Optimal, Distributed, Timer-Based Best Two Relay Discovery Scheme for Cooperative Systems

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***Abstract*—Multiple relay selection enables a cooperative system to obtain better performance than single relay selection and yet avoid challenging problems such as synchronization that are associated with having all the relays transmit. While its benefits have been well characterized, the problem of developing distributed, scalable schemes that discover the best subset of relays remains to be fully investigated. The problem is challenging because the relays are spatially separated from each other and have only local channel knowledge. We investigate the popular, low feedback, and distributed timer scheme and derive a novel, optimal timer mapping that maximizes the probability of selecting the best two relays. This has applications in several cooperative schemes proposed in the literature. We derive several novel structural properties about the optimal mapping, which reduce the complexity of finding it from the large space of all functions to a one-dimensional search that can be solved using a computationally efficient, iterative algorithm. Our extensive benchmarking shows that the optimal mapping outperforms several relay discovery schemes proposed in the literature. The approach can be generalized to selecting the best l relays, as well.**

I. Introduction

In cooperative communications, opportunistic relay selection is a popular solution for harnessing spatial diversity [1]. In it, one or a few relays are selected from the set of available relays based on their current channel conditions to forward the data from a source to a destination. Having fewer relays transmit ameliorates the problem of ensuring tight synchronization among them and makes relay selection practically appealing. On account of its simplicity, relay selection can be applied in a variety of cooperative communication protocols.

Broadly, two forms of relay selection have been investigated in the literature, namely, single relay selection and multiple relay selection. In single relay selection, only one relay, which we shall refer to as the best relay, is selected [2], [3]. In order to improve performance, single relay selection has been generalized to multiple relay selection, in which the best two relays, and, in general, the best l relays, are selected for forwarding data to the destination.

Examples of Multiple Relay Selection: In [4], selecting the best two relays was shown to improve the diversity-multiplexing trade-off of an amplify-and-forward (AF) relaying protocol. In [5] and [6], multi-relay selection was used to minimize the data transmission time and maximize rate, respectively. In [7], it was shown that the quality of 3-D video transmission over a cooperative relay network can be improved by using the best two relays and unequal error protection. Multiple relay selection for AF relaying has also been considered in [8], in cooperative beamforming [9], and in multiple sensor selection to maximize the energy efficiency of wireless sensor networks [10]. It also arises in distributed space-time coding, in which each selected relay forwards a column of the space-time codeword matrix [11].

In the above works, the focus has mostly been on characterizing the benefits of selection and on developing selection criteria. Given the channel realization, the best relays are assumed to have been identified perfectly and instantaneously. However, no distributed mechanism for identifying them is specified. This is a challenging and time-consuming task in practice because the relays are geographically separated from one another and each relay has only local channel knowledge. Thus, no relay in the cooperative system knows a priori who the l best relays are. Their identity needs to be discovered.

Given the local channel knowledge, relay discovery proceeds as follows. *A relay i generates a real-valued preference metric μ_i, which quantifies how useful it will be if selected.* The relays with the l largest metrics are then selected by a relay discovery algorithm. The metric depends on the cooperative communication protocol. For example, for an AF relay, its metric is a harmonic mean of its source-relay and relay-destination channel gains [2], as this is its contribution to the signal-to-noise-ratio at the destination.

Relay Discovery Schemes: One plausible relay discovery algorithm, which is called polling, proceeds as follows. The relays transmit their metrics one by one to a central controller, which then selects the l best relays. However, polling is not a scalable solution because the time required by it to discover the best relay(s) increases linearly with the total number of relays in the system. This is not desirable because the more the time spent on selection, the less the fraction of time available for data transmission using the selected relays and the lower the net spectral efficiency of the system. It also makes the system more sensitive to time-variations in the channel gains. Therefore, a distributed relay discovery scheme is desirable.

