SEP-Optimal Antenna Selection for Average Interference Constrained Underlay Cognitive Radios

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***Abstract*—In the underlay mode of cognitive radio, secondary users can transmit when the primary is transmitting, but under tight interference constraints, which limit the secondary system performance. Antenna selection (AS)-based multiple antenna techniques, which require less hardware and yet exploit spatial diversity, help improve the secondary system performance. In this paper, we develop the optimal transmit AS rule that minimizes the symbol error probability (SEP) of an average interference-constrained secondary system that operates in the underlay mode. We show that the optimal rule is a non-linear function of the power gains of the channels from secondary transmit antenna to primary receiver and secondary transmit antenna to secondary receive antenna. The optimal rule is different from the several ad hoc rules that have been proposed in the literature. We also propose a closed-form, tractable variant of the optimal rule and analyze its SEP. Several results are presented to compare the performance of the closed-form rule with the ad hoc rules, and interesting inter-relationships among them are brought out.**

I. Introduction

The increasing demand for high wireless data rates has increased the need for efficient spectrum utilization techniques and has led to the development of cognitive radio (CR) [1]. In one common paradigm of CR, two classes of users are defined, namely, primary users (PU) and secondary users (SU). The PU is the owner of the spectrum. A SU can use the same spectrum as the PU, but under constraints that are designed to protect the PU. For example, in the interweave mode of CR [2], the SU transmits only in the spectral regions not being used by PU. Hence, the SU does not cause any interference to the PU, except when it senses the spectrum incorrectly. Whereas, in the underlay mode of CR [2], the SU can access the spectrum even when the PU is transmitting, but under tight constraints on the average or peak interference that causes to the primary.

These constraints limit the performance of the secondary system, when measured in terms of throughput or symbol error probability (SEP). Employing multiple antennas at the SUs mitigates the impact of this limitation [3]–[5]. However, one drawback of a multiple antenna system is that each antenna element requires a dedicated, expensive radio frequency (RF) chain to process its signals. In order to reduce the hardware costs, a technique called antenna selection (AS) has been extensively studied; see, for example, [6], [7] and references therein. In single transmit AS, which is the focus of this paper, one of the transmit antennas is selected as a function of the channel conditions and is connected to the one available RF chain in the transmitter. Doing so reduces the hardware complexity, cost, and size of the transmitter. Yet, AS has been shown to harness the diversity benefits of multiple antennas [6], [8], [9].

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Given its promise and practical feasibility, AS has also been considered in CR systems [10]–[12], and has been shown to improve secondary system performance. In the interweave mode, since the SU does not interfere with the PU, the AS rule remains the same as that in a conventional AS system that is not subject to any interference constraint. For example, in a multiple input single output (MISO) secondary system, this involves selecting the secondary transmitter (STx) antenna that has the strongest channel power gain to the secondary receiver (SRx). We shall refer to this as the *unconstrained* AS rule henceforth. However, in the underlay mode, the primary interference constraint affects the choice of the transmit antenna. Intuitively, even though an STx antenna has a strong link to the SRx, it should not get selected if it causes significant interference to the primary receiver (PRx). Therefore, the AS rule must take into consideration both the STx to SRx (STx-SRx) and STx to PRx (STx-PRx) channel power gains in the underlay mode.

Related Work on AS in Underlay CR: Several interesting rules for selecting an antenna in a MISO CR, such as the minimum interference (MI) rule and the maximum signal power to leak interference power ratio (MSLIR) rule, were proposed in [10]. The MI rule selects the antenna that causes the least interference to the PRx. However, the selection is done only on the basis of the channel gains to the PRx. The MSLIR rule compromises between the MI and unconstrained rules, and selects the antenna with the highest ratio of STx-SRx and STx-PRx channel power gains. Note that the above rules do not consider the average interference constraint and may not always be feasible. A difference selection (DS) rule was proposed in [11]. It selects that antenna that maximizes a weighted difference between the STx-SRx and STx-PRx channel power gains. It was shown to outperform the MSLIR rule in many scenarios. All the above rules use fixed transmit power. They are ad hoc as they do not provably optimize an end objective such as SEP or capacity. The optimal AS rule for underlay CR even with fixed transmit power is an open problem, and is the problem that this paper solves.

Contributions: We systematically develop the optimal AS rule for a MISO underlay CR system that minimizes the SEP at the SRx when the STx is subject to an average primary interference constraint. In our model, the STx transmits with a fixed power or with zero power depending on the channel

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conditions. The SEP-optimal rule is shown to take a simple form; it is a linear combination of the STx-PRx channel power gain and an exponentially decaying function of the STx-SRx gain. This rule turns out to be in the form of a single integral. In order to obtain a closed-form characterization for the AS rule, we then consider a variant of it that minimizes SEP upper bound instead of the exact SEP. For brevity we shall refer to this as the *closed-form optimal* rule. We then analyze its SEP. To the best of our knowledge, such an SEP analysis is interesting even from an AS performance analysis point of view because it deals with a non-linear selection rule, while most of the literature on the SEP analysis of AS in MISO systems focuses on linear rules [13], [14].

