_i to X_m is g_{im}, where $1 \leq m \leq M$ and $1 \leq i \leq L$. We assume that the various links undergo Rayleigh fading and are mutually independent: $h_{SD} \sim CN(0, \mu_{SD})$, $h_{Si} \sim CN(0, \mu_{SR})$, $h_{iD} \sim CN(0, \mu_{RD})$, $g_{Sm} \sim CN(0, \mu_{SX})$, and $g_{im} \sim CN(0, \mu_{RX})$, where μ_{SD}, μ_{SR}, μ_{RD}, μ_{SX}, and μ_{RX} denote the respective mean channel power gains [2], [3], [8], [11], [15]. The above means are function of the path-loss between the corresponding nodes. The channel fades are assumed to remain constant over the duration of relay selection and data transmission [11], [13].

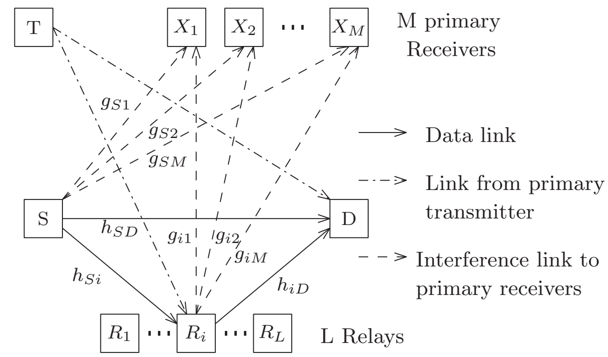

Fig. 1. An underlay CR system with a source S, a destination D, and L secondary relays $R_1, R_2, \ldots, R_L$ that co-exists with a primary network consisting of a transmitter T and receivers $X_1, X_2, \ldots, X_M$.

A. CSI Model

Let Φ_S denote the set of indices of primary receivers for whom the interference channel power gains are available at S. Similarly, let Φ_i denote the set of indices of primary receivers for whom the interference channel power gains are available at R_i, for $1 \leq i \leq L$. The sets Φ_S, $\Phi_1, \Phi_2, \ldots, \Phi_L$ have a cardinality of N, where $1 \leq N \leq M$.

We assume that the secondary nodes have perfect CSI of their respective secondary links [2], [5], [8]. Specifically, S knows $|h_{SD}|^2$ and $|h_{Si}|^2$, for $1 \leq i \leq L$, which it can acquire in practice by using a training protocol and exploiting channel reciprocity [16]. Similarly, R_i or D estimate $|h_{iD}|^2$ and feed it back to S for relay selection. For coherent demodulation, D is assumed to know h_{SD}, $h_{S\beta}$, and $h_{\beta D}$, where R_β denotes the selected relay [17]. It can estimate h_{SD} and $h_{\beta D}$ by using a training protocol [16], and it can learn about $h_{S\beta}$ from S.

B. Interference Outage Constraint and Power Control

Let O denote the the probability that the interference power at any primary receiver due to secondary transmissions exceeds a threshold I_{th}. The interference outage constraint requires that O cannot exceed p_{o}, where $0 \leq p_{\text{o}} \leq 1$ [1]. This is a generalization of the peak interference constraint, which corresponds to $p_{\text{o}} = 0$. Let $I_{Sk} = P_s|g_{Sk}|^2$ denote the instantaneous interference power at X_k, for $1 \leq k \leq M$, when S transmits with power P_s. Then, the interference outage constraint is given by

$$\Pr(I_{S1} \leq I_{\text{th}}, I_{S2} \leq I_{\text{th}}, \ldots, I_{SM} \leq I_{\text{th}}) \geq 1 - p_{\text{o}}. \quad (1)$$

Similarly, let $I_{ik} = P_i|g_{ik}|^2$ denote the instantaneous interference power at X_k when R_i transmits with power P_i. The corresponding interference outage constraint is given by

$$\Pr(I_{i1} \leq I_{\text{th}}, I_{i2} \leq I_{\text{th}}, \ldots, I_{iM} \leq I_{\text{th}}) \geq 1 - p_{\text{o}}, \ 1 \leq i \leq L. \quad (2)$$

We consider the following power control policy in which S sets its transmit power P_s as a function of the interference channel power gains $|g_{Sm}|^2$, for $m \in \Phi_S$, as follows:

$$P_s = \frac{I_{\text{th}}}{\tau_S \max\limits_{m \in \Phi_S} \{|g_{Sm}|^2\}}, \quad (3)$$

where τ_S (≥ 1) is the source power back-off factor that is chosen in order to satisfy the interference outage constraint in (1) with equality. In this case, the instantaneous interference power at X_k is $I_{Sk} = \frac{I_{\text{th}}|g_{Sk}|^2}{\tau_S \max_{m\in\Phi_S}\{|g_{Sm}|^2\}}$, $1 \leq k \leq M$. Similarly, the transmit power P_i of relay R_i is given by

$$P_i = \frac{I_{\text{th}}}{\tau_R \max_{m\in\Phi_i}\{|g_{im}|^2\}}, \quad \text{for } 1 \leq i \leq L, \tag{4}$$

where the relay power back-off factor τ_R (≥ 1) is chosen in order to satisfy (2) with equality. The power back-off factors for all the relays are identical because the interference links from the relays to the primary receivers are assumed to be statistically identical. In this case, the instantaneous interference power at X_k is $I_{ik} = \frac{I_{\text{th}}|g_{ik}|^2}{\tau_R \max_{m\in\Phi_i}\{|g_{im}|^2\}}$, for $1 \leq k \leq M$.

