Hybrid Energy Harvesting Wireless Systems: Performance Evaluation and Benchmarking

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***Abstract*—Energy harvesting sensor (EHS) nodes provide an attractive and green solution to the problem of limited lifetime of wireless sensor networks (WSNs). Unlike a conventional node that uses a non-rechargeable battery and dies once it runs out of energy, an EHS node can harvest energy from the environment and replenish its rechargeable battery. We consider hybrid WSNs that comprise of both EHS and conventional nodes; these arise when legacy WSNs are upgraded or due to EHS deployment cost issues. We compare conventional and hybrid WSNs on the basis of a new and insightful performance metric called k-outage duration, which captures the inability of the nodes to transmit data either due to lack of sufficient battery energy or wireless fading. The metric overcomes the problem of defining lifetime in networks with EHS nodes, which never die but are occasionally unable to transmit due to lack of sufficient battery energy. It also accounts for the effect of wireless channel fading on the ability of the WSN to transmit data. We develop two novel, tight, and computationally simple bounds for evaluating the k-outage duration. Our results show that increasing the number of EHS nodes has a markedly different effect on the k-outage duration than increasing the number of conventional nodes.**

I. Introduction

The nodes in a wireless sensor network (WSN) sense data, process it, and send it to a fusion node (FN). In several deployment scenarios, it is cumbersome to run cables to power the nodes. Therefore, sensor nodes are often equipped with pre-charged batteries, which supply the energy required for their operations. Over time, the node expends all the energy in its battery and becomes inoperable or dead. Eventually, the network itself fails to meet its sensing objective.

Improving network lifetime is, therefore, an important objective of WSN design. Depending on the service provided by the WSN and its network topology, different definitions of lifetime have been used [1]. In many papers, e.g., [2], the death of the first node in the WSN is defined as lifetime. This is pessimistic because the other nodes in the network may still carry out sensing and communication tasks. In [3], lifetime is defined as the death of a pre-specified fraction of nodes. However, these definitions are based purely on the battery energies of the nodes. The failure to communicate the sensed data to a FN due to channel fading is not accounted for. In [4], network lifetime is defined in the terms of number of nodes that run out of energy and also the number of communication failures that occurred due to deep fades in the channel between the sensor node and the FN. However, WSNs consisting of different types of sensor nodes are not discussed. Other definitions of lifetime are based on sensing coverage [5], connectivity of the nodes to the FN [6], and both coverage and connectivity in [7]. However, a deterministic path loss model without fading is assumed in [7].

Energy harvesting sensor (EHS) nodes, which replenish the energy they consume by harvesting it from the environment and storing it in their batteries, provide a promising and green alternative to tackle the problem of lifetime [8]–[11]. While EHS nodes are attracting considerable interest, several new challenges need to be overcome before they can be widely deployed. First, the energy harvesting process can be sporadic. Second, an EHS node needs additional circuitry to harvest, store, and to provide a regulated supply of the harnessed energy to its battery or supercapacitor [12]. Hence, EH nodes are likely to be more expensive than conventional nodes, which come equipped with pre-charged, non-rechargeable batteries.

Given the above challenges, hybrid WSNs, which comprise of a mixture of EHS nodes and conventional nodes, are likely. Upgradation of the legacy WSNs, in which conventional nodes are gradually replaced by EHS nodes, also naturally leads to hybrid WSNs. However, relatively less research has been done on hybrid WSNs. In the hybrid WSN considered in [13], the EH functionality is used only to relay information from a cluster head to the FN. In [14], solar-aware clustering for WSN is proposed. The choice of the cluster head is made on the basis of battery energy, position, and whether the node is EH or not. In [2], mobile rechargeable relay nodes are considered, but the randomness in the energy harvesting process or the temporary unavailability of the EHS nodes is not modeled. Furthermore, channel fading is not considered in [2], [13], [14].

The presence of EHS nodes in a network makes it even more challenging to define lifetime because these nodes do not die. Instead, they are occasionally unavailable, the probability of which depends upon the communication protocol used and the energy harvesting process. Thus, one can no longer define lifetime defined as the time until the death of a pre-specified fraction or number of nodes in a hybrid WSN. Hence, we introduce a new performance metric called *k-outage duration* that enables a direct comparison of conventional, hybrid, and all-EHS networks, which consist only of EHS nodes. An outage is an event in which data does not reach the FN either due to lack of battery energy for transmission or due to the communication failures caused by channel fades. The average

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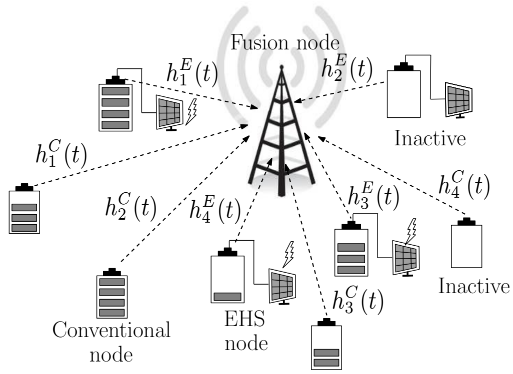

Fig. 1. A hybrid WSN consisting of $M_E = 4$ EHS nodes and $M_C = 4$ conventional nodes that transmit data to the FN over wireless links that undergo fading.

time required for k outages to occur in the WSN is called the k-outage duration. In sensing critical applications, k is small, while in routine monitoring applications, k is likely to be large.

