Trade-offs in Estimating Maximum of Sensor Readings in Energy Harvesting Wireless Networks

Shilpa Rao and Neelesh B. Mehta, *Senior Member, IEEE*
Dept. of Electrical Communication Eng.
Indian Institute of Science (IISc), Bangalore, India
Emails: devashilpa@gmail.com, nbmehta@ece.iisc.ernet.in

***Abstract*— Computing the maximum of the sensor readings across a wireless sensor network (WSN) has applications in environmental, health, and industrial monitoring. We characterize the novel trade-offs that arise when green energy harvesting (EH) WSNs are deployed for computing the maximum. In these WSNs, the nodes harvest random amounts of energy from the environment for communicating their readings to a fusion node over time-varying wireless channels that undergo fading. The fusion node then periodically estimates the maximum. For a transmission schedule in which randomly selected sensor nodes are scheduled for transmission in each sensor data collection round, we derive closed-form expressions for the mean absolute error (MAE), which is defined as the expectation of the absolute difference between the maximum sensor reading and that estimated by the fusion node in a data collection round. We optimize the transmit energy and the number of nodes that should transmit in each round. Our analysis holds for any probability distribution of the sensor readings, and for the general class of stationary and ergodic energy harvesting random processes. Our results show that the optimal number of nodes that transmit in each round and their transmit powers depend on the average rate of energy harvesting and the number of nodes in the WSN.**

I. INTRODUCTION

Battery-operated wireless sensor networks (WSNs) are finding increasing acceptance in a diverse range of applications such as health monitoring, home automation, industrial control, and environmental monitoring [1]. In a WSN, each node senses, processes, and transmits data over a wireless channel to a fusion node (FN). The nodes expend energy in the process and eventually die [2]. Hence, a lot of attention has been devoted to techniques that extend the lifetime of WSNs.

Unlike a data network, which transports data between nodes, a WSN is designed for a specific sensing task [3]. The performance of a WSN is measured not by the amount of data transported to the FN but the accuracy of the sensed data aggregated at the FN. One practical application of WSNs involves determining the maximum of the sensor readings in the network, and has been referred to as the max function computation problem in the literature [3]–[6]. It arises in early detection of an impending event such as a fire or pollution and health monitoring, where the highest reading across all the sensors must be determined and tracked over time. In general, the max function computation problem comes under the class of problems called network function computation [3]–[5], [7], [8]. Solutions such as block computation [4] and in-network filtering [9] improve the efficiency and lifetime of such WSNs.

Energy harvesting (EH) is a different, green, and promising solution to address the lifetime conundrum in WSNs. EH sensor nodes equipped with rechargeable batteries harvest energy from renewable resources such as light, heat, and wind, and replenish their batteries [10]. While EH can ensure perpetual operability of the WSN, the system designer needs to grapple with the new challenges that arise due to the randomness in the harvested energy. For example, in max function computation, the FN may occasionally fail to determine the maximum if the node with the highest reading does not have sufficient energy to transmit its data to the FN or if its channel to the FN is in a deep fade. In general, this is determined by the energy harvesting, channel fading, and sensor readings, all of which are random processes, and the sensing protocol.

Focus and Contributions: In this paper, we highlight and study the novel system design trade-offs that arise in designing protocols for EH WSNs for max function computation. The challenge lies in handling the randomness in the EH and channel fading processes, which together influence the accuracy of the max function computation. While several recent papers in the literature have considered minimizing estimation errors in EH WSNs [11]–[14], minimizing the estimation error for max function computation has not been addressed to the best of our knowledge. The minimization of estimation error in max function computation gives new insights into protocol design for EH WSNs.

We study an EH WSN with a star topology, which is a fundamental building block in WSNs [4], [12], [15], [16] and is supported by Zigbee [17]. Time is divided into data collection rounds (DCRs). The FN needs to estimate the max sensor reading in each DCR. In each DCR, a subset of the nodes is scheduled to transmit as per a pre-defined *transmission schedule*. This is a generalization of the model in which all the sensor nodes transmit to the FN [12]. We also model fading in the channels between the nodes and the FN and the randomness in the energy harvested by different nodes, due to which some transmissions are not decodable by the FN. Thus, whether the FN receives a node's measurement depends on the transmission schedule, the node's battery energy, and

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the channel gain between the node and the FN.