While a similar power back-off policy is assumed in [10], [18], the direct SD link is ignored and the secondary system is assumed to know the channel gains of all the interference links, albeit imperfectly. For analytical tractability, we do not model a peak transmit power constraint for any node [2], [15].

C. Data Transmission Protocol and Preliminaries

S can transmit data to D via a selected DF relay R_β, where $\beta \in \{1, 2, \ldots, L\}$, or directly, which we shall denote by a virtual relay with $\beta = 0$ and $h_{S0} = h_{0D} \triangleq 0$. We consider the proactive model of relaying, in which relay selection precedes data transmission by S [3], [13], [19]. This enables S to adapt its transmission rate as a function of the instantaneous SINRs of the SR, RD, and SD links.

If a relay R_β, for $\beta \in \{1, 2, \ldots, L\}$, is selected, then in the first time slot, S transmits a data symbol x_s with a power P_S, and R_β and D listen. In the second time slot, R_β retransmits the signal decoded by it to D with a power P_β. The destination coherently combines the signals received from S and R_β over two time slots using maximal ratio combining (MRC). The instantaneous SINR at D after MRC is $\gamma_{SD} + \gamma_{\beta D}$, where $\gamma_{SD} = \frac{P_s|h_{SD}|^2}{\sigma_0^2+\sigma_2^2}$ is the SINR of the direct SD link, $\gamma_{\beta D} = \frac{P_\beta|h_{\beta D}|^2}{\sigma_0^2+\sigma_2^2}$ is the SINR of the link between R_β and D, σ_0^2 is the Gaussian noise variance at the relays and D, and σ_2^2 is the variance of the interference at D due to transmissions by T.

The instantaneous rate C_β between S and D in bits/sec/Hz when R_β, for $\beta \in \{1, 2, \ldots, L\}$, is selected is given by [19][1]

$$C_\beta = \frac{1}{2}\log_2\left(1 + \min\{\gamma_{S\beta}, \gamma_{\beta D} + \gamma_{SD}\}\right), \tag{5}$$

where $\gamma_{S\beta} = \frac{P_s|h_{S\beta}|^2}{\sigma_0^2+\sigma_1^2}$ is the SINR of the link between S and R_β, and σ_1^2 is the variance of the Gaussian interference from T to R_β. If no relay is selected ($\beta = 0$), the source sends a new message in the second time slot, and the instantaneous rate C_0 equals

$$C_0 = \log_2\left(1 + \gamma_{SD}\right). \tag{6}$$

D. Rate-Optimal Relay Selection Rule

For the transmit power control policy given in Section II-B above, the rate-optimal relay selection rule is clearly

$$\beta = \underset{i\in\{0,1,\ldots,L\}}{\text{argmax}}\{C_i\}. \tag{7}$$

From (5) and (6), it can be shown that this rule is a non-linear, quadratic function of γ_{SD}.

III. Outage Probability with Incomplete CSI

We first derive τ_S and τ_R in terms of the system parameters.

A. Computing τ_S and τ_R

Lemma *1:* The probability $\mathfrak{I}_S$ that no interference outage occurs due to transmissions by S is given by

$$\mathfrak{I}_S = N\sum_{m=0}^{M-N}\binom{M-N}{m}\sum_{k=0}^{N-1}\binom{N-1}{k}\frac{(-1)^{m+k}}{m\tau_S + k + 1}. \tag{8}$$

The probability $\mathfrak{I}_R$ that no interference outage occurs due to transmissions by a relay is the same as (8) except that τ_S is replaced by τ_R.

Proof: The proof is relegated to Appendix A. ■

Since τ_S and τ_R are the solutions of the equations $\mathfrak{I}_S = 1 - p_{\text{o}}$ and $\mathfrak{I}_R = 1 - p_{\text{o}}$, respectively, they can be easily computed using routines such as `fsolve` in Matlab. We also see that $\tau_S = \tau_R$. This is because they are not functions of μ_{SX} or μ_{RX}. For the special case of complete CSI, i.e., $N = M$, the peak interference constraint at each primary receiver is always satisfied. Therefore, $p_{\text{o}} = 0$ and $\tau_S = \tau_R = 1$.

We now analyze the outage probability of the rate-optimal relay selection rule in (7) given $\tau_S = \tau_R = \tau$. It is defined as the probability $P_{\text{out}}(r)$ that the source transmission rate r exceeds the capacity of the secondary system. We present below a closed-form upper bound for $P_{\text{out}}(r)$.