We note that k-outage duration is not without its limitations as a performance metric. Ideally, the performance evaluation of the WSN must be made on the basis of whether the FN can fuse the sensed data with sufficient accuracy. However, the sensing, fusion, and communication processes then need to be specified in detail, which makes the problem intractable. The k-outage duration is a tractable way to compare WSNs, and, as we shall show, it provides valuable insights about the impact of the EHS nodes on the WSN.

We consider a star hybrid WSN with a time slotted transmission scheme. We analyze its k-outage duration. An important contribution of the paper is the development of two upper bounds on the k-outage duration. These bounds are based on two hypothetical systems whose time evolution is tightly coupled with the hybrid system. The bounds effectively circumvent the large computational complexity required to exactly analyze a hybrid WSN. The bounds are together shown to be tight and markedly easier to compute. Extensive numerical results that study the effect of the number of conventional and EHS nodes in the hybrid system are presented. We show that increasing the number of EHS nodes has a markedly different effect on the k-outage duration than increasing the number of conventional nodes. We note that the k-outage duration can be evaluated not just for the above model, but also other WSNs, in general. These include WSNs that use physical layer diversity techniques such as transmit diversity or cooperative relaying.

The paper is organized as follows. The system model is developed in Sec. II. In Sec. III, we analyze the k-outage duration. Numerical results in Sec. IV are followed by our conclusions in Sec. V.

II. Hybrid WSN System Model

We use the following notation henceforth. The probability of an event A is denoted by $\Pr(A)$. For a random variable (RV) X, its expected value is denoted by $\mathbb{E}[X]$ and its expected value conditioned on event A is denoted by $\mathbb{E}[X|A]$. The indicator function for an event A is denoted by $1_{\{A\}}$; it equals 1 if A occurs and is 0 otherwise. $\mathbf{v}^T$ denotes the transpose of the vector $\mathbf{v}$. For $b < a$, the sum $\sum_{i=a}^{b}$ is identically 0. $\mathbf{1}_n = (1, 1, \ldots, 1)^T$ denotes the all ones vector of size $n \times 1$. And, $\lfloor \cdot \rfloor$ denotes the floor function and $\mathbb{Z}^+$ the set of non-negative integers.

As shown in the Figure 1, the system has M_C conventional nodes, M_E EHS nodes, and a FN. Each conventional node has a non-rechargeable battery with an initial energy of B_0. Each EHS node has a rechargeable battery that can store a maximum of $B_{\max}^E$ units of energy. We assume that the battery of each EHS node is also pre-charged to B_0 at the time of deployment.

Time is divided into slots of duration T_{coh}, where T_{coh} is the coherence interval. We assume a frequency-flat, block-fading channel model. Let $h_i^C(t)$ and $h_j^E(t)$ denote the channel gains of the i^{th} conventional node and the j^{th} EHS node, respectively, in the t^{th} time slot. $h_i^C(t)$ and $h_j^E(t)$ are independent and identically distributed (i.i.d.), for all $t \geq 1$, $1 \leq i \leq M_C$, and $1 \leq j \leq M_E$. Let γ_0 denote the mean channel power gain.

The energy harvesting process of an EHS node is modeled as a Bernoulli injection process, as has also been modeled in [8], [9], [15]. An EHS node harvests E_h units of energy in every slot with probability ρ, independently of other EHS nodes. The energy harvested in a slot is available for transmission in the next slot. For tractability, B_0 is expressed as an integer multiple of E_h: $B_0 = uE_h$, where $u \in \mathbb{Z}^+$. Similarly, $B_{\max}^E = dE_h$, where $d \in \mathbb{Z}^+$.

A. Transmission Scheme

A node that has sufficient battery energy to transmit E_{tx} energy and whose channel gain exceeds the threshold γ_{th} is called an *active* node. Among the active nodes, the node with the largest battery energy is selected to transmit in the time slot [4]. If no node is active in a slot, no transmission takes place in that slot and an outage occurs. For tractability, we set E_{tx} to be an integer multiple of E_h, $E_{\text{tx}} = lE_h$, $l \in \mathbb{Z}^+$.

Selection can be implemented using distributed selection algorithms, which can rapidly select the node for transmission and require minimal energy [16]. We, therefore, do not consider the time and energy overhead of selection, as is typical in the literature [4]. The energy cost of sensing and data processing is neglected as radio transmission is often the dominant cause of energy consumption [4], [8].