We also determine the optimal setting of the transmit power. We then extensively benchmark the performance of the closed-form optimal rule with several rules proposed in the literature. In order to provide a fair comparison, we compare against enhanced versions of the MI and MSLIR rules that can always adhere to the average interference constraint. The comparisons bring out interesting inter-relationships among the rules. For example, we show that the closed-form optimal rule, the enhanced MI rule, and the DS rule are equivalent only for large values of transmit power, and that even the enhanced MSLIR rule is suboptimal in most scenarios of interest. Due to space constraints, we focus on the scenario where the STx has two transmit antennas. The approach can be easily generalized to handle more antennas at the STx [15].

The paper is organized as follows. Section II develops the system model. The optimal selection rule and SEP analysis are developed in Section III. Numerical results are presented in Section IV, and are followed by our conclusions in Section V. Mathematical derivations are relegated to the Appendix.

II. System Model and Problem Statement

We use the following notation henceforth. The absolute value of a complex number x is denoted by $|x|$. The probability of an event A and the conditional probability of A given B are denoted by $\Pr(A)$ and $\Pr(A|B)$, respectively. For a random variable (RV) X, $f_X(x)$ denotes its probability density function (PDF) and $\mathbf{E}_X[.]$ denotes its expectation. Scalar variables are shown in normal font and vector variables are shown in bold font. $I_{\{a\}}$ denotes the indicator function; it is 1 if a is true and is 0 otherwise.

As shown in Figure 1, we consider an underlay CR system in which an STx transmits data to an SRx, and in the process interferes with a PRx. The SRx and STx constitute the secondary system. The PRx and the SRx have one receive antenna each. The STx has two transmit antennas and one RF chain; it, therefore, selects one of its antennas for transmission. For $i \in \{1,2\}$, h_i denotes the instantaneous channel power gain between the i^{th} antenna of the STx and the SRx antenna, and g_i denotes the instantaneous channel power gain between the i^{th} antenna of the STx and the PRx antenna. All channels undergo Rayleigh fading. The STx-SRx channels are independent and identically distributed (i.i.d.) random variables (RVs), and so are the STx-PRx channels. The independence of the channel gains is justified when the antennas at the STx are spaced sufficiently apart in a rich scattering environment. Thus, h_i and g_i are i.i.d. exponential RVs with means μ_h and μ_g, respectively. Let $\mathbf{h} = [h_1,\ h_2]$ and $\mathbf{g} = [g_1,\ g_2]$.

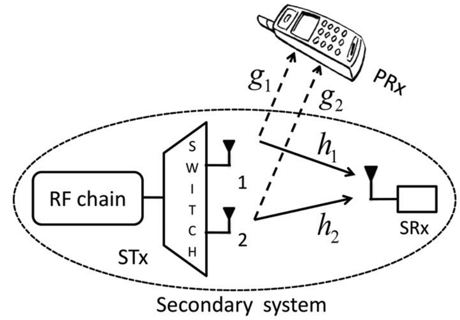

Fig. 1. System model with one PRx and a secondary system consisting of a STx with two transmit antennas and one RF chain that communicates with a SRx with one receive antenna.

A. Selection Options and Data Transmission

The STx transmits a symbol x that is drawn with equal probability from an M-ary PSK (MPSK) constellation. It can transmit using one out of two antennas with a fixed symbol energy E_t. Further, it may choose to transmit with zero power in order to avoid interfering with the primary. We shall represent the zero transmit power option by 0, and shall define the corresponding channel power gains as zero, i.e., $h_0 \triangleq 0$ and $g_0 \triangleq 0$. Note that the SRx cannot know when the STx has used the zero transmit power option. When the STx transmits a symbol with zero power, the SRx can correctly decode it with probability $\frac{1}{M}$. Thus, the SEP for this option is equal to $m \triangleq 1 - \frac{1}{M}$ [16]. Transmission using antenna i is represented by option i, for $i = 1, 2$.

Let $s \in \{0,1,2\}$ be the option selected. Then the signal r received by the SRx and the interference signal i_p seen by the PRx are given by

$$r = \sqrt{E_t}\sqrt{h_s}e^{j\theta_{h_s}}x + n + w_{ps}, \tag{1}$$

$$i_p = \sqrt{E_t}\sqrt{g_s}e^{j\theta_{g_s}}x, \tag{2}$$

where $|x|^2 = 1$, θ_{h_s} and θ_{g_s} are the phases of the complex baseband STx-SRx and STx-PRx channel gains, respectively, for the selection option s, and n is circular symmetric complex additive white Gaussian noise at the SRx. The interference seen by the SRx due to primary transmission is w_{ps}, and is assumed to be Gaussian. This corresponds to a worst case model for the interference and makes the problem of finding the optimal AS rule tractable. Therefore, $n + w_{ps}$ is a circular symmetric complex Gaussian RV, whose variance we denote by σ^2.[1]

We assume that the STx knows $\mathbf{h}$ and $\mathbf{g}$, i.e., its channel power gains to the SRx and to the PRx, as has been assumed

[1]The primary transmitter (PTx) affects the secondary system performance entirely through σ^2. Therefore, no additional details about PTx are given.

in [10], [11], [17]. Note also that no knowledge of the phases of any complex baseband channel gains is required at the STx.[2] The SRx uses a coherent receiver, and is assumed to know h_s and θ_{h_s}.[3] No knowledge of $\mathbf{g}$ or any other channel gain is required at the SRx.