Result *1:* The outage probability $P_{\text{out}}(r)$ under incomplete CSI of the interference links is upper bounded by

$$P_{\text{out}}(r) \leq 1 - N\sum_{k_1=0}^{N-1}\frac{\binom{N-1}{k_1}(-1)^{k_1}}{k_1 + 1 + \frac{(2^r-1)\,\mu_{SX}}{q_{SD}}} + \frac{N^2}{\mu_{SX}\,\mu_{RX}\,\mu_{SD}}\sum_{k=1}^{L}\sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1}\binom{L}{k}\binom{N-1}{k_1}\binom{N-1}{k_2} \times (-1)^{k+k_1+k_2}J(k,k_1,k_2), \tag{9}$$

where

$$J(k,k_1,k_2) = \frac{q_{RD}\mu_{SD}}{2kq_{SD}}\left[\frac{{}_2F_1\left(1,2;3;\frac{\mathcal{B}}{\mathcal{C}}\right)}{\mathcal{C}^2} - \frac{{}_2F_1\left(1,2;3;\frac{\mathcal{B}}{\mathcal{A}}\right)}{\mathcal{A}^2}\right], \tag{10}$$

$q_{SD} = \frac{I_{\text{th}}\,\mu_{SD}}{\tau\left(\sigma_0^2+\sigma_2^2\right)}$, $q_{SR} = \frac{I_{\text{th}}\,\mu_{SR}}{\tau\left(\sigma_0^2+\sigma_1^2\right)}$, $q_{RD} = \frac{I_{\text{th}}\,\mu_{RD}}{\tau\left(\sigma_0^2+\sigma_2^2\right)}$, $\mathcal{A} = \frac{k_1+1}{\mu_{SX}} + \frac{k\left(2^{2r}-1\right)}{q_{SR}}$, $\mathcal{B} = \mathcal{A} + \frac{2^{2r}-1}{q_{SD}} + \frac{(k_2+1)q_{RD}}{k\,\mu_{RX}\,q_{SD}}$, $\mathcal{C} = \mathcal{A} + \frac{2^r-1}{q_{SD}}$,

[1] In order to arrive at this expression, the interferences at R_β and D due to transmissions by T are assumed to be Gaussian, which has been assumed in [1], [5], [17]. It is justified with one primary transmitter when it transmits a constant amplitude signal over a Rayleigh fading link [1], and with many primary transmitters by the central limit theorem. We refer the reader to [17] for a comparison of this Gaussian interference model with other models.

and ${}_2F_1(\alpha, \beta; \gamma; z)$ is the Gauss hypergeometric function [20, (9.111)].

Proof: The proof is relegated to Appendix B. ■

The first term $1 - N\sum_{k_1=0}^{N-1} \frac{\binom{N-1}{k_1}(-1)^{k_1}}{k_1+1+\frac{(2^r-1)\mu_{SX}}{q_{SD}}}$ in (9) is the contribution from the direct SD link transmissions. The second term is the contribution from the L relays. At first sight, $P_{\text{out}}(r)$ depends only on the number of primary receivers N whose CSI is available. However, it implicitly depends on M since τ is a function of M. Substituting $N = M$ in (9) yields the outage probability upper bound for the complete CSI model.

B. Asymptotic Outage Probability for High SINR

In order to get more insights, we investigate the high SINR asymptotic regime. Let $\sigma_1^2 = \sigma_2^2 = \sigma^2$ and $\gamma = 1/\left(\sigma_0^2 + \sigma^2\right)$ denote the system SINR [9], [13]–[15].

Corollary *1:* In the high SINR regime, i.e., as $\gamma \to \infty$, the outage probability upper bound in (9) simplifies to

$$P_{\text{out}}(r) \approx \frac{N^2}{\mu_{SX}\mu_{RX}\mu_{SD}} \sum_{k=0}^{L}\sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1}\binom{L}{k}\binom{N-1}{k_1}\binom{N-1}{k_2} \times (-1)^{k_1+k_2}\ J'(k,k_1,k_2), \quad (11)$$

where

$$J'(k,k_1,k_2) \approx \frac{\tau^{L+1}(2^{2r}-1)^k\ \Gamma(L-k+1)}{(I_{\text{th}}\gamma)^{L+1}\ \mu_{SR}^k\ \mu_{RD}^{L-k}\ \left(\frac{k_2+1}{\mu_{RX}}\right)^{L-k+1}} \times \left[\frac{\left((2^{2r}-1)^{L-k+1} - (2^{2r}-2^r)^{L-k+1}\right)\Gamma(k+2)}{(L-k+1)\left(\frac{k_1+1}{\mu_{SX}} + \frac{2^{2r}-1}{q_{SD}}\right)^{k+2}} + \frac{\left((2^{2r}-1)^{L-k+2} - (2^{2r}-2^r)^{L-k+2}\right)\Gamma(k+3)}{q_{SD}\ (L-k+2)\left(\frac{k_1+1}{\mu_{SX}} + \frac{2^{2r}-1}{q_{SD}}\right)^{k+3}}\right], \quad (12)$$

and $\Gamma(z)$ denotes the Gamma function [20, (8.310)].