The timer scheme is a popular and elegant example of such a scheme, and has been extensively used for discovering the best relay. In it, each relay maps its metric to a timer using a deterministic metric-to-timer mapping and transmits a packet, which contains its identity, when its timer expires. The main idea is that the metric-to-timer mapping is a monotone non-increasing (MNI) function. This ensures that the timer of the best relay expires earlier than those of the other relays, and can, thus, be identified by the sink. However, the timer scheme

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can fail to select the best relay when another relay's timer expires within a vulnerability duration Δ of the expiry of the timer of the best relay [1], [12], [13]. In this case, the best relay's timer packet cannot be decoded by the sink. Several metric-to-timer mappings have been proposed in the literature for discovering the single best relay, and do so with varying degrees of effectiveness [1], [13]–[16].

However, the design of the timer mapping for selecting the best l relays, where $l \geq 2$, has not received as much attention in the literature, and is the focus of this paper. In [2], [17], the l best relays are selected by running the timer scheme, which uses the inverse metric mapping, l times. In the j^{th} run, the j^{th} best relay gets selected. In [18], a feedback-intensive approach based on the splitting algorithm is instead pursued.

Contributions: In this paper we present an optimal timer mapping that maximizes the probability of discovering the best two relays. To this end, we first derive several novel insights into the special discrete structure of the optimal mapping. These show that finding it in the large space of all MNI functions requires just a one-dimensional search. These then lead to a computationally efficient, iterative algorithm to find the optimal mapping.

We show that the optimal mapping markedly outperforms several ad hoc timer mappings presented in the literature. It outperforms repeated selection, and serves as a fundamental benchmark for evaluating multi-relay discovery schemes. The proposed scheme can also be adapted to accommodate fairness and quality of service constraints. This is done by redefining the metric, along the lines of [2], [19].

We note that our approach can be generalized to solve the problem of discovering the $l > 2$ best relays [20]. Furthermore, the best two relay discovery problem is interesting in its own right because it has not been investigated in the literature. It is applicable when the Alamouti code is used as the space-time code [11], as a special case of the multi-relay selection problem in [5], [8]–[10], [21], and in [4], [7]. As we shall see, determining the optimal timer mapping for discovering the best two relays is much more involved than determining the optimal mapping for discovering just the single best relay, which was solved in [13]. This is a key challenge that this paper tackles.

Outline: The paper is organized as follows. Section II describes the system model. The optimal timer scheme and its performance are studied in Sec. III. Simulation results and our conclusions follow in Sec. IV and Sec. V, respectively.

II. System Model and Timer Scheme

Consider a cooperative system with K relays out of which the best two are to be discovered, as illustrated in Figure 1. A relay i locally computes a real-valued metric μ_i. Without loss of generality μ_i is assumed to be uniformly distributed over $[0, 1)$.[1] Relay i sets its timer T_i as a function of its metric μ_i as $T_i = f(\mu_i)$. As mentioned, f is an MNI function. The relay transmits a small timer packet to the sink if its timer expires before $T_{\max}$, which is the maximum time allocated for selection. The timer packet contains the relay's identity to enable the sink to identify it. When arranged in descending order, the metrics can be written as

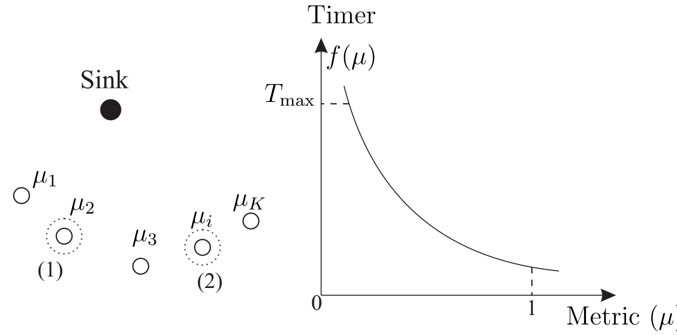

Fig. 1. A system consisting of K relays in which the best two relays need to be selected. Each relay i sets its timer depending on its metric μ_i, and transmits a small timer packet when its timer expires.

$$\mu_{(1)} \geq \mu_{(2)} \geq \cdots \geq \mu_{(N)}, \tag{1}$$

where, as per standard order statistics notation, (i) denotes the index of the node with the i^{th} largest metric.