III. k-Outage Duration Analysis

The battery evolution of the network can be shown to follow a Markov model. The transmission and harvesting process start from time slot $t = 1$. In time slot t, let $\mathcal{X}_i^C(t)$ be the event that the i^{th} conventional node transmits, $\mathcal{X}_j^E(t)$ be the event that the j^{th} EHS node transmits, and $\mathcal{H}_j^E(t)$ be the event that the j^{th} EHS node harvests energy. The number of outages in the network by the end of time slot t is represented by $O(t)$. Clearly, $O(0) = 0$. Let $\mathscr{T}_k$ be the time for k outages to occur.

For the i^{th} conventional node, the battery state $B_i^C(t+1)$ at the beginning of time slot $t+1$ evolves as

$$B_i^C(t+1) = B_i^C(t) - 1_{\{\mathcal{X}_i^C(t)\}} E_{\text{tx}}. \quad (1)$$

For the j^{th} EHS node, the battery state $B_j^E(t+1)$ at the beginning of time slot $t+1$ evolves as

$$B_j^E(t+1) = B_j^E(t) - 1_{\{\mathcal{X}_j^E(t)\}} E_{\text{tx}} + 1_{\{\mathcal{H}_j^E(t)\}} E_h. \quad (2)$$

At the beginning of time slot $t \geq 1$, the state of the network $\mathbf{S}(t)$ can be represented as

$$\mathbf{S}(t) = (B_1^C(t), B_2^C(t), \ldots, B_{M_C}^C(t), B_1^E(t), B_2^E(t), \ldots, B_{M_E}^E(t), O(t-1)).$$

Therefore, $\{\mathbf{S}(t), t \geq 1\}$ is a discrete time Markov chain (DTMC) that takes values in the state space $\mathcal{S} \bigcup \mathcal{A}$, where

$$\begin{aligned}\mathcal{S} = \{&(s_1, s_2, \ldots, s_{M_C}, s'_1, \ldots, s'_{M_E}, o) : \\ &0 \leq s_i \leq uE_h, 1 \leq i \leq M_C, \\ &0 \leq s'_j \leq dE_h, 1 \leq j \leq M_E, 0 \leq o \leq k-1\}, \quad (3)\\ \mathcal{A} = \{&(s_1, s_2, \ldots, s_{M_C}, s'_1, \ldots, s'_{M_E}, o) : \\ &0 \leq s_i \leq uE_h, 1 \leq i \leq M_C, \\ &0 \leq s'_j \leq dE_h, 1 \leq j \leq M_E, o = k\}. \quad (4)\end{aligned}$$

Here, $\mathcal{A}$ are the states of the system in which k outages have occurred. Hence, $\mathcal{A}$ forms the set of absorbing states of the DTMC. Once the system reaches any state in $\mathcal{A}$, its further evolution need not be analyzed.

The number of states is $(k+1)d^{M_E}u^{M_C}$, which is exponential in both M_C and M_E. Analyzing such a high dimensional Markov chain is computationally challenging and memory intensive. We, therefore, present two novel upper bounds for the k-outage duration $\mathbb{E}[\mathscr{T}_k]$ that effectively circumvent the problem. Henceforth, we will refer to the system described above as the *original system*.

A. Single Pooled Battery System Based Upper Bound

In this hypothetical system, there is a single node called a *single pooled battery (SP) node* that transmits data to the FN. Its battery energy at the beginning of time slot t is denoted by $B_{\text{SP}}(t)$. At start-up, $B_{\text{SP}}(1) = (M_C + M_E)B_0$, which is the total battery energy of all the nodes in the original system. The battery size of the SP node is the sum of the battery sizes of all nodes in the original system and equals $B_0 M_C + B_{\max}^E M_E$.

As in the original system, each time slot is of length T_{coh}. The channel gain $h_{\text{SP}}(t)$ seen by the SP node in the t^{th} time slot is the maximum of the channel gains seen by all the nodes in the original system in that time slot:

$$h_{\text{SP}}(t) = \max\{h_1^C(t), h_2^C(t), \ldots, h_{M_C}^C(t), h_1^E(t), h_2^E(t), \ldots, h_{M_E}^E(t)\}. \quad (5)$$

The energy harvested by the SP node in time slot t is the sum of the energies harvested by all the EHS nodes in the original system in that slot, and is given by $\sum_{j=1}^{M_E} 1_{\{\mathcal{H}_j^E(t)\}} E_h$. As in the original system, the SP node transmits in a slot if it is active, i.e., if $B_{\text{SP}}(t) \geq E_{\text{tx}}$ and $h_{\text{SP}}(t) \geq \gamma_{\text{th}}$. Let $\mathbb{E}[\mathscr{T}_k^{\text{SP}}]$ denote the k-outage duration of the SP system.