Proof: The proof is relegated to Appendix C. ■

Since $q_{SD} \to \infty$ as $\gamma \to \infty$, we have $\frac{k_1+1}{\mu_{SX}} + \frac{2^{2r}-1}{q_{SD}} \approx \frac{k_1+1}{\mu_{SX}}$ in (12). Furthermore, the term in the third line of (12) can be neglected compared to the term in the second line of it when $q_{SD} \to \infty$. In this case, (11) simplifies to

$$P_{\text{out}}(r) \approx \frac{N^2}{(I_{\text{th}}\gamma)^{L+1}} \sum_{k=0}^{L}\sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1}\binom{L}{k}\binom{N-1}{k_1}\binom{N-1}{k_2} \times \frac{\tau^{L+1}\ (2^{2r}-1)^k\ \Gamma(L-k+1)\ (-1)^{k_1+k_2}}{\mu_{SX}\ \mu_{RX}\ \mu_{SD}\ \mu_{SR}^k\ \mu_{RD}^{L-k}\left(\frac{k_2+1}{\mu_{RX}}\right)^{L-k+1}} \times \frac{\left((2^{2r}-1)^{L-k+1} - (2^{2r}-2^r)^{L-k+1}\right)\Gamma(k+2)}{(L-k+1)\left(\frac{k_1+1}{\mu_{SX}}\right)^{k+2}}. \quad (13)$$

It is clear from (13) that the diversity order is $L+1$, and is independent of M, N, I_{th}, and p_{o}. Hence, the system achieves full diversity even with incomplete CSI. However, the coding gain is indeed a function of M, N, L, I_{th}, p_{o}, and the average channel power gains of the various links.

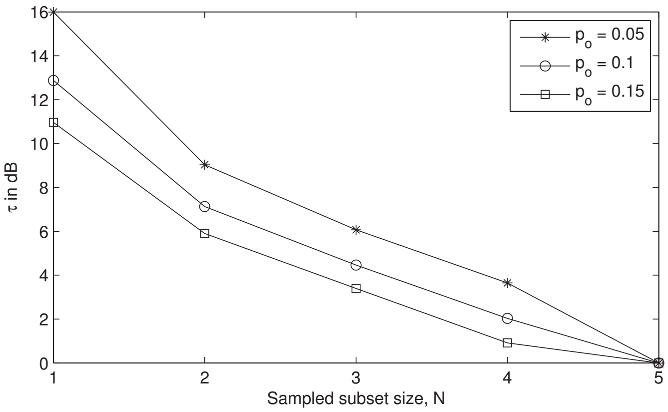

Fig. 2. Power back-off factor τ in dB as a function of sampled subset size N for different values of p_{o} ($M = 5$ and $I_{\text{th}} = 5$ dB).

IV. Numerical Results and Benchmarking

In order to verify the analysis and gain quantitative insights, we now present Monte Carlo simulation results that are averaged over 10^6 channel fades. We vary the system SINR $\gamma = 1/\left(\sigma_0^2 + \sigma^2\right)$. The average channel power gains of the various links are kept fixed. For the sake of illustration, we set $\mu_{SX} = \mu_{SD} = 1$, $\mu_{RX} = 2\mu_{SD}$, and $\mu_{SR} = \mu_{RD} = 10\mu_{SD}$.

Fig. 2 plots the power back-off factor $\tau_S = \tau_R = \tau$ in dB as a function of the sampled subset size N. When $N = M$, the CSI is complete and we get $\tau = 1$ (0 dB). As N decreases, the source and relays have CSI of fewer interference links. Hence, τ increases to meet the interference outage constraints. As the interference outage threshold p_{o} increases, τ decreases because the interference constraints have become more relaxed.

Fig. 3 plots the outage probability from simulations and its upper bound in (9) as a function of γ for different numbers of primary receivers M and the sampled subset size N. We see that the upper bound is within 0.5 dB of the curve obtained from simulations. For a fixed M, as N decreases, the outage probability increases because the CSI becomes more incomplete. This increases τ and reduces the transmit powers of S and the relays, which increases the outage probability. For example, for an outage probability of 0.001 and $M = 5$, the required γ increases by 1.7 dB when N is reduced from 5 to 4 and by an additional 4.0 dB when N is reduced to 2. Notice that the diversity order of 5 is achieved.

To study the effect of the number of relays L and the peak interference threshold I_{th}, Fig. 4 plots the outage probability from simulations, its upper bound in (9), and its high SINR asymptote in (11) as a function of γ. For a fixed I_{th}, as L increases, the outage probability decreases and the system diversity order $L+1$ increases. Thus, increasing the number of relays can mitigate the performance loss due to incomplete CSI. For a fixed L, as I_{th} increases, the outage probability decreases because the interference constraint is relaxed.

V. Conclusions

Acquiring CSI of all the interference links to multiple primary receivers in a timely manner is practically challenging for

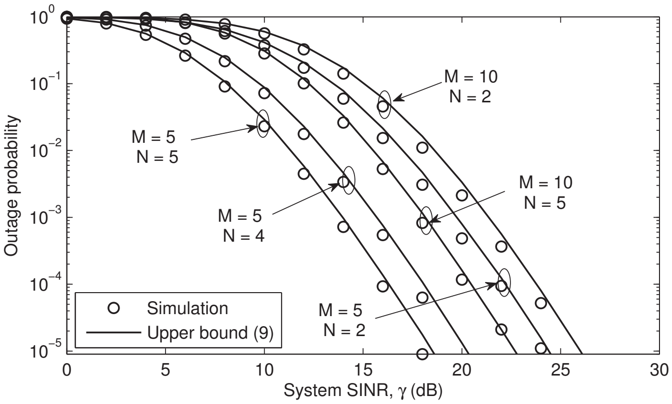

Fig. 3. Outage probability as a function of system SINR for different values of M and N ($p_{\text{o}} = 0.1$, $I_{\text{th}} = -5$ dB, $r = 1$ bit/sec/Hz, and $L = 4$).