If two timers expire within a duration less than Δ of each other, then their timer packets collide and cannot be decoded by the sink. The physical layer capabilities of the wireless system determine Δ. It includes the maximum propagation delay, the maximum delay spread in the channels seen by the relays, and time synchronization errors, if any, among relays. It also accounts for the duration of the timer packet.[2] The reader is referred to [1], [12] for an in-depth discussion of Δ.

We, thus, see that the sink can successfully discover the best two relays if and only if the timers of the best two relays: (i) expire before $T_{\max}$, and (ii) do not collide with each other and with that of any other relay. Otherwise, we say that the timer scheme has failed to discover the best two relays. Our goal is to maximize the success probability, i.e., the probability that the sink discovers the best two relays.

Alternative Models: We note that the success criterion above is stringent in that it requires that the best two relays must both be selected. This requirement can be relaxed in some systems. For example, the sink can wait for the first two timer packets that it can reliably decode – even if collisions occur in between – and select the relays responsible for them. However, in this case, the problem formulation needs to capture the system-specific performance penalty associated with selecting sub-optimal relays. Another possibility is that the sink can use feedback to resolve the collisions [12]. However, incorporating collision resolution is beyond the scope of the paper.

[1] If μ_i follows any other distribution, then one can transform it to a uniform random variable by using its cumulative distribution function [12], [13].

[2] Carrier sensing can also be incorporated in this framework by not accounting for the timer packet duration in Δ. Carrier sensing, thus, makes Δ smaller because the relays avoid transmitting and colliding as soon as they sense that the channel is busy. In this case, each relay freezes its timer once it senses the channel to be busy and continues to decrement it once it senses channel to be idle. Now, $T_{\max}$ is the maximum time duration over which the timers can be decremented.

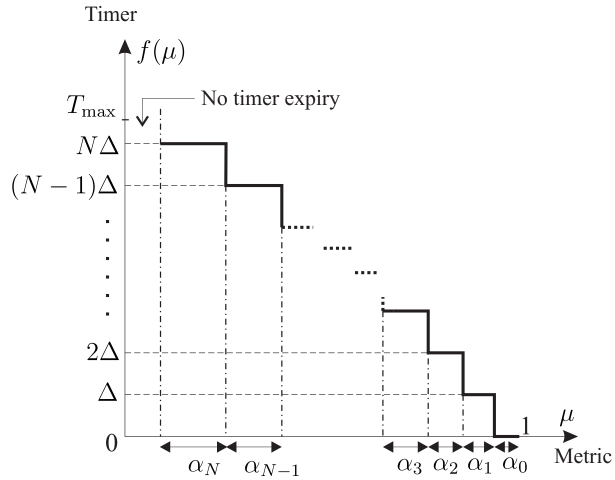

Fig. 2. Illustration of optimal metric-to-timer mapping.

III. Determining Optimal Timer Mapping

We shall use the following notation henceforth. The floor and ceil functions are denoted by $\lfloor\cdot\rfloor$ and $\lceil\cdot\rceil$, respectively. The optimal value of a parameter x is denoted by x^*.

A. Structural Insights

We now develop four useful and novel results about the structure of the optimal timer mapping to discover the best two relays. These then culminate in an iterative algorithm to compute it in Sec. III-B.

Result *1:* An optimal metric-to-timer mapping $f^*(\mu)$ that maximizes the probability of successfully discovering the best two relays within a maximum time $T_{\max}$ maps μ into $N+1$ discrete timer values $\{0, \Delta, 2\Delta, \ldots, N\Delta\}$, where $N = \left\lfloor \frac{T_{\max}}{\Delta} \right\rfloor$.

Proof: The proof is given in Appendix A. ■

Intuitively, the result is similar to the well-known result in the multiple access control literature that slotted Aloha is better than unslotted Aloha. Since the timer mapping is an MNI function, Result 1 implies that it is entirely characterized by $N+1$ interval lengths, which we denote by $\alpha_0, \ldots, \alpha_N$, such that the timer of a relay whose metric lies in $[1-\alpha_0, 1)$ expires immediately at time 0, the timer of a relay whose metric lies in $[1-\alpha_0-\alpha_1, 1-\alpha_0)$ expires at Δ, and so on. Furthermore, the timer of a relay whose metric is less than $1-\sum_{i=1}^{N}\alpha_i$ does not expire. Mathematically, the optimal mapping can be written as:

$$f^*(\mu) = \begin{cases} i\Delta, & 1-\sum_{j=0}^{i}\alpha_j \le \mu < 1-\sum_{j=0}^{i-1}\alpha_j, \\ T_{\max}^{+}, & \text{otherwise}, \end{cases} \tag{2}$$

where $T_{\max}^{+}$ indicates that the timer does not expire before $T_{\max}$. We shall refer to α_i as the i^{th} interval length. The intervals and the optimal mapping are illustrated in Fig. 2. This result generalizes the result in [13], which was only for discovering the single best relay.

Thus, we now only need to determine the $N+1$ interval lengths $\alpha_0, \ldots, \alpha_N$ to find the optimal mapping. Given the discrete structure, the probability of success $P_{K,N}$ of the optimal mapping takes the following simple form.

Result *2:* The probability of successfully discovering the best two relays is given by

$$P_{K,N} = K(K-1)\sum_{r=1}^{N}\alpha_r\left(1-\sum_{j=0}^{r}\alpha_j\right)^{K-2}\sum_{i=0}^{r-1}\alpha_i. \tag{3}$$

Proof: The proof is given in Appendix B. ■

Our objective is to maximize $P_{K,N}$ subject to two constraints: $\alpha_i \ge 0$, for $0 \le i \le N$, and $\sum_{j=0}^{N}\alpha_j \le 1$. Clearly, $N \ge 1$ since two relays have to be discovered. For $N=1$, the probability of success expression reduces to

$$K(K-1)\,\alpha_0\alpha_1\left(1-\alpha_0-\alpha_1\right)^{K-2}.$$

It can be easily shown that its optimum value is $\left(1-\frac{1}{K}\right)\left(1-\frac{2}{K}\right)^{K-2}$, and is attained when $\alpha_0 = \alpha_1 = \frac{1}{K}$. Hereon, we study the case $N \ge 2$.

The following result shows that optimal point lies in the interior of the constrained region.

Result *3:* The optimal value of α_i that maximizes $P_{K,N}$ satisfies $0 < \alpha_i^* < 1$, for $0 \le i \le N$, and $0 < \sum_{i=0}^{N}\alpha_i^* < 1$.

Proof: The proof is relegated to Appendix C. ■

Now we define a new set of variables $\{t_0, \ldots, t_N\}$, where

$$t_i = 1-\sum_{r=0}^{i}\alpha_r, \quad \text{for } 0 \le i \le N. \tag{4}$$

Using these new variables, the expression for $P_{K,N}$ becomes

$$P_{K,N} = K(K-1)\sum_{r=1}^{N}\left(t_{r-1}-t_r\right)\left(t_r\right)^{K-2}\left(1-t_{r-1}\right). \tag{5}$$

The next result shows that once t_N^* is known, then $t_0^*, \ldots, t_{N-1}^*$ are also known.

Result *4:* Let $q \triangleq K-2$. The optimal values of t_i satisfy the following recursion:

$$t_{i-1}^* = \begin{cases} \left(1+q^{-1}\right)t_i^*, & \text{for } i = N, \\ \frac{q+(1+q)t_i^*}{2q} - \frac{\sqrt{\left(q-(1+q)t_i^*\right)^2 + 4q\frac{\left(t_{i+1}^*\right)^q}{\left(t_i^*\right)^{q-1}}\left(1-2t_i^*+t_{i+1}^*\right)}}{2q}, & \text{for } 1 \le i \le N-1, \end{cases} \tag{6}$$

and

$$2t_0^* - t_1^* = 1, \tag{7}$$

Proof: The proof is given in Appendix D. ■

Thus, once t_N^* is known, $t_{N-1}^*, \ldots, t_0^*$ can be determined sequentially. The optimization problem now reduces to finding the optimal value of t_N, which is a one-dimensional search over the interval $(0,1)$. We now propose an iterative algorithm for this purpose.