We analyze the mean absolute error (MAE), which is defined as the expected value of the absolute difference between the maximum sensor reading in the WSN and that estimated by the FN with the expectation being taken over the channel fading, battery energy, transmission schedule, and sensor readings. A small MAE implies better tracking of the maximum with time. For a randomized transmission schedule in which a pre-specified number of randomly selected sensor nodes are scheduled for transmission, we derive closed-form expressions for the minimum MAE and the optimal transmit energy given the number of nodes K that transmit in a DCR, and then optimize K itself. The randomized transmission schedule ensures that all the nodes get, on average, equal opportunity to transmit their sensed data. Our analysis holds for any distribution of the sensor readings, and for the general class of all stationary and ergodic energy harvesting random processes, which covers several models considered in the literature [18]–[20]. For ease of exposition, we focus on readings with a non-negative support. Insightful simplifications for a specific statistical model of the sensor readings are also presented.

Outline and Notation: The system model is developed in Sec. II. We minimize the MAE in Sec. III. Numerical results are presented in Sec. IV, and our conclusions follow in Sec. V.

We use the following notation henceforth. The probability of an event A is denoted by $\Pr(A)$. For a random variable (RV) X, its expected value is denoted by $\mathbb{E}[X]$.

II. System Model

We consider a star network with N EH nodes and an FN. The measurement model, the battery evolution, and channel fading processes are as follows.

Measurement Model: A DCR is of duration T_{rd}. Let $Y_i[t]$ denote the sensor reading at the i^{th} node in the t^{th} DCR. The sensor readings are assumed to remain unchanged within a DCR. As mentioned, we focus on the scenario $Y_i[t] \geq 0$, for $t \geq 1$ and $1 \leq i \leq N$. In practice, this case arises in measuring quantities such as energy of vibrations, chemical concentrations, or in any counting process. Further, we assume that $Y_i[t]$ are independent and identically distributed (i.i.d.) across time [11], [12], [15] and nodes. Therefore, the maximum sensor reading $Y_{\max}[t]$ in the t^{th} DCR is equal to

$$Y_{\max}[t] = \max\{Y_1[t], Y_2[t], \ldots, Y_N[t]\}. \tag{1}$$

EH and Storage Model: Every node has a start-up battery energy of B_0. We assume that the battery capacity is infinite, as has been assumed in [12], [21]–[23]. The EH process at a node is assumed to be stationary and ergodic. The EH process is i.i.d. across nodes [16], [20]. The energy harvested in a DCR is available for transmission only in the subsequent DCR [16], [20]. Let $\overline{H}$ denote the average harvested energy by a node.

Channel Model: Let $h_i[t]$ denote the frequency-flat channel power gain of the i^{th} EH node in the t^{th} DCR. We assume Rayleigh fading. Furthermore, $h_i[t]$ are independent, for $1 \leq i \leq N$ and $t \geq 1$. For ease of exposition, we focus on the case where the channel gains are statistically identical [15], [16], [24]. Let γ_0 denote the mean channel power gain, which includes the path-loss.

Transmission Model: Every DCR is sub-divided into N time slots, one for each node. In each DCR, as per a pre-specified randomized transmission schedule, K nodes transmit their measured data sequentially in slots of duration $T_s = \frac{T_{\text{rd}}}{N}$ each.[1] While we do not model quantization noise in this paper, it can be easily incorporated into our model. When $K = N$, this model reduces to the case considered in [12], in which all the nodes transmit data to the FN. As mentioned, the transmission schedule is not a function of the instantaneous battery energies or the channel gains of the nodes. We neglect the energy required for sensing and computation, as radio communication is a major source of energy consumption in WSNs in which the nodes are sufficiently far away from the FN [12], [25].