B. Problem Statement

A selection rule ϕ is a mapping $\phi : (\mathbb{R}^+)^2 \times (\mathbb{R}^+)^2 \rightarrow \{0, 1, 2\}$ that selects one of the three options for every realization of $\mathbf{h}$ and $\mathbf{g}$. Our goal is to develop the optimal transmit AS rule that minimizes the average SEP of the secondary system and also ensures that the average interference caused to the PRx is below a threshold I_{ave}. We first consider the case where E_t is given. Let $\text{SEP}(h_s)$ denote the instantaneous SEP of the secondary system as a function of the channel power gain h_s of the selected option s. Using (2), the average interference at the PRx due to secondary transmission is given by $E_t \mathbf{E}_{\mathbf{h},\mathbf{g}}[g_s]$.

Terminology: We define a *feasible selection rule* to be a selection rule whose average interference is less than or equal to I_{ave}. Let $\mathcal{Z}$ be the set of all feasible selection rules.

Our problem can be mathematically stated as a minimization over the space of all selection rules $\mathcal{Z}$:

$$\begin{aligned} \min_{\phi \in \mathcal{Z}} \quad & \mathbf{E}_{\mathbf{h},\mathbf{g}}[\text{SEP}(h_s)] \\ \text{such that} \quad & E_t \mathbf{E}_{\mathbf{h},\mathbf{g}}[g_s] \leq I_{\text{ave}}, \\ & s = \phi(\mathbf{h}, \mathbf{g}). \end{aligned} \quad (3)$$

III. Optimal AS Rule and SEP Analysis

We now derive and analyze the optimal selection rule.

A. Optimal Selection Rule

Let us first consider the unconstrained AS rule that minimizes the SEP at the SRx without taking into consideration the interference caused to the PRx. It selects the antenna with the highest channel power gain to the SRx. It is given by $s = \arg\max_{i \in \{1,2\}} \{h_i\}$. The average interference I_{un} caused to the PRx by this rule is $I_{\text{un}} = E_t \mathbf{E}_{\mathbf{h},\mathbf{g}}[g_s] = E_t \mu_g$. The second equality follows because the choice of s does not depend on $\mathbf{g}$.

When $I_{\text{un}} > I_{\text{ave}}$, the unconstrained AS rule is not a feasible rule and, thus, cannot be optimal. The following result completely characterizes the optimal AS rule.

Theorem *1:* The optimal selection rule ϕ^*, where $s^* = \phi^*(\mathbf{h}, \mathbf{g})$, that minimizes the SEP under the average interference constraint is given as follows:

$$s^* = \begin{cases} \arg\max_{i \in \{1,2\}} \{h_i\}, & \text{if } I_{\text{un}} \leq I_{\text{ave}} \\ \arg\min_{i \in \{0,1,2\}} \{\text{SEP}(h_i) + \lambda g_i\}, & \text{if} I_{\text{un}} > I_{\text{ave}} \end{cases}. \quad (4)$$

where $\lambda > 0$ is chosen to satisfy the average interference constraint with equality: $E_t \mathbf{E}_{\mathbf{h},\mathbf{g}}[g_{s^*}] = I_{\text{ave}}$.

Proof: The proof is given in Appendix A. ■

The SEP as a function of h_s for MPSK is given by [18, (40)]

$$\text{SEP}(h_s) = \frac{1}{\pi} \int_0^{m\pi} \exp\left(\frac{-h_s E_t}{\alpha \sigma^2 \sin^2 \theta} \right) d\theta,$$

where $\alpha \triangleq \csc^2(\frac{\pi}{M})$. Thus, the SEP-optimal AS rule is a nonlinear function of h_s. This is unlike the MI, MSLIR, and the DS rules.

Since the optimal rule requires evaluation of an integral, we use the Chernoff upper bound of the SEP, i.e., $\text{SEP}(h_s) \leq m \exp\left(\frac{-h_s E_t}{\alpha \sigma^2} \right)$, in the selection rule in (4) in order to get an explicit and tractable characterization of the rule in terms of the channel power gains. Let

$$y_i \triangleq m \exp\left(\frac{-h_i E_t}{\alpha \sigma^2} \right), \quad i \in \{0, 1, 2\}. \quad (5)$$

In this case, for $I_{\text{un}} > I_{\text{ave}}$, (4) gets modified to

$$s^* = \arg \min_{i \in \{0,1,2\}} \{y_i + \lambda g_i\}. \quad (6)$$

As mentioned, we shall refer to this as the *closed-form optimal* rule for brevity. Note that optimal rule depends on λ which has to be computed numerically. This is typical for optimization problems that handle average power constraints, e.g., rate adaptation and water filling in space, time, or frequency [19]. We also see that $\lambda = 0$ makes it equivalent to unconstrained rule; its SEP is given in [9, (36)].

B. Performance Analysis of Closed-form Optimal AS Rule

We now analyze the SEP of the rule in (6) for $\lambda > 0$.