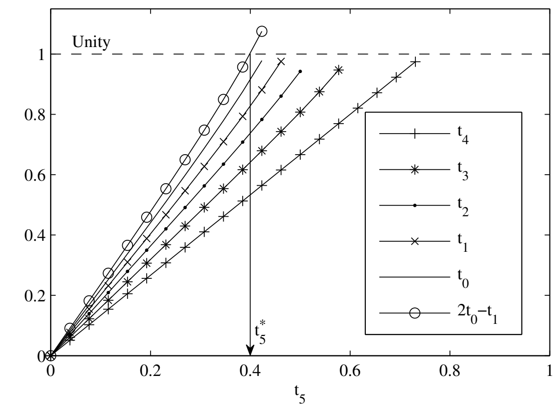

Fig. 3. Variation of t_i and $2t_1 - t_2$ with t_N for $K = 5$ and $N = 5$.

B. Iterative Algorithm

The algorithm proceeds as follows. It first chooses a value for $t_N \in (0, 1)$. Using (6), it obtains the value of t_{N-1} followed by t_{N-2} and so on up to t_0. In case t_i turns out to be negative, complex, or is greater than unity, then the algorithm chooses another value for t_N and restarts. Similarly, it chooses another value for t_N in case the constraint in (7) is not satisfied. For brevity, we shall say that t_i is feasible if it lies in $(0, 1)$. Else, we shall say that it is infeasible.

The only question that remains to be answered is whether t_N should be increased or decreased when the above constraints are not satisfied. To understand this, we plot the values of t_i and $2t_0 - t_1$ for $K = 5$ and $N = 5$ in Fig. 3. The figure leads to the following two useful empirical observations:

1) If $t_N > t_N^*$, then at least one among $t_{N-1}, \ldots, t_0$ is infeasible, or all are feasible but $2t_0 - t_1 > 1$.
2) If $t_N < t_N^*$, then $t_{N-1}, \ldots, t_0$ are all feasible but $2t_0 - t_1 < 1$.

We have found that these observations hold for all K and N.

These observations help us formulate a bisection algorithm to determine the optimal value of t_N. The pseudo-code for the algorithm is given in Figure 4. The value for t_N is chosen as the midpoint of the search interval $(0, 1)$. Thereafter, the values of $t_{N-1}, \ldots, t_0$ are computed sequentially. Once one of them becomes infeasible or $2t_0 - t_1 > 1$, then we know that $t_N > t_N^*$. In this case, the search interval is updated to $(t_N, 1)$. In all other cases, we know that $t_N < t_N^*$. Therefore, the search interval is updated to $(0, t_N)$. In the next step, we take the midpoint of the updated search interval as t_N and proceed in a similar manner.

Complexity of the Algorithm: If we consider the computation of a single value of t_i using (6) to have a complexity of $O(1)$, then the computational complexity for obtaining t_N^* with a precision of ψ can be shown to be at most $O\left(-N \lceil \log_2(\psi) \rceil\right)$ [22].

IV. Simulations

We now evaluate the performance of the optimal timer mapping and benchmark it with several timer schemes proposed in the literature. This includes the following: (i) Inverse metric timer mapping [1]: A relay i sets its timer as c/μ_i, where, in order to ensure as fair a comparison as possible, $c > 0$ is optimized using a brute-force numerical search for each value of K and N. (ii) Equal stair length discrete timer mapping [16]: The $N + 1$ interval lengths are set to be equal to $1/(N + 1)$. (iii) Affine mapping: A relay i sets its timer as $T_{\max}(1 - \mu_i)$, which is a special case of the class of mappings considered in [15]. Also plotted is the performance of repeated selection, in which optimal timer scheme for discovering the single best relay [13] is rerun twice. The first run uses $\left\lceil \frac{N-1}{2} \right\rceil$ timer levels and attempts to discover the best relay. After a time gap of Δ, the second run uses $\left\lfloor \frac{N-1}{2} \right\rfloor$ timer levels and attempts to discover the second best relay.

```
Data: K, N, ψ ;                          /*precision: ψ*/
begin
    l = 0, u = 1 ;      /*search interval: (l, u)*/
    repeat
        m = (l+u)/2
        t_N = m
        for i = N − 1 to 0 do
            Use (6) to compute t_i.
            if t_i is infeasible then
                u = m
                break
            end
        end
        if t_0 is feasible then
            if 2t_0 − t_1 > 1 then
                u = m
            end
            else
                l = m
            end
        end
    until u − l < ψ;
    Declare current values of t_0, ..., t_N as optimal.
end
```

Fig. 4. Iterative bisection algorithm to compute optimal best two relay timer mapping's parameters.