Let the transmit power be P. Then the energy E required by a node to transmit in a DCR is $E = PT_s$. A node that has an energy of at least E in its battery is called an *active node*. The power spectral density of noise is $\frac{k_B T_e}{2}$ where k_B is the Boltzmann constant and T_e the environment temperature. The noise energy in a slot of duration T_s with system bandwidth W is then $k_B T_e W T_s$. The FN decodes the transmission if the signal-to-noise-ratio (SNR) $\frac{h_i[t]E}{k_B T_e W T_s}$ exceeds a threshold ω. A node can transmit in a DCR only if it is included in the transmission schedule for that DCR and is active. Else, no transmission occurs in the slot and the FN sets the measurement reported in this slot to be zero as this has no effect on the maximum of the received measurements.

Measurement Error: Let $\mathcal{S}_t$ denote the set of nodes whose measurements are decoded by the FN in DCR t. Let $Y_{\text{rc}}[t]$ denote the maximum of the measurements received by the FN in the t^{th} round:

$$Y_{\text{rc}}[t] = \begin{cases} \max\limits_{i \in \mathcal{S}_t} Y_i[t], & \mathcal{S}_t \neq \phi, \\ 0, & \mathcal{S}_t = \phi, \end{cases} \tag{2}$$

where ϕ denotes the null set. The absolute measurement error $\mathcal{X}[t]$ in the t^{th} round is $\mathcal{X}[t] = |Y_{\max}[t] - Y_{\text{rc}}[t]|$. It is an RV since both $Y_{\max}[t]$ and $Y_{\text{rc}}[t]$ are RVs.

III. MAE Minimization

We focus on the steady state, in which the channel fading, the energy harvested, the energy consumed, and the measurements are all RVs and vary across DCRs, but their probability distributions have become stationary. We henceforth drop the time index t and consider the system behavior in an arbitrary DCR. Our goal is to find the optimal K and E that give the infimum of the MAE. The optimization problem can be stated

[1] Note that in this model, the slot duration is fixed and the entire DCR is not used for transmission when $K \neq N$. A variant of this model is to utilize the entire DCR for transmission, in which the slot duration is based on the scheduled subset size and is given by $T_s = \frac{T_{\text{rd}}}{K}$. The minimum MAE and optimal transmit energy for this alternate model can be determined along lines similar to the analysis presented in this paper.

as

$$\inf_{E,K} \mathbb{E}[\mathcal{X}], \quad \text{s.t. } E \geq 0,\ 1 \leq K \leq N. \tag{3}$$

We solve this optimization problem in two stages. Initially, for any given K, we determine the infimum of the MAE, which is denoted by $\overline{\mathcal{X}}_K^*$. Further, we show that the infimum is achievable. Thus, $\overline{\mathcal{X}}_K^*$ is the minimum MAE for a given K. Thereafter, we numerically find the optimal K, denoted by K^*, in Sec. IV.

For the N RVs $Y_1, Y_2, \ldots, Y_N$, let the RV $Y_{r:N}$ denote the r^{th} smallest value of the Y_i's [26, Chapter 1]. Clearly, $Y_{1:N} \leq Y_{2:N} \leq \ldots \leq Y_{N:N}$.

Theorem *1:* When K EH nodes are scheduled in each data collection round, $\overline{\mathcal{X}}_K^*$ is given by

$$\overline{\mathcal{X}}_K^* = \begin{cases} \mathbb{E}[Y_{N:N}], & \overline{H} = 0, \\ \mathbb{E}[Y_{N:N}] - \sum_{l=1}^{K} \mathbb{E}[Y_{l:K}] \frac{\overline{H}}{K\xi e}\left(1 - \frac{\overline{H}}{K\xi e}\right)^{K-l}, & 0 < \overline{H} < K\xi, \\ \mathbb{E}[Y_{N:N}] - \sum_{l=1}^{K} \mathbb{E}[Y_{l:K}] e^{-\frac{K\xi}{\overline{H}}}\left(1 - e^{-\frac{K\xi}{\overline{H}}}\right)^{K-l}, & \overline{H} \geq K\xi. \end{cases} \tag{4}$$

where $\xi = \frac{\omega k_B T_e W T_s}{N\gamma_0}$. The optimal transmission energy E_K^* that achieves it is given by

$$E_K^* = \begin{cases} 0, & \overline{H} = 0, \\ N\xi, & 0 < \overline{H} < K\xi, \\ \frac{N\overline{H}}{K}, & \overline{H} \geq K\xi. \end{cases} \tag{5}$$