Theorem *2:* The average SEP of the SU of the closed-form optimal rule is given by

$$\begin{aligned} \text{SEP} = & \, m \left(\frac{\alpha (\lambda \mu_g)^{\frac{\alpha}{\Omega}} e^{\frac{-m}{\lambda \mu_g}}}{\Omega m^{\frac{\alpha}{\Omega}}} \widetilde{\gamma}\left(\frac{\alpha}{\Omega}, \frac{m}{\lambda \mu_g} \right) \right)^2 + \left(\frac{\alpha}{\Omega m^{\frac{\alpha}{\Omega}}} \right)^2 \\ & \times \frac{(\lambda \mu_g)^{\frac{\alpha}{\Omega}}}{\pi} \int_0^{m\pi} \int_0^m \left[e^{\frac{-y_1}{\lambda \mu_g}} \widetilde{\gamma}\left(\frac{\alpha}{\Omega}, \frac{y_1}{\lambda \mu_g} \right) - e^{\frac{y_1 - 2m}{\lambda \mu_g}} \widetilde{\gamma}\left(\frac{\alpha}{\Omega}, \frac{m}{\lambda \mu_g} \right) \right. \\ & \left. + e^{\frac{y_1}{\lambda \mu_g}} \left(\gamma\left(\frac{\alpha}{\Omega}, \frac{y_1}{\lambda \mu_g} \right) - \gamma\left(\frac{\alpha}{\Omega}, \frac{m}{\lambda \mu_g} \right) \right) \right] \left(\frac{y_1}{m} \right)^{\csc^2(\theta)} y_1^{\frac{\alpha}{\Omega} - 1} dy_1 d\theta \\ & + \frac{1}{\pi} \int_0^{m\pi} \frac{2\alpha^2 \sin^4(\theta)}{\left(\Omega + \alpha \sin^2(\theta) \right) \left(\Omega + 2\alpha \sin^2(\theta) \right)} d\theta, \end{aligned} \quad (7)$$

where $\Omega \triangleq \frac{E_t \mu_h}{\sigma^2}$ and $\widetilde{\gamma}(s, x) \triangleq \int_0^x t^{s-1} e^t \, dt$.

Proof: The proof is given in Appendix B. ■

Note that $\widetilde{\gamma}(.,.)$ is a modified version of the lower incomplete gamma function [20, (6.5.2)] and can be evaluated using standard routines available for the latter.

The expression in (7) is in the form of a double integral. The Chernoff bound for the SEP, which is denoted by SEP_{UB}, is obtained by using the inequality $\sin^2 \theta \leq 1$. It is in the form of a single integral, and is not shown here to conserve

[2] In the time division duplex (TDD) mode of operation, information about $\mathbf{h}$ and $\mathbf{g}$ can be obtained by the STx by exploiting reciprocity. Since phase information is not required, simple signal strength-based estimation techniques can be used. We note that these results also serve as bounds on the performance of AS in average interference-constrained CR systems that have partial or imperfect knowledge of $\mathbf{g}$.

[3] In practice, this can be achieved by embedding a pilot once in every coherence interval along with the data symbols since the channel does not change within a coherence interval.

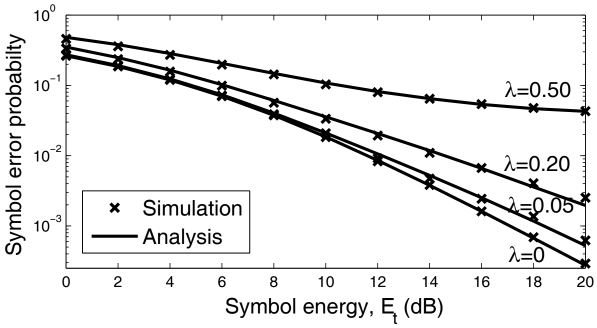

Fig. 2. SEP as a function of E_t for different values of λ, each of which corresponds to a different value of $\frac{I_{\text{ave}}}{E_t}$ (QPSK) .

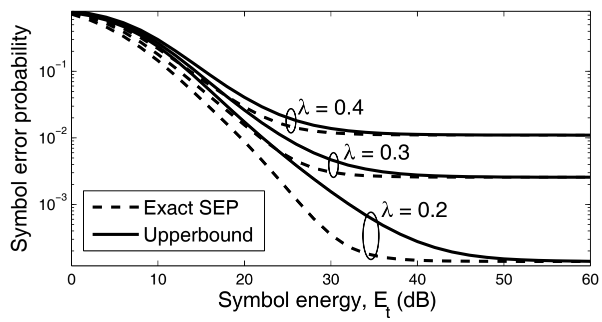

Fig. 3. SEP as a function of large E_t for different λ (8PSK).