Figure 5 plots the probability of discovering the best two relays as a function of the number of timer levels N for $K = 20$ relays. We see that as N increases, the probability of success increases. Notice that the optimal timer scheme markedly outperforms the optimized inverse metric, affine, and equal stair length timer schemes. For example, at $N = 35$, the probability that the optimal timer scheme discovers the best two relays is 222%, 182%, and 57% better than those of the inverse metric, affine, and equal stair length timer schemes, respectively. It is also 6% better than repeated selection, which demonstrates the importance of accounting for the number of relays to be selected in the optimization problem formulation.

Figure 6 plots the probability of discovering the best two relays as a function of the number of relays K for $N = 20$. We see that the optimal timer scheme is scalable because its

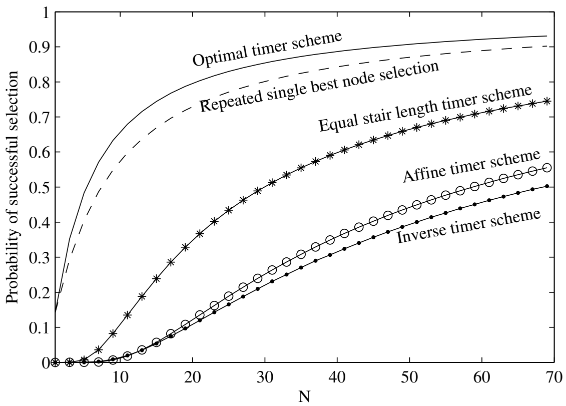

Fig. 5. Probability of selecting the best two relays out of $K = 20$ relays as a function of the number of timer levels N.

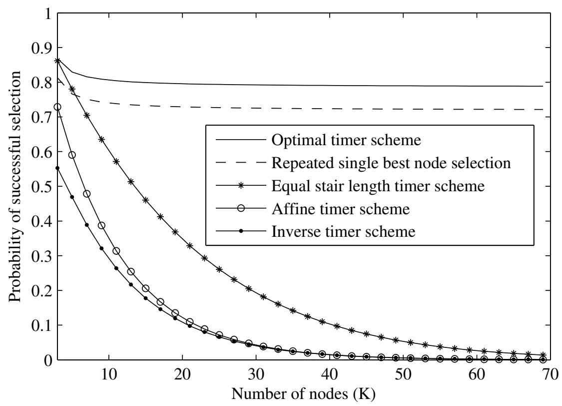

Fig. 6. Probability of selecting best two relays as a function of the number of relays in the system ($N = 20$).

success probability decreases only marginally from 0.81 to 0.79 when the number of relays increases seven-fold from 10 to 70. This is unlike the inverse metric scheme, whose probability of success decreases to 0 as K increases. As before, the optimal mapping again markedly outperforms all the ad hoc mappings. It also outperforms repeated selection, whose probability of success, for large K, stabilizes at 0.72, which is 9% lower than that of the optimal mapping.

V. Conclusions

A variety of cooperative communication systems need a distributed algorithm to discover the best two relays from the set of available relays because the information about the usefulness of a relay is local and no node knows a priori who the best relays are. For such systems, we derived an optimal timer mapping that maximizes the probability of discovering the best two relays. We proved that it has a special discrete structure in which the timers can expire only at integer multiples of the vulnerability window. This makes it easy to implement since a relay only needs to store a simple lookup table with $N + 1$ entries to map its metric to the timer value. We saw that finding the optimal mapping reduces to a one-dimensional search, for which an efficient bisection algorithm was developed. The optimal mapping markedly outperformed several timer mappings that have been proposed in the literature, and also outperformed repeated selection.