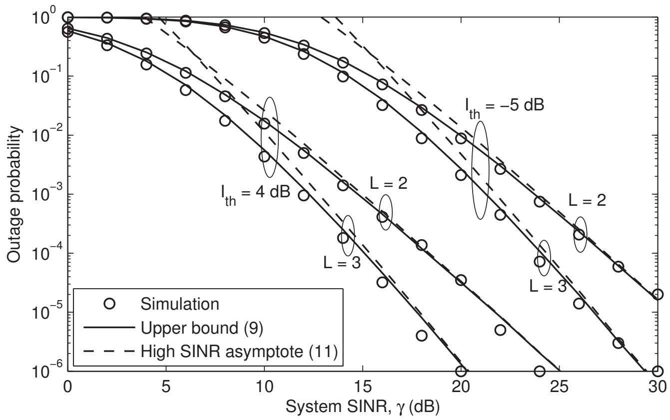

Fig. 4. Effect of L and I_{th} on the outage probability ($p_{\text{o}} = 0.1$, $M = 5$, $N = 2$, and $r = 1$ bit/sec/Hz).

a CR system. We studied a model in which the source and the selected relay backed-off their transmit powers on the basis of incomplete CSI of the interference links in order to continue to adhere to an interference outage constraint. We derived a tight upper bound on the outage probability for the rate-optimal relay selection rule, and showed that full diversity order was achieved even with incomplete CSI. We saw that increasing the number of relays could ameliorate the performance loss due to the incompleteness of the CSI. Interesting avenues for future work include incorporating noisy channel estimates, alternate relaying paradigms, and peak transmit power constraint.

Appendix

A. Brief Proof of Lemma 1

The probability $\mathfrak{I}_S$ that no interference outage occurs due to transmissions by S is given by

$$\begin{aligned}\mathfrak{I}_S &= \Pr(I_{S1} \leq I_{\text{th}}, I_{S2} \leq I_{\text{th}}, \ldots, I_{SM} \leq I_{\text{th}}),\\ &= \sum_{i=1}^{\binom{M}{N}} \Pr(I_{S1} \leq I_{\text{th}}, I_{S2} \leq I_{\text{th}}, \ldots, I_{SM} \leq I_{\text{th}} | \Phi_S = \phi_{Si})\\ &\qquad \times \Pr(\Phi_S = \phi_{Si}), \end{aligned} \tag{14}$$

where I_{Sk} is defined in Section II-B and $\phi_{S1}, \phi_{S2}, \ldots, \phi_{S\binom{M}{N}}$ are all possible equi-probable realizations of Φ_S. Since $|g_{S1}|^2, |g_{S2}|^2, \ldots, |g_{SM}|^2$ are i.i.d. RVs, (14) simplifies to

$$\begin{aligned}\mathfrak{I}_S &= \Pr(I_{S1} \leq I_{\text{th}}, I_{S2} \leq I_{\text{th}}, \ldots, I_{SM} \leq I_{\text{th}} | \Phi_S = \mathbb{N}),\\ &= \mathbb{E}_{U_0}[\Pr(I_{S1} \leq I_{\text{th}}, I_{S2} \leq I_{\text{th}}, \ldots, I_{SM} \leq I_{\text{th}} | \Phi_S = \mathbb{N}, U_0)],\end{aligned} \tag{15}$$

where $\mathbb{N} = \{1, 2, \ldots, N\}$ and $U_0 = \max_{m \in \mathbb{N}} \{|g_{Sm}|^2\}$. Given $\Phi_S = \mathbb{N}$, we have $I_{Sk} = \frac{I_{\text{th}} |g_{Sk}|^2}{\tau_S U_0}$. Since, $\tau_S \geq 1$ and $\frac{|g_{Sk}|^2}{U_0} \leq 1$, for $1 \leq k \leq N$, we have $I_{Sk} \leq \frac{I_{\text{th}}}{\tau_S} \leq I_{\text{th}}$, for $1 \leq k \leq N < M$. Hence, (15) simplifies to

$$\begin{aligned}\mathfrak{I}_S = \mathbb{E}_{U_0} [&\Pr(I_{SN+1} \leq I_{\text{th}}, I_{SN+2} \leq I_{\text{th}}, \ldots, I_{SM} \leq I_{\text{th}}\\ &| \Phi_S = \mathbb{N}, U_0)].\end{aligned} \tag{16}$$

Since $|g_{S1}|^2, |g_{S2}|^2, \ldots, |g_{SM}|^2$ are i.i.d. exponential RVs with mean μ_{SX}, we have $\mathfrak{I}_S = \mathbb{E}_{U_0}\left[\left(1 - e^{\frac{\tau_S U_0}{\mu_{SX}}}\right)^{M-N}\right]$.