space. Using Gauss-Legendre quadrature [20], SEP_{UB} can be evaluated accurately as the following sum of a few terms:

$$\begin{aligned}\text{SEP}_{UB} \approx\, & m\left(\frac{\alpha e^{\frac{-m}{\lambda\mu_g}}}{\Omega m^{\frac{\alpha}{\Omega}}}\left(\lambda\mu_g\right)^{\frac{\alpha}{\Omega}}\widetilde{\gamma}\left(\frac{\alpha}{\Omega},\frac{m}{\lambda\mu_g}\right)\right)^2 + \frac{m}{2}\left(\frac{\alpha}{\Omega m^{\frac{\alpha}{\Omega}}}\right)^2 \\ & \times\left[\sum_{k=1}^{N} w_k z_k^{\frac{\alpha}{\Omega}}\left(\lambda\mu_g\right)^{\frac{\alpha}{\Omega}}\left(e^{\frac{z_k}{\lambda\mu_g}}\gamma\left(\frac{\alpha}{\Omega},\frac{z_k}{\lambda\mu_g}\right)+e^{\frac{-z_k}{\lambda\mu_g}}\widetilde{\gamma}\left(\frac{\alpha}{\Omega},\frac{z_k}{\lambda\mu_g}\right)\right)\right] \\ & -\left(\frac{\alpha}{\Omega m^{\frac{\alpha}{\Omega}}}\right)^2\left(\lambda\mu_g\right)^{1+\frac{2\alpha}{\Omega}}\widetilde{\gamma}\left(1+\frac{\alpha}{\Omega},\frac{m}{\lambda\mu_g}\right)\left[\gamma\left(\frac{\alpha}{\Omega},\frac{m}{\lambda\mu_g}\right)\right. \\ & \left.+\, e^{\frac{-2m}{\lambda\mu_g}}\widetilde{\gamma}\left(\frac{\alpha}{\Omega},\frac{m}{\lambda\mu_g}\right)\right]+\frac{2m\alpha^2}{\left(\Omega+\alpha\right)\left(\Omega+2\alpha\right)}.\end{aligned}$$

Here, $Z_k \triangleq \frac{m}{2}(x_k+1)$ and x_k and w_k are the N Gauss-Legendre abscissas and weights, respectively. As N increases, the approximation becomes tighter. We have found that $N=3$ terms are sufficient for $\lambda \geq 0.20$, $N=5$ terms are sufficient for $0.05 < \lambda < 0.20$. For $\lambda \leq 0.05$ more terms are required.

IV. Performance Evaluation and Benchmarking

We now present Monte Carlo simulations that use 10^6 samples to verify our analytical results and quantitatively understand the behavior of the optimal selection rule under different conditions. The mean channel powers and noise variance are set as unity: $\mu_h = \mu_g = \sigma^2 = 1$.

Figure 2 plots the SEP as a function of E_t for a fixed value of λ on each curve. From the average interference constraint in (3), a fixed λ implies that the ratio $\frac{I_{\text{ave}}}{E_t}$ is kept constant. The $\lambda = 0$ curve corresponds to the unconstrained AS rule. As λ increases, the SEP increases due to a tighter average interference constraint. Notice that the analysis and simulation results match each other very well. Given the good match between the two, we no longer distinguish between them in the results that follow below.

Figure 3 plots the SEP and its upper bound for large E_t for different λ. At large E_t, when the STx transmits, the SEP is close to zero, but the SEP due to the zero transmit power option is always m. Thus, at higher E_t, the SEP saturates to $me^{\frac{-2m}{\lambda\mu_g}}$. Thus, an error floor occurs due to the average interference constraint. As expected, the SEP increases as λ increases. Notice also that the gap between the exact SEP and its upper bound disappears at larger E_t.

A. Benchmark Selection Rules

We now state the MI, MSLIR, and DS rules, which have been proposed in the literature. We shall enhance the MI and MSLIR rules in order to make them feasible and serve as useful performance benchmarks for all average interference thresholds.

1) MI Rule: The MI rule always selects the transmit antenna with the smallest channel power gain to the PRx [10]. It selects the antenna $s_{\text{mi}} = \arg\min_{i\in\{1,2\}}\{g_i\}$. If the average interference I_{mi} caused to the PRx by the MI rule is greater than I_{ave}, then the MI rule is infeasible.

To overcome this we introduce the zero transmit power option; the STx transmits with zero power in case $g_i \geq \tau$, i.e., when the power gains of all its channels that interfere with the PRx are large. Thus, the enhanced MI (EMI) rule selects the option s_{emi} as follows:

$$s_{\text{emi}} = \begin{cases} 0, & \text{if } g_1 \geq \tau, g_2 \geq \tau \\ \arg\min_{i\in\{1,2\}}\{g_i\}, & \text{otherwise} \end{cases}. \quad (8)$$

Note that when $I_{\text{mi}} < I_{\text{ave}}$, $\tau = \infty$. Thus, the EMI and MI rules are equivalent whenever the latter is feasible. However, τ is finite when $I_{\text{mi}} > I_{\text{ave}}$, in which case the MI rule is infeasible but the EMI is still feasible.