Proof: The proof is relegated to Appendix A. ∎

In order to gain more insights, we study the special case in which $Y_1, Y_2, \ldots, Y_N$ are uniformly distributed in the interval $[0, 1]$. In this case,

$$\mathbb{E}[Y_{l:K}] = \frac{l}{K+1} \text{ and } \mathbb{E}[Y_{N:N}] = \frac{N}{N+1}. \tag{6}$$

The minimum MAE with K nodes simplifies to

$$\overline{\mathcal{X}}_K^* = \begin{cases} \frac{N}{N+1}, & \overline{H} = 0, \\ \frac{K\xi e}{\overline{H}(K+1)} - \frac{1}{N+1} - \frac{1+\frac{K\xi e}{\overline{H}}}{K+1}\left(1 - \frac{\overline{H}}{K\xi e}\right)^{K}, & 0 < \overline{H} < K\xi, \\ \frac{e^{\frac{K\xi}{\overline{H}}}}{K+1} - \frac{1}{N+1} - \frac{1+e^{\frac{K\xi}{\overline{H}}}}{K+1}\left(1 - e^{-\frac{K\xi}{\overline{H}}}\right)^{K}, & \overline{H} \geq K\xi. \end{cases}$$

IV. Numerical Results

We now evaluate the minimum MAE of the system using Monte Carlo simulations that average over 10^5 DCRs and compare it with our analytical results. We set $\omega = 8$ dB, $T_s = 10$ ms, $W = 5$ MHz, $T_e = 300$ K, and the carrier frequency to 2.4 GHz. Nodes are at a distance of 83 m from the FN and the path-loss exponent is 4. Using the simplified path-loss model [Chapter 2] [27] with a 10 m reference distance, we get $\gamma_0 = 2.06 \times 10^{-10}$. In the simulations, every node experiences the Bernoulli energy harvesting process, in which 7 μJ of energy is injected with probability ρ in a DCR. The measurements are uniformly distributed in $[0, 1]$.

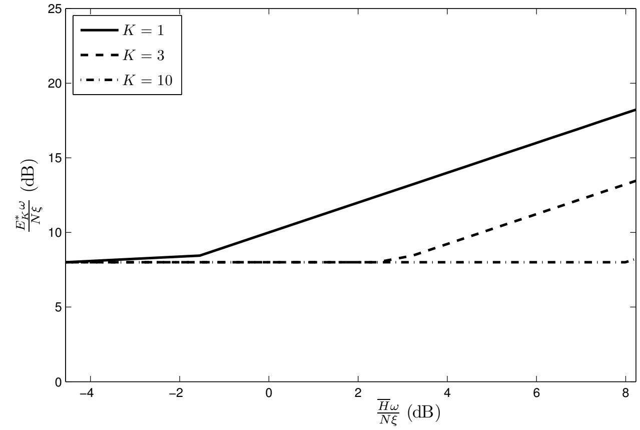

Fig. 1. $\frac{E_K^*\omega}{N\xi}$ as a function of $\frac{\overline{H}\omega}{N\xi}$ ($N = 10$).

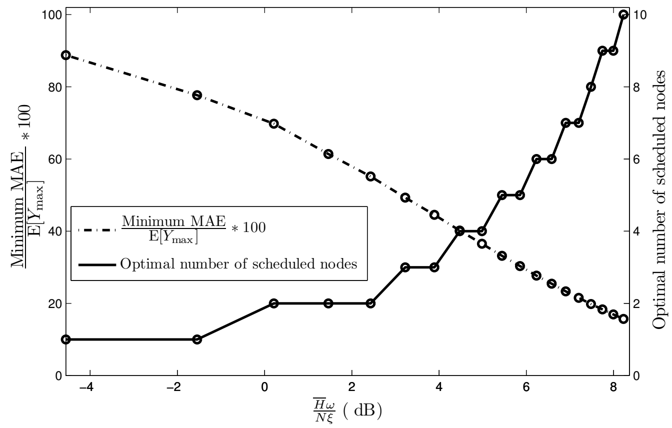

Fig. 2. Optimal number of scheduled nodes and the minimum MAE as a function of $\frac{\overline{H}\omega}{N\xi}$ ($N = 10$). The lines indicate the optimal analytical values obtained by numerically optimizing $\overline{\mathcal{X}}_K^*$, which is given in (4). Simulation results are shown by the marker ∘.