Theorem *1:* The k-outage duration of the original system is upper bounded by the k-outage duration of the SP system

$$\mathbb{E}[\mathscr{T}_k] \leq \mathbb{E}[\mathscr{T}_k^{\text{SP}}]. \quad (6)$$

Proof: The proof is relegated to Appendix A. ■

1) Analysis of $\mathbb{E}[\mathscr{T}_k^{\text{SP}}]$: Let $O_{\text{SP}}(t)$ denote the number of outages that have occurred in the SP system by the end of time slot t. Clearly, $O_{\text{SP}}(0) = 0$. The state of the SP system at the beginning of time slot t is $\mathbf{S}_{\text{SP}}(t) = \{B_{\text{SP}}(t), O_{\text{SP}}(t-1)\}$. Then, $\{\mathbf{S}_{\text{SP}}(t), t \geq 1\}$ is a DTMC that takes values in the state space $\mathcal{S}_{\text{SP}} \bigcup \mathcal{A}_{\text{SP}}$, where

$$\mathcal{S}_{\text{SP}} = \{(sE_h, o) : 0 \leq s \leq uM_C + dM_E, 0 \leq o \leq k-1\}, \quad (7)$$

$$\mathcal{A}_{\text{SP}} = \{(sE_h, o) : 0 \leq s \leq uM_C + dM_E, o = k\}. \quad (8)$$

Here, $\mathcal{A}_{\text{SP}}$ is the set of absorbing states of the DTMC in which k outages have occurred. Hence, the SP system is only a two-dimensional DTMC, while the original system is of dimension $M_C + M_E + 1$.

The probability of moving from state (wE_h, x) to state $(w'E_h, x')$ is represented by $p_{(w,x),(w',x')}$. The transition probability matrix $\mathbf{P}_k^{\text{SP}}$ of the DTMC is given in Appendix B. Let the restriction of $\mathbf{P}_k^{\text{SP}}$ on $\mathcal{S}_{\text{SP}}$ be denoted by $\mathbf{Z}_k^{\text{SP}}$. $p_{(w,x),(w',x')}$ is the $(x(uM_C + dM_E + 1) + w + 1)^{\text{th}}$ row and $(x'(uM_C + dM_E + 1) + w' + 1)^{\text{th}}$ column element of $\mathbf{Z}_k^{\text{SP}}$.

Let $\mathbb{E}[\mathscr{T}_k^{\text{SP}} | (w, x)]$ denote the k-outage duration of the SP system given that it starts from state $(wE_h, x) \in \mathcal{S}_{\text{SP}}$ and let

$$\mathbf{v}^{\text{SP}} = (\mathbb{E}[\mathscr{T}_k^{\text{SP}} | (0,0)], \mathbb{E}[\mathscr{T}_k^{\text{SP}} | (1,0)], \ldots, \mathbb{E}[\mathscr{T}_k^{\text{SP}} | ((uM_C + dM_E), k-1)])^T, \quad (9)$$

be the vector of k-outage durations given that the DTMC starts from state $(wE_h, x) \in \mathcal{S}_{\text{SP}}$.

As shown in Appendix C, $\mathbb{E}[\mathscr{T}_k^{\text{SP}}]$ is given by

$$\mathbb{E}[\mathscr{T}_k^{\text{SP}}] = \mathbb{E}[\mathscr{T}_k^{\text{SP}} | (uM_C + dM_E, 0)]. \quad (10)$$

It is the $(uM_C + dM_E + 1)^{\text{th}}$ element of $\mathbf{v}^{\text{SP}}$, which is given by

$$\mathbf{v}^{\text{SP}} = (\mathbf{I}_k^{\text{SP}} - \mathbf{Z}_k^{\text{SP}})^{-1} \mathbf{1}_{k(uM_c + dM_E + 1)}, \quad (11)$$

where $\mathbf{I}_k^{\text{SP}}$ is an identity matrix of the same size as $\mathbf{Z}_k^{\text{SP}}$.

B. Dual Pooled Battery System Based Upper Bound

The SP system leads to a weak upper bound in the regime in which conventional nodes have drained out their batteries since the SP node always sees the best of $M_C + M_E$ channel gains. We present below an alternative upper bound that is better suited for this regime.

The bound is based on *the dual pooled battery (DP) system*, in which there are two nodes called *conventional pooled* *battery (CDP) node* and *EHS pooled battery (EDP) node* that have data to transmit to the FN. At start-up, the battery energy of the CDP node is $M_C B_0$, which is the sum of the start-up battery energies of all the conventional nodes in the original system. Each time slot is of duration T_{coh}.

In time slot t, the channel gains of the CDP and EDP nodes are the maximum of the channel gains of the conventional nodes and the maximum of channel gains of the EHS nodes in the original system, respectively. The energy harvested by the EDP node in any time slot is $M_E E_h$, which is the maximum energy that can be harvested by all the EHS nodes in the original system in a time slot. When both the EDP and CDP nodes are active, the EDP node is chosen for transmission. Thus, in a slot, an outage occurs if neither the CDP node nor the EDP node can transmit.

Let $\mathbb{E}[\mathscr{T}_k^{\text{DP}}]$ be the average time needed for k outages to occur in the DP system.