2) MSLIR Rule: The MSLIR rule selects the antenna with the highest ratio of the STx-SRx and the STx-PRx channel power gains [10]. It, therefore, selects the option $s_{\text{mslir}} = \arg\max_{i\in\{1,2\}}\left\{\frac{h_i}{g_i}\right\}$. As before, we introduce the zero transmit power option in order to make the MSLIR rule feasible for all I_{ave}. The enhanced MSLIR (EMSLIR) rule is as follows:

$$s_{\text{emslir}} = \begin{cases} 0, & \text{if } \frac{h_1}{g_1} \leq \eta, \frac{h_2}{g_2} \leq \eta \\ \arg\max_{i\in\{1,2\}}\{\frac{h_i}{g_i}\}, & \text{otherwise} \end{cases}.$$

3) Difference AS Rule: The difference selection (DS) rule is given by [11]

$$s_{\text{ds}} = \arg\max_{\{1,2\}}\{\delta h_i - (1-\delta)g_i\}, \quad (9)$$

where $\delta \in [0,1]$. Notice that the DS rule behaves as the unconstrained AS rule when $\delta = 1$ and as the MI rule when $\delta = 0$.

The values of τ, η and δ in the above rules are chosen such that interference constraint (3) is met with equality.

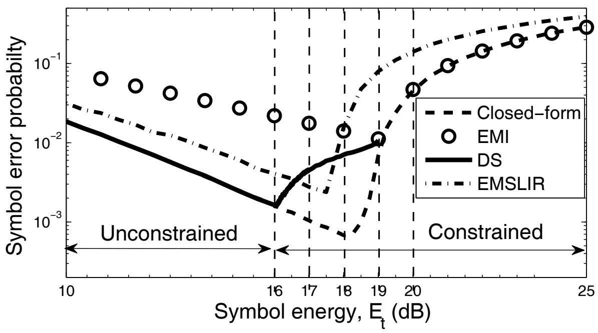

Fig. 4. Comparison of the SEPs of the closed-form optimal AS rule and several benchmark rules ($I_{\text{ave}} = 16$ dB and QPSK).

Comparison with other rules and optimization of E_t: Figure 4 plots the SEP of the closed-form optimal rule and the benchmark rules as a function of E_t, for $I_{\text{ave}} = 16$ dB. We see that there are three regions of operation for the closed-form optimal rule: (i) $E_t \le 16$ dB: In this case, the interference constraint is not active. Hence, $\lambda = 0$. In this regime, the closed-form optimal rule, the DS rule, and the unconstrained rule are equivalent. However, the EMI and EMSLIR rules are sub-optimal. (ii) 16 dB $< E_t \le 18$ dB: In this case λ is non-zero, but small. The SEP of the closed-form optimal rule decreases as E_t increases. In this regime, the DS rule performs worse than the closed-form optimal rule. (iii) $E_t > 18$ dB: The SEP of the closed-form optimal rule increases as E_t increases. This is because the probability that the SU does not transmit increases as E_t increases. The SEPs of the closed-form optimal, EMI, and DS rules match at $E_t = 19$ dB. Beyond this the DS rule becomes infeasible and the closed-form optimal rule is equivalent to the EMI rule. Further, the EMSLIR rule is again sub-optimal. *Thus, $E_t = 18$ dB is the optimal transmit symbol energy for the closed-form optimal rule when $I_{ave} = 16$ dB.* At the optimal E_t, the SEP of the closed-form optimal rule is lower by a factor of 16.5, 3.5, and 2.4 than the minimum SEPs of the EMI, EMSLIR, and DS rules.

V. Conclusions

We developed the optimal AS rule that minimizes the SEP of a secondary system that operates in the underlay CR mode under an average interference constraint. The STx can transmit at a fixed power or with zero power. For the closed-form variant of the optimal AS rule, we derived expressions for the SEP and its upper bound. The optimal rule turns out to be functionally different from the many ad hoc rules that have been proposed in the literature, and is inherently non-linear in nature. A comparison of the closed-form optimal rule with enhanced versions of the ad hoc rules showed that it behaves as the unconstrained rule for small values of E_t and as the EMI rule for very large values of E_t. We saw that for a fixed $\frac{I_{\text{ave}}}{E_t}$ an error floor arises. While this paper focuses on on-off power control, in which the STx transmits with a fixed power or with zero power, a natural generalization is to allow continuous power control along with AS at the STx.

Appendix

A. Proof of Theorem 1

When $I_{\text{un}} \le I_{\text{ave}}$, the unconstrained rule is feasible. Therefore, it must be the SEP-optimal rule. Now, consider the case when $I_{\text{un}} > I_{\text{ave}}$. A selection rule that always chooses the zero transmit power option causes zero interference to the PRx. It is, therefore, feasible for any I_{ave}. Therefore, the set of all feasible selection rules $\mathcal{Z}$ is a non-empty set.

Let $\phi \in \mathcal{Z}$ be a feasible rule. For a given $\lambda > 0$, let

$$L_\phi(\lambda) \triangleq \mathbf{E}_{\mathbf{h},\mathbf{g}}\left[\text{SEP}(h_s) + \lambda g_s\right], \tag{10}$$

where $s = \phi(\mathbf{h}, \mathbf{g})$. From the definition of ϕ^* in (4), it follows that $L_{\phi^*}(\lambda) \le L_\phi(\lambda)$. Therefore,

$$\mathbf{E}_{\mathbf{h},\mathbf{g}}\left[\text{SEP}(h_{s^*})\right] + \lambda \mathbf{E}_{\mathbf{h},\mathbf{g}}\left[g_{s^*}\right] \le \mathbf{E}_{\mathbf{h},\mathbf{g}}\left[\text{SEP}(h_s)\right] + \lambda \mathbf{E}_{\mathbf{h},\mathbf{g}}\left[g_s\right],$$

where $s^* = \phi^*(\mathbf{h}, \mathbf{g})$. Choose λ such that $\mathbf{E}_{\mathbf{h},\mathbf{g}}\left[g_{s^*}\right] = \frac{I_{\text{ave}}}{E_t}$.[4] Thus, ϕ^* is also feasible.