The probability density function (PDF) of U_0 can be shown to be $f_{U_0}(u) = \frac{N}{\mu_{SX}} \sum_{k_1=0}^{N-1} \binom{N-1}{k_1} (-1)^{k_1}\, e^{-\frac{(k_1+1)u}{\mu_{SX}}}$, for $u \geq 0$. Using this to evaluate the expression for $\mathfrak{I}_S$ in (16) yields (8). The derivation for $\mathfrak{I}_R$ follows along the similar lines.

B. Proof of Result 1

From (7) and using the law of total probability, we have

$$\begin{aligned}P_{\text{out}}(r) = \sum_{j=1}^{\binom{M}{N}} &\Pr\left(\max\left\{C_0, \max_{1 \leq i \leq L}\{C_i\}\right\} \leq r \,\middle|\, \Phi_S = \phi_{Sj}\right)\\ &\times \Pr(\Phi_S = \phi_{Sj}).\end{aligned} \tag{17}$$

As in Appendix A, this simplifies to

$$P_{\text{out}}(r) = \Pr\left(\max\left\{C_0, \max_{1 \leq i \leq L}\{C_i\}\right\} \leq r \,\middle|\, \Phi_S = \mathbb{N}\right), \tag{18}$$

where $\mathbb{N} = \{1, 2, \ldots, N\}$, as defined before. Note that $C_0 = \log_2(1 + \gamma_{SD})$ and $C_i = \frac{1}{2} \log_2(1 + \min\{\gamma_{Si}, \gamma_{iD} + \gamma_{SD}\})$, for $1 \leq i \leq L$, are correlated RVs because they all depend on γ_{SD}. The analysis below tackles this inter-dependency.

Let $W = |h_{SD}|^2$ and $U_0 = \max_{m \in \mathbb{N}} \{|g_{Sm}|^2\}$. Given $\Phi_S = \mathbb{N}$, the SINRs of SD and S-to-R_i links are respectively given by

$$\gamma_{SD} = \frac{I_{\text{th}} W}{\tau\left(\sigma_0^2 + \sigma_2^2\right) U_0} \quad \text{and} \quad \gamma_{Si} = \frac{I_{\text{th}} |h_{Si}|^2}{\tau\left(\sigma_0^2 + \sigma_1^2\right) U_0}. \tag{19}$$

Conditioned on U_0 and W, i.e., conditioned on γ_{SD}, C_0 is independent of $C_1, \ldots, C_L$, which are now i.i.d. RVs. Thus,

$$\begin{aligned}P_{\text{out}}(r) = \mathbb{E}_{U_0, W} &\left[\Pr\left(C_0 \leq r \,\middle|\, \Phi_S = \mathbb{N}, U_0, W\right)\right.\\ &\left.\times \left(\Pr\left(C_1 \leq r \,\middle|\, \Phi_S = \mathbb{N}, U_0, W\right)\right)^L\right].\end{aligned} \tag{20}$$

Let $q_{SD} = \frac{I_{\text{th}} \mu_{SD}}{\tau\left(\sigma_0^2 + \sigma_2^2\right)}$. From the expression for γ_{SD} in (19), we have $\Pr\left(C_0 \leq r \,\middle|\, \Phi_S = \mathbb{N}, U_0, W\right) = 1_{\left\{W \leq \frac{(2^r - 1)\mu_{SD} U_0}{q_{SD}}\right\}}$.

Evaluating $\Pr\left(C_1 \leq r \,\middle|\, \Phi_S = \mathbb{N}, U_0, W\right)$ *in* (20)*:* As in Appendix A, we can show that $\Pr\left(C_1 \leq r \,\middle|\, \Phi_S = \mathbb{N}, U_0, W\right) =$

$\Pr\big(C_1 \le r \,\big|\, \Phi_S = \mathbb{N}, \Phi_1 = \mathbb{N}, U_0, W\big)$. Let $a = 2^{2r} - 1$ and $T_1 = \min\{\gamma_{S1}, \gamma_{1D} + \gamma_{SD}\}$. Since $C_1 = \frac{1}{2}\log_2(1+T_1)$, we get

$$\begin{aligned}\Pr\big(C_1 \le r \,\big|\, \Phi_S = \mathbb{N}, \Phi_1 = \mathbb{N}, U_0, W\big) &= \Pr\big(T_1 \le a \,\big|\, \Phi_S = \mathbb{N}, \Phi_1 = \mathbb{N}, U_0, W\big), \\ &= \mathbb{E}_{U_1}\big[\Pr\big(T_1 \le a \,\big|\, \Phi_S = \mathbb{N}, \Phi_1 = \mathbb{N}, U_0, W, U_1\big)\big]. \quad (21)\end{aligned}$$

where $U_1 = \max\limits_{m \in \mathbb{N}} \left\{|g_{1m}|^2\right\}$.