Theorem *2:* When $E_{\text{tx}} \leq M_E E_h$, the k-outage duration of the DP system upper bounds that of the original system

$$\mathbb{E}[\mathscr{T}_k] \leq \mathbb{E}[\mathscr{T}_k^{\text{DP}}]. \tag{12}$$

Proof: The proof uses concepts similar to that in Appendix A, and is omitted due to space constraints. ∎

Note that the upper bounds given in Theorems 1 and 2 hold for any probability distribution of the channel gains seen by the nodes. They also hold for any energy harvesting process.

Intuitively, when the average energy harvested by the original system per slot is greater than its average transmission energy per slot, the EHS nodes of the original system will always have sufficient energy to transmit, just like the EDP node. Hence, in this regime, the DP system based upper bound in (12) is tighter than that given by the SP system in (6). On the other hand, when the original system harvests less energy on average than it expends, the EHS nodes will be energy constrained and the SP system based upper bound is tighter.

For $E_{\text{tx}} \leq M_E E_h$, we know from Theorems 1 and 2 that $\mathbb{E}[\mathscr{T}_k] \leq \mathbb{E}[\mathscr{T}_k^{\text{DP}}]$ and $\mathbb{E}[\mathscr{T}_k] \leq \mathbb{E}[\mathscr{T}_k^{\text{SP}}]$. Thus, we get the following tighter upper bound:

$$\mathbb{E}[\mathscr{T}_k] \leq \min\{\mathbb{E}[\mathscr{T}_k^{\text{DP}}], \mathbb{E}[\mathscr{T}_k^{\text{SP}}]\}. \tag{13}$$

1) Analysis of $\mathbb{E}[\mathscr{T}_k^{\text{DP}}]$*:* Let $B_{\text{CDP}}(t)$ denote the battery energy of the CDP node at the beginning of time slot t. The number of outages in the DP system at the end of time slot t is denoted by $O_{\text{DP}}(t)$. Clearly, $O_{\text{DP}}(0) = 0$. The state of the DP system at the beginning of time slot t is given by $\mathbf{S}_{\text{DP}}(t) = \{B_{\text{CDP}}(t), O_{\text{DP}}(t-1)\}$. Note that the battery energy of the EDP node need not be tracked since $M_E E_h \geq E_{\text{tx}}$ implies that it harvests enough energy to transmit in any time slot. Thus, $\{\mathbf{S}_{\text{DP}}(t), t \geq 1\}$ is a DTMC with state space $\mathcal{S}_{\text{DP}} \bigcup \mathcal{A}_{\text{DP}}$, where

$$\mathcal{S}_{\text{DP}} = \{(sE_h, o) : 0 \leq s \leq uM_C, 0 \leq o \leq k-1\}, \tag{14}$$

$$\mathcal{A}_{\text{DP}} = \{(sE_h, o) : 0 \leq s \leq uM_C, o = k\}. \tag{15}$$

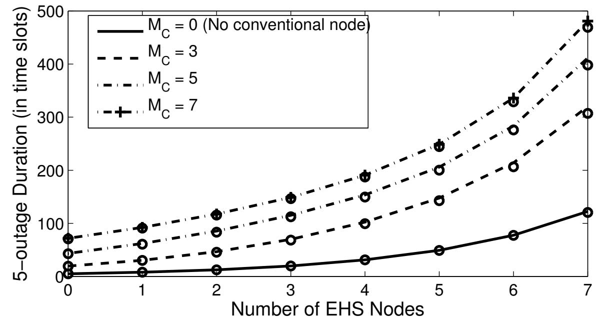

Fig. 2. Effect of M_C on 5-outage duration ($\rho = 0.1$ and $\gamma_{\text{th}} = 1$). Simulation results are shown using the marker ∘.

As before, $\mathcal{A}_{\text{DP}}$ is the set of absorbing states of the DP system. Hence, the DP system is also a two-dimensional DTMC. Its state transition matrix $\mathbf{P}_k^{\text{DP}}$ can be written along lines similar to $\mathbf{P}_k^{\text{SP}}$ in Appendix B, and is not shown here.

Let $\mathbf{Z}_k^{\text{DP}}$ be the restriction of $\mathbf{P}_k^{\text{DP}}$ on $\mathcal{S}_{\text{DP}}$. Then, the probability $q_{(w,x),(w',x')}$ of moving from state (wE_h, x) to $(w'E_h, x')$ is the $(x(uM_C+1)+w+1)^{\text{th}}$ row and $(x'(uM_C+1)+w'+1)^{\text{th}}$ column element of $\mathbf{Z}_k^{\text{DP}}$. The k-outage duration of the DP system, given that it starts from state $(wE_h, x) \in \mathcal{S}_{\text{DP}}$, is denoted by $\mathbb{E}[\mathscr{T}_k^{\text{DP}}|(w,x)]$. Let

$$\mathbf{v}^{\text{DP}} = (\mathbb{E}[\mathscr{T}_k^{\text{DP}}|(0,0)], \mathbb{E}[\mathscr{T}_k^{\text{DP}}|(1,0)], \ldots, \mathbb{E}[\mathscr{T}_k^{\text{DP}}|(uM_C, k-1)])^T. \tag{16}$$