By rearranging the terms in the above inequality we get

$$\mathbf{E}_{\mathbf{h},\mathbf{g}}\left[\text{SEP}(h_{s^*})\right] \le \mathbf{E}_{\mathbf{h},\mathbf{g}}\left[\text{SEP}(h_s)\right] + \lambda\left(\mathbf{E}_{\mathbf{h},\mathbf{g}}\left[g_s\right] - \frac{I_{\text{ave}}}{E_t}\right).$$

However, since ϕ is feasible, we know that $\mathbf{E}_{\mathbf{h},\mathbf{g}}\left[g_s\right] \le \frac{I_{\text{ave}}}{E_t}$. Hence, for any feasible rule ϕ, $\mathbf{E}_{\mathbf{h},\mathbf{g}}\left[\text{SEP}(h_{s^*})\right] \le \mathbf{E}_{\mathbf{h},\mathbf{g}}\left[\text{SEP}(h_s)\right]$. Thus, ϕ^* is the SEP-optimal selection rule.

B. Proof of Theorem 2

The probability of error conditioned on $\mathbf{h}$ and $\mathbf{g}$ can be written in terms of all the possible selection options as follows:

$$\Pr\left(\text{Err}|\mathbf{h},\mathbf{g}\right) = \sum_{i=0}^{2} \Pr\left(s = i|\mathbf{h},\mathbf{g}\right) \Pr\left(\text{Err}|\mathbf{h},\mathbf{g}, s = i\right). \tag{11}$$

When option s is selected, the conditional error probability depends only on h_s. Further, for the zero transmit power option, $\Pr\left(\text{Err}|h_0\right) = m$. Averaging over $\mathbf{h}$ and $\mathbf{g}$, we get

$$\begin{aligned}\text{SEP} &= m\mathbf{E}_{\mathbf{h},\mathbf{g}}\left[\Pr\left(s = 0|\mathbf{h},\mathbf{g}\right)\right] \\ &\quad + 2\mathbf{E}_{\mathbf{h},\mathbf{g}}\left[\Pr\left(s = 1|\mathbf{h},\mathbf{g}\right) \frac{1}{\pi}\int_0^{m\pi} \exp\left(\frac{-h_1 E_t}{\alpha\sigma^2 \sin^2\theta}\right) d\theta\right],\end{aligned}$$

where the factor 2 arises because the channel gains of the two transmit antennas are i.i.d. From the definition of y_i in (5), the above expression for the SEP can be recast as

$$\begin{aligned}\text{SEP} &= m\mathbf{E}_{\mathbf{h},\mathbf{g}}\left[\Pr\left(s = 0|\mathbf{h},\mathbf{g}\right)\right] \\ &\quad + \frac{2}{\pi}\int_0^{m\pi} \mathbf{E}_{\mathbf{y},\mathbf{g}}\left[\Pr\left(s = 1|\mathbf{y},\mathbf{g}\right)\left(\frac{y_1}{m}\right)^{\csc^2(\theta)}\right] d\theta,\end{aligned} \tag{12}$$

[4]Such a unique choice of λ is possible since $0 \le I_{\text{ave}} < I_{\text{un}}$ and the average interference decreases monotonically as λ increases.

where $\mathbf{y} = [y_1,\ y_2]$. Using the fundamental theorem of expectation, we get $\mathbf{E}_{\mathbf{h},\mathbf{g}}\left[\Pr\left(s=0|\mathbf{h},\mathbf{g}\right)\right] = \Pr\left(s=0\right)$ and

$$\mathbf{E}_{\mathbf{y},\mathbf{g}}\left[\Pr\left(s=1|\mathbf{y}\right)\left(\frac{y_1}{m}\right)^{\csc^2(\theta)}\right] = \mathbf{E}_{\mathbf{y}}\left[\Pr\left(s=1|\mathbf{y}\right)\left(\frac{y_1}{m}\right)^{\csc^2(\theta)}\right].$$

We evaluate the above two terms separately below.