Substituting (21) in (20) and using the Jensen's inequality $(\mathbb{E}[X])^L \le \mathbb{E}\left[X^L\right]$, for $L \ge 1$, we get

$$P_{\text{out}}(r) \le \mathbb{E}_{U_0,W,U_1}\Bigg[1_{\left\{W \le \frac{(2^r-1)\mu_{SD} U_0}{q_{SD}}\right\}} \times \bigg(\Pr\big(T_1 \le a \,\big|\, \Phi_S = \mathbb{N}, \Phi_1 = \mathbb{N}, U_0, W, U_1\big)\bigg)^L\Bigg]. \quad (22)$$

The conditional cumulative distribution function (CDF) of T_1 in (22) can be shown to be

$$\Pr(T_1 \le a | \Phi_S = \mathbb{N}, \Phi_1 = \mathbb{N}, U_0, W, U_1) = \begin{cases} 1 - e^{-\frac{aU_0}{q_{SR}}}, & 0 \le a < \frac{q_{SD} W}{\mu_{SD} U_0}, \\ 1 - e^{-\left[\frac{aU_0}{q_{SR}} + \left(a - \frac{q_{SD} W}{\mu_{SD} U_0}\right)\frac{U_1}{q_{RD}}\right]}, & \text{otherwise}, \end{cases}$$

where $q_{SR} = \frac{I_{\text{th}}\mu_{SR}}{\tau\left(\sigma_0^2 + \sigma_1^2\right)}$ and $q_{RD} = \frac{I_{\text{th}}\mu_{RD}}{\tau\left(\sigma_0^2 + \sigma_2^2\right)}$. In (22), we need to average over the range $\frac{q_{SD} W}{\mu_{SD} U_0} \le (2^r - 1) \le a$. Thus, in this range, the conditional CDF of T_1 is given by the second case of the above equation. Hence, (22) simplifies to

$$P_{\text{out}}(r) \le \mathbb{E}_{U_0,W,U_1}\Bigg[1_{\left\{W \le \frac{(2^r-1)\mu_{SD} U_0}{q_{SD}}\right\}} \times \bigg(1 - e^{-\left[\frac{aU_0}{q_{SR}} + \left(a - \frac{q_{SD} W}{\mu_{SD} U_0}\right)\frac{U_1}{q_{RD}}\right]}\bigg)^L\Bigg]. \quad (23)$$

Similar to Appendix A, the PDF of U_1 can be shown to be $f_{U_1}(v) = \frac{N}{\mu_{RX}} \sum\limits_{k_2=0}^{N-1} \binom{N-1}{k_2} (-1)^{k_2}\, e^{-\frac{(k_2+1)v}{\mu_{RX}}}$, $v \ge 0$. Using the binomial expansion of $(.)^L$ in (23), averaging over U_0, U_1, and W, and simplifying further yields (9), where

$$J(k, k_1, k_2) = \int_0^\infty \int_0^{\frac{(2^r-1)\mu_{SD} u}{q_{SD}}} \int_0^\infty e^{-u\left(\frac{k_1+1}{\mu_{SX}} + \frac{ka}{q_{SR}}\right)}\, e^{-\frac{w}{\mu_{SD}}} \times e^{-v\left(\frac{k_2+1}{\mu_{RX}} + \left(a - \frac{q_{SD} w}{\mu_{SD} u}\right)\frac{k}{q_{RD}}\right)}\, dv\, dw\, du.$$

Integrating with respect to v, using the variable transformations $a - \frac{q_{SD} w}{\mu_{SD} u} = t$ and $t + \frac{(k_2+1) q_{RD}}{k\, \mu_{RX}} = t_1$, and the identities in [20, (2.325.1)] and [20, (6.228.2)] yields (10).

C. Brief Proof of Corollary 1

As $\sigma_1^2 = \sigma_2^2 = \sigma^2$ and $\gamma = 1/\left(\sigma_0^2 + \sigma^2\right) \to \infty$, we have $q_{SR}, q_{RD}, q_{SD} \to \infty$. Hence, using $1 - e^{-x} \approx x$, for $x \ll 1$, the argument inside the expectation in (23) can be replaced by $1_{\left\{W \le \frac{(2^r-1)\mu_{SD} U_0}{q_{SD}}\right\}} \left[\frac{aU_0}{q_{SR}} + \left(a - \frac{q_{SD} W}{\mu_{SD} U_0}\right)\frac{U_1}{q_{RD}}\right]^L$. Expanding this in terms of a binomial series and after taking the expectation over U_0, W, and U_1, as in (23), yields (11), where

$$J'(k, k_1, k_2) = \int_0^\infty \int_0^{\frac{(2^r-1)\mu_{SD} u}{q_{SD}}} \int_0^\infty e^{-\frac{(k_1+1)u}{\mu_{SX}}}\, e^{-\frac{(k_2+1)v}{\mu_{RX}}}\, e^{-\frac{w}{\mu_{SD}}} \times \left(\frac{au}{q_{SR}}\right)^k \left[\left(a - \frac{q_{SD} w}{\mu_{SD} u}\right)\frac{v}{q_{RD}}\right]^{L-k} dv\, dw\, du.$$

Integrating with respect to v, using the variable transformation $a - \frac{q_{SD} w}{\mu_{SD} u} = t$, and approximating $e^{\frac{ut}{q_{SD}}} e^{-u\left(\frac{k_1+1}{\mu_{SX}} + \frac{a}{q_{SD}}\right)}$ as $\left(1 + \frac{ut}{q_{SD}}\right) e^{-u\left(\frac{k_1+1}{\mu_{SX}} + \frac{a}{q_{SD}}\right)}$ when $q_{SD} \to \infty$ yields (12).