The k-outage duration of the DP system is given by

$$\mathbb{E}[\mathscr{T}_k^{\text{DP}}] = \mathbb{E}[\mathscr{T}_k^{\text{DP}}|(uM_C, 0)], \tag{17}$$

where $\mathbb{E}[\mathscr{T}_k^{\text{DP}}|(uM_C, 0)]$ is the $(uM_C+1)^{\text{th}}$ element of the vector $\mathbf{v}^{\text{DP}}$ and

$$\mathbf{v}^{\text{DP}} = (\mathbf{I}_k^{\text{DP}} - \mathbf{Z}_k^{\text{DP}})^{-1} \mathbf{1}_{k(uM_c+1)}. \tag{18}$$

Here, $\mathbf{I}_k^{\text{DP}}$ is an identity matrix of the same size as $\mathbf{Z}_k^{\text{DP}}$. The proof is similar to that in Appendix C and is omitted.

IV. Numerical Results and Discussion

We now study the behaviour of the original system using Monte Carlo simulations that use 10^5 sample paths. The analytically derived upper bound is compared with the simulation results. We set $\gamma_0 = 1$, $\frac{B_0}{E_h} = 10$, $\frac{B_{\max}}{E_h} = 40$, and $\frac{E_{\text{tx}}}{E_h} = 1$.

Figure 2 plots the upper bound on the 5-outage duration as a function of M_E for different M_C for Rayleigh fading. Also plotted are the values measured from simulations. We see that the upper bound is tight for all M_E. As M_E increases, the 5-outage duration increases due to two reasons: (i) An increase in M_E increases the odds that at least one node sees a channel gain that is better than γ_{th}, and (ii) When more EHS nodes are available to transmit, the batteries of the conventional nodes get drained less. Similarly, as M_C increases the 5-outage duration increases because more nodes are available in the system to transmit data to the FN.

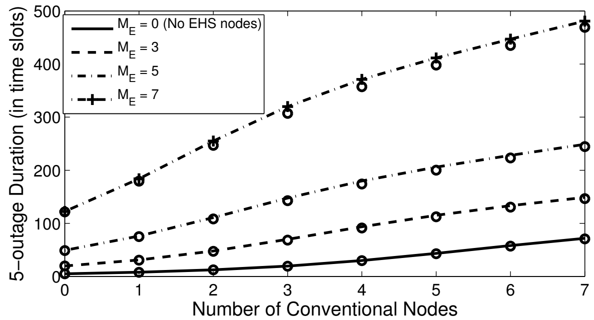

Fig. 3. Effect of M_E on 5-outage duration ($\rho = 0.1$ and $\gamma_{\text{th}} = 1$). Simulation results are shown using the marker $\circ$.

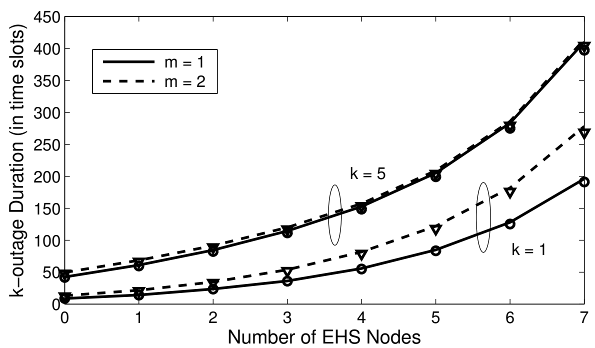

Fig. 4. Effect of k and channel fading statistics on the k-outage duration ($M_C = 5$, $\rho = 0.1$, and $\gamma_{\text{th}} = 1$). Simulation results for Nakagami-m fading are shown using the markers $\circ$ and $\triangledown$ for $m = 1$ and $m = 2$, respectively.

Figure 3 plots the upper bound on the 5-outage duration and its exact value measured from simulations as a function of M_C for different M_E for Rayleigh fading channel. As M_C increases, the 5-outage duration increases because more nodes are available in the network to transmit data to the FN. The upper bound is again tight. Unlike Figure 2, in which the 5-outage duration increases more rapidly as M_E increases, here the increase in the 5-outage duration is marginal as M_C increases.

Figure 4 plots the k-outage duration as a function of M_E for different values of k for Nakagami-m fading channels. Two values of m, namely, 1 and 2, are considered. As k increases, the k-outage duration increases, which is intuitive. However, it begins to saturate. This is because for large k, the odds that all the conventional nodes have drained out their batteries by the time k outages occur is high. Hence, the k-outage duration becomes primarily a function of the number of EHS nodes in the system. The bounds are tight for both values of m.

V. Conclusions

EHS nodes offer a green alternative to tackle the challenging problem of limited lifetime of conventional WSNs. We studied hybrid WSNs that consist of both conventional and EHS nodes, and which are likely arise in the near future given the challenges of deploying EHS nodes in current WSNs. We proposed the use of the k-outage duration as a performance metric for comparing hybrid and conventional WSNs. It avoids the pitfalls associated with defining a lifetime-type metric for WSNs with EHS nodes, and considers both battery energy and the wireless channel conditions in determining the ability of the WSN to transmit sensed data to the fusion node.