Fig. 1 plots the scaled optimal transmission energy $\frac{E_K^*\omega}{N\xi}$ as a function of $\frac{\overline{H}\omega}{N\xi}$, which is the average SNR if the node were to transmit with energy $\overline{H}$. We observe that when $\overline{H} < K\xi$, the node transmits with a constant energy so that the SNR is fixed at ω (cf. Theorem 1). Here, the probability that the node is active is less than 1. When the average harvested energy increases to $K\xi$, the node's probability of being active increases to 1. Once $\overline{H} \geq K\xi$, the node's transmission energy becomes proportional to $\overline{H}$ so that the probability that the transmitted packet gets decoded increases, and probability that the node is active is 1.

Fig. 2 plots the optimal number of nodes and the minimum MAE as a function of $\frac{\overline{H}\omega}{N\xi}$. We observe that when the average energy harvested by the nodes is very less, a transmission schedule with fewer nodes is optimal as it conserves the scarce energy for future transmissions. As $\overline{H}$ increases, the number of active nodes and probability that the packet gets decoded at the FN increase. The optimal number of scheduled nodes increases since it is the number of received measurements that now drives the EH WSN's measurement accuracy.

V. Conclusions

We considered an EH WSN deployed for the purpose of max function computation. In it, the FN periodically collects measurements from the EH sensor nodes and estimates the maximum. We analyzed the mean absolute error, which measures the accuracy in estimating the maximum, for the general class of stationary and ergodic EH random processes and for any distribution of the sensor readings. Our optimization of the MAE characterized for the first time how the optimal number of nodes that transmit and their transmit power depends on the mean energy harvested by them and the number of nodes in the WSN. Future work involves investigating the impact of channel state information, spatial correlation, and quantization error on estimating the maximum in such EH WSNs.

Appendix

A. Proof of Theorem 1

Since $Y_{\max} \geq Y_{\text{rc}}$, evaluating $\overline{\mathcal{X}}_K^*$ is equivalent to determining $\sup \mathbb{E}[Y_{\text{rc}}]$. Let ψ be the probability of the event that the channel power gain of a node exceeds $\frac{\omega k_B T_e W T_s}{E}$. For Rayleigh fading, $\psi = e^{-\frac{\omega k_B T_e W T_s}{\gamma_0 E}} = e^{-\frac{N\xi}{E}}$. Let ζ be the probability of the event that a node's battery energy is at least E. As these two events are mutually independent, a scheduled node's measurement is decoded with probability $\zeta\psi$.

In a DCR, let $\mathscr{C}_l^K$ denote the event that given that K nodes are scheduled for transmission, the measurement of the scheduled node with the l^{th} smallest value among the K nodes is decoded by the FN and the measurements of the scheduled nodes with $(l+1)^{\text{th}}, (l+2)^{\text{th}}, \ldots, K^{\text{th}}$ smallest measurements are not decoded. As the transmission schedule, the energy harvesting process, and the channel fading process of the nodes are mutually independent, it follows that

$$\Pr\left(\mathscr{C}_l^K\right) = \zeta\psi(1-\zeta\psi)^{K-l}. \tag{7}$$

From the law of total probability, we then get

$$\begin{aligned}\mathbb{E}[Y_{\text{rc}}] &= \sum_{l=1}^{K} \mathbb{E}[Y_{\text{rc}} \mid \mathscr{C}_l^K]\Pr\left(\mathscr{C}_l^K\right), \\ &= \sum_{l=1}^{K} \mathbb{E}[Y_{l:K}]\zeta\psi(1-\zeta\psi)^{K-l}. \end{aligned} \tag{8}$$

Let $\overline{U}$ denote the average transmit energy consumed by a node in a DCR. From the law of conservation of energy, it follows that

$$\overline{H} \geq \overline{U} = \frac{K}{N}\zeta E. \tag{9}$$

The latter follows because a node is chosen in a DCR with probability $\frac{K}{N}$, and it transmits with energy E with probability ζ.