References

[1] S. Kashyap and N. B. Mehta, "SEP-optimal transmit power policy for peak power and interference outage probability constrained underlay cognitive radios," *IEEE Trans. Wireless Commun.*, vol. 12, no. 12, pp. 6371–6381, Dec. 2013.

[2] H. Tran, T. Q. Duong, and H. J. Zepernick, "Performance analysis of cognitive relay networks under power constraint of multiple primary users," in *Proc. Globecom*, Dec. 2011, pp. 1–6.

[3] H. Huang, Z. Li, N. C. Beaulieu, and J. Si, "Underlay cognitive proactive DF relay networks with multiple primary transmitters and receivers," in *Proc. Globecom*, Dec. 2015, pp. 1–6.

[4] M. Xia and S. Aissa, "Underlay cooperative AF relaying in cellular networks: Performance and challenges," *IEEE Commun. Mag.*, vol. 51, no. 12, pp. 170–176, Dec. 2013.

[5] I. Trigui, S. Affes, and A. Stphenne, "Capacity of cognitive AF relay networks with multiple primary receivers," in *Proc. WCNC*, Apr. 2014, pp. 133–138.

[6] R. Zhang, "On peak versus average interference power constraint for protecting primary users in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 4, pp. 2112–2120, Apr. 2009.

[7] P. J. Smith, P. A. Dmochowski, H. A. Suraweera, and M. Shafi, "The effects of limited channel knowledge on cognitive radio system capacity," *IEEE Trans. Veh. Technol.*, vol. 62, no. 2, pp. 927–933, Feb. 2013.

[8] T. Q. Duong, P. L. Yeoh, V. N. Q. Bao, M. Elkashlan, and N. Yang, "Cognitive relay networks with multiple primary transceivers under spectrum-sharing," *IEEE Signal Process. Lett.*, vol. 19, no. 11, pp. 741–744, Nov. 2012.

[9] J. Chen, Z. Li, J. Si, H. Huang, and R. Gao, "Outage probability analysis of partial AF relay selection in spectrum-sharing scenario with multiple primary users," *Electron. Lett.*, vol. 48, no. 19, pp. 1211–1212, Sept. 2012.

[10] Q. Wu, Z. Zhang, and J. Wang, "Outage analysis of cognitive relay networks with relay selection under imperfect CSI environment," *IEEE Commun. Lett.*, vol. 17, no. 7, pp. 1297–1300, Jul. 2013.

[11] H. Huang, Z. Li, J. Si, and L. Guan, "Underlay cognitive relay networks with imperfect channel state information and multiple primary receivers," *IET Commun.*, vol. 9, no. 4, pp. 460–467, Mar. 2015.

[12] T. Q. Duong, K. J. Kim, H. J. Zepernick, and C. Tellambura, "Opportunistic relaying for cognitive network with multiple primary users over Nakagami-m fading," in *Proc. ICC*, Jun. 2013, pp. 5668–5673.

[13] F. R. V. Guimarães, D. B. Da Costa, M. Benjillali, T. A. Tsiftsis, and G. K. Karagiannidis, "Cooperative spectrum sharing systems with relay selection in the presence of multiple primary receivers," *IET Commun.*, vol. 8, no. 4, pp. 546–553, Mar. 2014.

[14] S. Majhi and A. Banerjee, "Asymptotic outage analysis of incremental decode and forward cognitive radio relay network," in *Proc. Int. Conf. Commun. Syst. and Netw.*, Jan. 2015, pp. 1–8.

[15] H. Huang, Z. Li, and J. Si, "Multi-source multi-relay underlay cognitive radio networks with multiple primary users," in *Proc. ICC*, Jun. 2015, pp. 7558–7563.

[16] S. Bharadwaj and N. B. Mehta, "Accurate performance analysis of single and opportunistic AF relay cooperation with imperfect cascaded channel estimates," *IEEE Trans. Commun.*, vol. 61, no. 5, pp. 1764–1775, May 2013.

[17] P. Das, N. B. Mehta, and G. Singh, "Novel relay selection rules for average interference-constrained cognitive AF relay networks," *IEEE Trans. Wireless Commun.*, vol. 14, no. 8, pp. 4304–4315, Aug. 2015.

[18] J. Chen, J. Si, Z. Li, and H. Huang, "On the performance of spectrum sharing cognitive relay networks with imperfect CSI," *IEEE Commun. Lett.*, vol. 16, no. 7, pp. 1002–1005, Jul. 2012.

[19] K. Woradit, T. Quek, W. Suwansantisuk, H. Wymeersch, L. Wuttisittikulkij, and M. Z. Win, "Outage behavior of selective relaying schemes," *IEEE Trans. Wireless Commun.*, vol. 8, no. 8, pp. 3890–3895, Aug. 2009.

[20] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, 4th ed. Academic Press, 1980.