We developed two new upper bounds for the k-outage duration. The bounds are useful because they are tight and require analyzing much simpler two-dimensional Markov chains than that required by an exact analysis. We saw that as the number of EHS nodes increases, the k-outage duration increases more rapidly, while as the number of conventional nodes increases, the k-outage duration begins to saturate.

Appendix

A. Proof of Theorem 1

We compare the evolution of the original system and the SP system in parallel for a given sample path of the energy harvesting and channel fading processes. The total energy provided to the SP node prior to time slot $t \geq 1$ is $(M_C + M_E)B_0 + \sum_{n=1}^{t-1}\sum_{j=1}^{M_E} 1_{\mathcal{H}_j^E(n)} E_h$. Since each transmission consumes energy E_{tx}, the number transmissions by the SP node until time t, $T_{\text{SP}}(t)$, cannot exceed $T_{\max}(t)$, where

$$T_{\text{SP}}(t) \leq T_{\max}(t) = \left\lfloor \frac{(M_C + M_E)B_0 + \sum_{n=1}^{t-1}\sum_{j=1}^{M_E} 1_{\mathcal{H}_j^E(n)} E_h}{E_{\text{tx}}} \right\rfloor .$$

By the same reasoning, the number of transmissions by the original system by time t, $T(t)$, obeys the inequality

$$T(t) \leq T_{\max}(t). \tag{19}$$

In both the original system and the SP system, either a transmission or an outage occurs in every slot. Hence,

$$t = T_{\text{SP}}(t) + O_{\text{SP}}(t) = T(t) + O(t), \quad \text{for } t \geq 1. \tag{20}$$

The SP system evolves in one of the following two ways: *i) $T_{\text{SP}}(t) < T_{\max}(t)$, for all $t \geq 1$:* In this case, the SP node has at least $(T_{\max}(t) - T_{\text{SP}}(t-1))E_{\text{tx}} \geq E_{\text{tx}}$ units of energy in its battery. Since $h_{\text{SP}}(t)$ is at least as good as the channel seen by the transmitting node in the original system, the SP node will transmit whenever a node in the original system transmits. Hence, $T_{\text{SP}}(t) - T(t) \geq 0$, for $t \geq 1$. Using (20), we get

$$O(t) - O_{\text{SP}}(t) = T_{\text{SP}}(t) - T(t) \geq 0, \quad \text{for } t \geq 1. \tag{21}$$

ii) $T_{\text{SP}}(t) = T_{\max}(t)$, for $t = t_0, t_1, t_2, \ldots$ and $T_{\text{SP}}(t) < T_{\max}(t)$ otherwise: Using (20), for $r \geq 0$, we get

$$O(t_r) - O_{\text{SP}}(t_r) = T_{\text{SP}}(t_r) - T(t_r) = T_{\max}(t_r) - T(t_r) \geq 0.$$

For $0 \leq t < t_0$, since $T_{\text{SP}}(t) < T_{\max}(t)$, the reasoning in the previous case implies that $O(t) \geq O_{\text{SP}}(t)$.

We will now prove that $O(t) \geq O_{\text{SP}}(t)$, for $t_r < t < t_{r+1}$, for all $r \geq 0$. The following two cases can occur: a) $T_{\text{SP}}(t_{r+1}) = T_{\text{SP}}(t_r)$, or b) $T_{\text{SP}}(t_{r+1}) > T_{\text{SP}}(t_r)$.

a) When $T_{\text{SP}}(t_r) = T_{\text{SP}}(t_{r+1})$: This implies, for $t_r < t < t_{r+1}$, that $T_{\text{SP}}(t) = T_{\max}(t) = T_{\max}(t_r) = T_{\max}(t_{r+1})$. Hence, from (20) we get, $O(t) - O_{\text{SP}}(t) = T_{\text{SP}}(t) - T(t) = T_{\max}(t) - T(t) \geq 0$.

b) When $T_{\text{SP}}(t_{r+1}) > T_{\text{SP}}(t_r)$: Consider $r = 0$. If $t_1 = 1 + t_0$, then we are done. Else, in time slots $t_0 + 1 \leq t < t_1$, we are given that $T_{\text{SP}}(t) < T_{\max}(t)$. Then, in each of these slots, the SP node has sufficient transmission energy since $B_{\text{SP}}(t) \geq (T_{\max}(t) - T_{\text{SP}}(t-1))E_{\text{tx}} \geq E_{\text{tx}}$. Hence, using the same reasoning as before, the number of transmissions by the SP system is greater than or equal to those by the original system, i.e., $T(t) \leq T_{\text{SP}}(t)$, $t_0+1 \leq t < t_1$. Hence, from (20), we can again show that $O(t) - O_{\text{SP}}(t) \geq 0$, for $t_0+1 \leq t < t_1$.