We now calculate the supremum of $\mathbb{E}[Y_{\text{rc}}]$ in each of the following three regimes: $0 < E < \frac{N\overline{H}}{K}$, $E = \frac{N\overline{H}}{K}$, and $\frac{N\overline{H}}{K} < E < \infty$. As we shall see, ζ changes from 1 in the first two regimes to strictly less than 1 in the last regime.

1) When $0 < E < \frac{N\overline{H}}{K}$: In this regime, it can be shown that, in steady state, the battery energy of the EH node becomes infinite. Hence, $\zeta = 1$, and

$$\mathbb{E}[Y_{\text{rc}}] = \sum_{l=1}^{K} \mathbb{E}[Y_{l:K}]\psi(1-\psi)^{K-l}. \tag{10}$$

It can also be shown that $\mathbb{E}[Y_{\text{rc}}]$ is monotonically increasing in ψ, which lies in the interval $\left[0, e^{-\frac{K\xi}{\overline{H}}}\right)$. Therefore,

$$\sup_{0 \leq E < \frac{N\overline{H}}{K}} \mathbb{E}[Y_{\text{rc}}] = \sum_{l=1}^{K} \mathbb{E}[Y_{l:K}]e^{-\frac{K\xi}{\overline{H}}}\left(1 - e^{-\frac{K\xi}{\overline{H}}}\right)^{K-l}. \tag{11}$$

2) When $E = \frac{N\overline{H}}{K}$: Here $\overline{U} = \overline{H}\zeta$. In this regime, we can show that $\zeta = 1$. Hence, from (10), with ψ set as $e^{-\frac{K\xi}{\overline{H}}}$, we get

$$\mathbb{E}[Y_{\text{rc}}] = \sum_{l=1}^{K} \mathbb{E}[Y_{l:K}]e^{-\frac{K\xi}{\overline{H}}}\left(1 - e^{-\frac{K\xi}{\overline{H}}}\right)^{K-l}. \tag{12}$$

3) When $\frac{N\overline{H}}{K} < E < \infty$: In this regime, it can be shown that $0 \leq \zeta < 1$ and $\overline{U} = \overline{H}$. From (9), we then get $\zeta = \frac{N\overline{H}}{KE}$.

It can be shown that $\mathbb{E}[Y_{\text{rc}}]$ is a monotonically increasing function of $\zeta\psi$. If $\overline{H} < K\xi$, $\zeta\psi$ increases when E increases from $\frac{N\overline{H}}{K}$ to $N\xi$, and thereafter decreases. If $\overline{H} \geq K\xi$, $\zeta\psi$ decreases as E increases from $\frac{N\overline{H}}{K}$ to ∞. Therefore,

$$\begin{aligned}&\sup_{\frac{N\overline{H}}{K} < E < \infty} \mathbb{E}[Y_{\text{rc}}] \\ &= \begin{cases} \sum_{l=1}^{K} \mathbb{E}[Y_{l:K}]\frac{\overline{H}}{K\xi e}\left(1 - \frac{\overline{H}}{K\xi e}\right)^{K-l}, & \overline{H} < K\xi, \\ \sum_{l=1}^{K} \mathbb{E}[Y_{l:K}]e^{-\frac{K\xi}{\overline{H}}}\left(1 - e^{-\frac{K\xi}{\overline{H}}}\right)^{K-l}, & \overline{H} \geq K\xi. \end{cases}\end{aligned} \tag{13}$$

We now compare (11), (12), and (13) to get $\sup_{E \geq 0} \mathbb{E}[Y_{\text{rc}}]$.