Using the AS rule in (6), the first term of (12), denoted by T_1, can be written as

$$\begin{aligned} T_1 = m\Pr\left(s=0\right) &= m\Pr\left(y_1+\lambda g_1 > m, y_2+\lambda g_2 > m\right), \\ &= m\left(\Pr\left(y_1+\lambda g_1 > m\right)\right)^2. \end{aligned} \tag{13}$$

Here, (13) follows because the channel power gains of antennas 1 and 2 are i.i.d. Further, the PDF of y_1 can be shown to be $f_{y_1}(y_1) = \frac{\alpha y_1^{\frac{\alpha}{\Omega}-1}}{\Omega m^{\frac{\alpha}{\Omega}}}$, for $y_1 \in (0, m]$. Hence, we get

$$\begin{aligned} \Pr\left(y_1+\lambda g_1 > m\right) &= \int_0^m \int_{\frac{m-y_1}{\lambda}}^{\infty} \frac{e^{-\frac{g_1}{\mu_g}}}{\mu_g} \frac{\alpha y_1^{\frac{\alpha}{\Omega}-1}}{\Omega m^{\frac{\alpha}{\Omega}}} dg_1\, dy_1, \\ &= \int_0^m e^{\left(\frac{y_1-m}{\lambda\mu_g}\right)} \frac{\alpha y_1^{\frac{\alpha}{\Omega}-1}}{\Omega m^{\frac{\alpha}{\Omega}}}\, dy_1, \\ &= \frac{\alpha e^{\frac{-m}{\lambda\mu_g}}}{\Omega m^{\frac{\alpha}{\Omega}}} \left(\lambda\mu_g\right)^{\frac{\alpha}{\Omega}} \widetilde{\gamma}\left(\frac{\alpha}{\Omega}, \frac{m}{\lambda\mu_g}\right). \end{aligned} \tag{14}$$

Here, the last equality follows from the definition of $\widetilde{\gamma}(.,.)$ in the theorem statement. Substituting $\Pr\left(y_1+\lambda g_1 > m\right)$ in (13) yields

$$T_1 = m\left(\frac{\alpha e^{\frac{-m}{\lambda\mu_g}}}{\Omega m^{\frac{\alpha}{\Omega}}} \left(\lambda\mu_g\right)^{\frac{\alpha}{\Omega}} \widetilde{\gamma}\left(\frac{\alpha}{\Omega}, \frac{m}{\lambda\mu_g}\right)\right)^2. \tag{15}$$

Let the integrand in the second term of (12) be denoted by T_2. It can be written as

$$\begin{aligned} T_2 = \mathbf{E}_{\mathbf{y}}\left[\Pr\left(s=1|\mathbf{y}\right)\left(\frac{y_1}{m}\right)^{\csc^2(\theta)}\right] = \left(\frac{\alpha}{\Omega m^{\frac{\alpha}{\Omega}}}\right)^2 \\ \times \int_0^m \int_0^m \left(\frac{y_1}{m}\right)^{\csc^2(\theta)} \Pr\left(s=1|\mathbf{y}\right)\left(y_1 y_2\right)^{\frac{\alpha}{\Omega}-1} dy_2\, dy_1. \end{aligned} \tag{16}$$

For the closed-form optimal antenna selection rule in (6), $\Pr\left(s=1|\mathbf{y}\right) = \Pr\left(y_1+\lambda g_1 < y_2+\lambda g_2, y_1+\lambda g_1 < m|\mathbf{y}\right)$. By rearranging terms and summing over the mutually exclusive events $y_2 < y_1$ and $y_2 > y_1$, we get

$$\begin{aligned} \Pr(s=1|\mathbf{y}) = \Pr\left(y_2 < y_1, g_1 - g_2 < \frac{y_2-y_1}{\lambda}, g_1 < \frac{m-y_1}{\lambda}|\mathbf{y}\right) \\ + \Pr\left(y_2 > y_1, g_1 - g_2 < \frac{y_2-y_1}{\lambda}, g_1 < \frac{m-y_1}{\lambda}|\mathbf{y}\right). \end{aligned}$$

Since g_1 and g_2 are i.i.d. exponential RVs, we can show that

$$\begin{aligned} \Pr\left(s=1|\mathbf{y}\right) = \frac{1}{2}\left(e^{\frac{y_2-y_1}{\lambda\mu_g}} - e^{\frac{y_1+y_2-2m}{\lambda\mu_g}}\right) I_{\{y_2<y_1\}} \\ + \frac{1}{2}\left(2 - e^{\frac{y_1-y_2}{\lambda\mu_g}} - e^{\frac{y_1+y_2-2m}{\lambda\mu_g}}\right) I_{\{y_2>y_1\}}. \end{aligned} \tag{17}$$

Recall that $I_{\{.\}}$ denotes the indicator function. Substituting (17) in (16) and updating the integration limits of y_2 yields

$$\begin{aligned} T_2 = \frac{1}{2}\left(\frac{\alpha}{\Omega m^{\frac{\alpha}{\Omega}}}\right)^2 \int_0^m \left[\int_0^{y_1}\left(e^{\frac{y_2-y_1}{\lambda\mu_g}} - e^{\frac{y_1+y_2-2m}{\lambda\mu_g}}\right) y_2^{\frac{\alpha}{\Omega}-1} dy_2 \right. \\ \left. + \int_{y_1}^m \left(2 - e^{\frac{y_1-y_2}{\lambda\mu_g}} - e^{\frac{y_1+y_2-2m}{\lambda\mu_g}}\right) y_2^{\frac{\alpha}{\Omega}-1} dy_2\right] \left(\frac{y_1}{m}\right)^{\csc^2(\theta)} y_1^{\frac{\alpha}{\Omega}-1} dy_1. \end{aligned}$$

Substituting T_1 and T_2 in (12) and writing the integral over y_2 in terms of incomplete gamma functions yields the desired result in (7).

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