Similarly, by induction it can be shown that the same holds for any $1+t_r \leq t < t_{r+1}$, for all $r \geq 1$. Hence, $O(t) \geq O_{\text{SP}}(t)$, for $t \geq 1$. Since $O(\mathscr{T}_k) = k$, this implies that $O_{\text{SP}}(\mathscr{T}_k) \leq k$. Hence, $\mathscr{T}_k \leq \mathscr{T}_k^{\text{SP}}$, which implies that $\mathbb{E}[\mathscr{T}_k] \leq \mathbb{E}[\mathscr{T}_k^{\text{SP}}]$.

B. Transition Probability Matrix $\mathbf{P}_k^{\text{SP}}$ of the SP System

The probability $p_{(w,x),(w',x')}$ that the SP node moves from state (wE_h, x) to state $(w'E_h, x')$ is obtained as follows. Once the SP system is in an absorbing state, then it remains there. Else, if $(wE_h, x) \notin \mathcal{A}_{\text{SP}}$, then the following two cases arise.

a) SP node does not transmit: The outage count increases to $x + 1$. This happens if: i) The SP node has no energy for transmission, $B_{\text{SP}}(t) < E_{\text{tx}}$, or ii) $h_{\text{SP}}(t) < \gamma_{\text{th}}$, which happens with probability $1-\zeta$, where ζ is the probability that the SP node's channel gain is at least γ_{th}. For Rayleigh fading, $\zeta = 1 - \left(1 - \exp\left(-\frac{\gamma_{\text{th}}}{\gamma_0}\right)\right)^{M_C+M_E}$. Also, if yE_h energy is harvested, $B_{\text{SP}}(t)$ increases from wE_h to $(w+y)E_h$. This happens with probability σ_y, where $\sigma_y = \binom{M_E}{y}\rho^y(1-\rho)^{M_E-y}$.

The probability of moving from the state (wE_h, x) to $((w+y)E_h, x+1)$, for $w + y < d$, is given by

$$p_{(w,x),(w+y,x+1)} = \begin{cases} \sigma_y, & wE_h < E_{\text{tx}}, \\ (1-\zeta)\sigma_y, & wE_h \geq E_{\text{tx}}. \end{cases} \tag{22}$$

The probability of moving from (wE_h, x) to $(dE_h, x+1)$ is

$$p_{(w,x),(d,x+1)} = \begin{cases} \sum_{y=d-w}^{M_E} \sigma_y, & wE_h < E_{\text{tx}}, \\ (1-\zeta)\sum_{y=d-w}^{M_E} \sigma_y, & wE_h \geq E_{\text{tx}}. \end{cases} \tag{23}$$

b) SP node transmits: The SP node transmits if: i) It has sufficient battery energy, $B_{\text{SP}}(t) \geq E_{\text{tx}}$, and ii) $h_{\text{SP}}(t) \geq \gamma_{\text{th}}$, which happens with probability ζ. As a transmission has occurred, the outage count remains unchanged. After a transmission, the battery energy of the SP node is $(w-l+y)E_h$ with probability σ_y. Therefore, for $wE_h \geq E_{\text{tx}}$ and $(w-l+y)E_h < dE_h$, $p_{(w,x),(w-l+y,x)} = \zeta\sigma_y$. Similarly, the probability of moving from state (wE_h, x) to (dE_h, x) is $p_{(w,x),(d,x)} = \zeta\sum_{y=d-w+l}^{M_E}\sigma_y$. All other transition probabilities are 0.

C. Deriving $\mathbb{E}[\mathscr{T}_k^{\text{SP}}]$

When the SP system starts from $(wE_h, x) \in \mathcal{A}_{\text{SP}}$, k outages have already occured. Hence, the k-outage duration given that the SP system starts from state $(wE_h, x) \in \mathcal{A}_{\text{SP}}$ is zero. Therefore, the absorbing states need not be considered in the analysis. If the system is in state $(wE_h, x) \in \mathcal{S}_{\text{SP}}$, it transits into $(w'E_h, x')$ in the next time slot with probability $p_{(w,x),(w',x')}$. Given the Markovian evolution of the SP system, the k-outage duration, given that the current state is $(wE_h, x) \in \mathcal{S}_{\text{SP}}$, is equal to

$$\mathbb{E}[\mathscr{T}_k^{\text{SP}}|(w,x)] = 1 + \sum_{w'=0}^{uM_C+dM_E} \sum_{x'=x}^{\min(x+1,k-1)} \mathbb{E}[\mathscr{T}_k^{\text{SP}}|(w',x')]p_{(w,x),(w',x')}.$$

Writing in terms of the vector $\mathbf{v}^{\text{SP}}$, we get $(\mathbf{I}_k^{\text{SP}} - \mathbf{Z}_k^{\text{SP}})\mathbf{v}^{\text{SP}} = \mathbf{1}_{k(uM_c+dM_E+1)}$. Hence, (11) follows.

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