i) When $\overline{H} \geq K\xi$: We find that $E_K^* = \frac{N\overline{H}}{K}$ with

$$\sup_{E \geq 0} \mathbb{E}[Y_{\text{rc}}] = \sum_{l=1}^{K} \mathbb{E}[Y_{l:K}]e^{-\frac{K\xi}{\overline{H}}}\left(1 - e^{-\frac{K\xi}{\overline{H}}}\right)^{K-l}. \tag{14}$$

ii) When $\overline{H} < K\xi$: We know that $\mathbb{E}[Y_{\text{rc}}]$ is monotonically increasing in $\zeta\psi$. From the discussion preceding (11), $\sup_{0 \leq E < \frac{N\overline{H}}{K}} \zeta\psi = e^{-\frac{K\xi}{\overline{H}}}$, and that preceding (13), $\sup_{\frac{N\overline{H}}{K} < E < \infty} \zeta\psi = \frac{\overline{H}}{K\xi e}$. From (12), $\zeta\psi = e^{-\frac{K\xi}{\overline{H}}}$ when $E = \frac{N\overline{H}}{K}$.

Using the inequality $e^{-x} \leq \frac{1}{xe}$, $x \geq 0$, we get $e^{-\frac{K\xi}{\overline{H}}} \leq \frac{\overline{H}}{K\xi e}$. Hence, $\sup_{0 \leq E < \infty} \zeta\psi = \frac{\overline{H}}{K\xi e}$. From (13), we find that in this regime, $E_K^* = N\xi$ and

$$\sup_{E \geq 0} \mathbb{E}[Y_{\text{rc}}] = \sum_{l=1}^{K} \mathbb{E}[Y_{l:K}]\frac{\overline{H}}{K\xi e}\left(1 - \frac{\overline{H}}{K\xi e}\right)^{K-l}.$$

Thus, $\sup \mathbb{E}[Y_{\text{rc}}]$ is achievable, i.e., the maximum of $\mathbb{E}[Y_{\text{rc}}]$ exists, and $\overline{\mathcal{X}}_K^*$ is the minimum MAE. Hence, (4) follows.

References

[1] I. F. Akyildiz, T. Melodia, and K. R. Chowdury, "Wireless multimedia sensor networks: A survey," *IEEE Wireless Commun. Mag.*, vol. 14, no. 6, pp. 32–39, Dec. 2007.

[2] Y. Chen, Q. Zhao, V. Krishnamurthy, and D. Djonin, "Transmission scheduling for optimizing sensor network lifetime: A stochastic shortest path approach," *IEEE Trans. Signal Process.*, vol. 55, no. 5, pp. 2294–2309, May 2007.

[3] A. Giridhar and P. Kumar, "Toward a theory of in-network computation in wireless sensor networks," *IEEE Commun. Mag.*, vol. 44, no. 4, pp. 98–107, Apr. 2006.

[4] ——, "Computing and communicating functions over sensor networks," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 4, pp. 755–764, Apr. 2005.

[5] N. Khude, A. Kumar, and A. Karnik, "Time and energy complexity of distributed computation of a class of functions in wireless sensor networks," *IEEE Trans. Mobile Comput.*, vol. 7, no. 5, pp. 617–632, May 2008.

[6] A. Anand and N. B. Mehta, "Quick, decentralized, one shot max function computation using timer based selection," in *Proc. ICC*, Jun. 2014, pp. 1990–1995.

[7] R. Sappidi, A. Girard, and C. Rosenberg, "Maximum achievable throughput in a wireless sensor network using in-network computation for statistical functions," *IEEE/ACM Trans. Netw.*, vol. 21, no. 5, pp. 1581–1594, Oct. 2013.

[8] M. Li, Y. Wang, and Y. Wang, "Complexity of data collection, aggregation, and selection for wireless sensor networks," *IEEE Trans. Comput.*, vol. 60, no. 3, pp. 386–399, Mar. 2011.

[9] G. Riva, J. Finochietto, and G. Leguizamon, "Bio-inspired in-network filtering for wireless sensor monitoring systems," in *IEEE Cong. Evol. Comput. (CEC)*, Jun. 2013, pp. 3379–3386.

[10] S. Sudevalayam and P. Kulkarni, "Energy harvesting sensor nodes: Survey and implications," *IEEE Commun. Surveys Tuts.*, vol. 13, no. 3, pp. 443–461, Third Quarter 2011.

[11] Y. Zhao, B. Chen, and R. Zhang, "Optimal power allocation for an energy harvesting estimation system," in *Proc. ICASSP*, May 2013, pp. 4549–4553.

[12] C. Huang, Y. Zhou, T. Jiang, P. Zhang, and S. Cui, "Power allocation for joint estimation with energy harvesting constraints," in *Proc. ICASSP*, May 2013, pp. 4804–4808.

[13] J. Wu and G. Zhou, "A new ultra-low power wireless sensor network with integrated energy harvesting, data sensing, and wireless communication," in *Proc. ICC*, Jun. 2011, pp. 1–5.

[14] L. Vijayandran, M. Brandt-Pearce, K. Kansanen, and T. Ekman, "Efficient state estimation with energy harvesting and fairness control using stochastic optimization," in *Proc. Globecom*, Dec. 2011, pp. 1–6.

[15] S. Cui, J.-J. Xiao, A. Goldsmith, Z.-Q. Luo, and H. Poor, "Estimation diversity and energy efficiency in distributed sensing," *IEEE Trans. Signal Process.*, vol. 55, no. 9, pp. 4683–4695, Sept 2007.

[16] F. Iannello, O. Simeone, and U. Spagnolini, "Medium access control protocols for wireless sensor networks with energy harvesting," *IEEE Trans. Commun.*, vol. 60, no. 5, pp. 1381–1389, May 2012.

[17] *IEEE standard for local and metropolitan area networks, IEEE Std 802.15.4e–2012 (Amendment to IEEE Std 802.15.4–2011)*, IEEE Std. 802.15.4e, 2012.

[18] V. Sharma, U. Mukherji, V. Joseph, and S. Gupta, "Optimal energy management policies for energy harvesting sensor nodes," *IEEE Trans. Wireless Commun.*, vol. 9, no. 4, pp. 1326–1336, Apr. 2010.

[19] A. Seyedi and B. Sikdar, "Energy efficient transmission strategies for body sensor networks with energy harvesting," *IEEE Trans. Commun.*, vol. 58, no. 7, pp. 2116–2126, Jul. 2010.

[20] K. Ramachandran and B. Sikdar, "A population based approach to model the lifetime and energy distribution in battery constrained wireless sensor networks," *IEEE J. Sel. Areas Commun.*, vol. 28, no. 4, pp. 576–586, Apr. 2010.

[21] R. Vaze, "Transmission capacity of wireless ad hoc networks with energy harvesting nodes," in *Proc. GlobalSIP*, Dec. 2013, pp. 353–358.

[22] K. Huang, "Spatial throughput of mobile ad hoc networks powered by energy harvesting," *IEEE Trans. Inf. Theory*, vol. 59, no. 11, pp. 7597–7612, Nov. 2013.

[23] S. Park, H. Kim, and D. Hong, "Cognitive radio networks with energy harvesting," *IEEE Trans. Wireless Commun.*, vol. 12, no. 3, pp. 1386–1397, Mar. 2013.

[24] R. Niu, B. Chen, and P. Varshney, "Fusion of decisions transmitted over Rayleigh fading channels in wireless sensor networks," *IEEE Trans. Signal Process.*, vol. 54, no. 3, pp. 1018–1027, Mar. 2006.

[25] V. Raghunathan, C. Schurgers, S. Park, and M. Srivastava, "Energy-aware wireless microsensor networks," *IEEE Signal Process. Mag.*, vol. 19, no. 2, pp. 40–50, Mar. 2002.

[26] B. C. Arnold, N. Balakrishnan, and H. N. Nagaraja, *A First Course in Order Statistics (Classics in Applied Mathematics)*. SIAM, 2008.

[27] A. J. Goldsmith, *Wireless Communications*, 1st ed. Cambridge Univ. Press, 2005.