Appendix

A. Proof of Result 1

Let $f(\mu)$ be an optimal MNI metric-to-timer mapping. Consider the new mapping $g(\mu)$ that is defined in terms of $f(\mu)$ as follows:

$$g(\mu) = \begin{cases} \left\lfloor \frac{f(\mu)}{\Delta} \right\rfloor \Delta, & \text{if } f(\mu) \le T_{\max} \\ f(\mu), & \text{if } f(\mu) > T_{\max} \end{cases}. \quad (8)$$

Therefore, $g(\mu) = 0$, if $0 \le f(\mu) < \Delta$, $g(\mu) = \Delta$, if $\Delta \le f(\mu) < 2\Delta$, and so on. Since $f(\mu)$ is MNI, it is easy to verify that $g(\mu)$ is well-defined and MNI.

We prove below that in every realization of metrics for which f successfully discovers the best two relays, g also successfully discovers the same best two relays. Therefore, if f is optimal, then g, which has the desired discrete structure, must also be optimal.

Consider a realization of the K metrics. For this realization, let the corresponding values of timers for the K relays when f is used as the metric-to-timer mapping be denoted by $t^f_{(1)}, \ldots, t^f_{(K)}$. Recall that (i) denotes the index of the relay with the i^{th} largest metric, which is the same as the relay with the i^{th} smallest timer. Similarly, the timers of the K relays when g is used as the metric-to-timer mapping are denoted by $t^g_{(1)}, \ldots, t^g_{(K)}$.

The best two relays are successfully discovered by f only under the following two mutually exclusive events:

i) Timers $t^f_{(1)}$, $t^f_{(2)}$, and $t^f_{(3)}$ expire before $T_{\max}$: In this case, successful discovery occurs if and only if these three timers expire at least Δ duration apart from each other:

$$t^f_{(2)} - t^f_{(1)} \ge \Delta,\ t^f_{(3)} - t^f_{(2)} \ge \Delta,\ \text{and } t^f_{(3)} \le T_{\max}. \quad (9)$$

Therefore, $\frac{t^f_{(j)}}{\Delta} \ge \frac{t^f_{(j-1)}}{\Delta} + 1$, for $j = 2, 3$.

From the properties of the floor function, it follows that

$$\left\lfloor \frac{t^f_{(j)}}{\Delta} \right\rfloor \ge \left\lfloor \frac{t^f_{(j-1)}}{\Delta} \right\rfloor + 1, \quad \text{for } j = 2, 3. \quad (10)$$

Multiplying both sides by Δ, it follows from the definition of g in (8) that $t^g_{(j)} \ge t^g_{(j-1)} + \Delta$, for $j = 2, 3$. Furthermore, from (9), we know that $t^f_{(2)} < T_{\max}$. Therefore, g also successfully discovers the best two relays.

ii) Timers $t^f_{(1)}$ and $t^f_{(2)}$ expire before $T_{\max}$ and the timers $t^f_{(3)}, \ldots, t^f_{(K)}$ do not expire: Since a successful discovery has occurred in f, we must have $t^f_{(2)} \ge t^f_{(1)} + \Delta$ and $t^f_{(2)} \le T_{\max}$. As before, from the definition of g in (8), this implies that $t^g_{(2)} \ge t^g_{(1)} + \Delta$, $t^g_{(2)} \le T_{\max}$, and $t^g_{(3)} > T_{\max}$. Thus, the third best relay's timer does not expire under g and, consequently, does not collide with the timers of the best two relays, which have expired before $T_{\max}$ and have not collided with each other.

B. Proof of Result 2

Maintaining the order of the sorted metrics, the best two relays can be chosen from the K relays in $K(K-1)$ ways. When both relays are successfully discovered, let the second best relay lie in the r^{th} interval, which is of length α_r. Since a success occurs only if the best two relays lie in different intervals, we must have $r \geq 1$. For successful discovery of the best two relays to occur: (i) The best relay must lie in any of the intervals $\alpha_0, \ldots, \alpha_{r-1}$. This occurs with probability $\sum_{i=0}^{r-1} \alpha_i$. (ii) And, the remaining $K-2$ relays must lie in the interval $[0, 1 - \sum_{j=0}^{r} \alpha_j)$. This occurs with probability $\left(1 - \sum_{j=0}^{r} \alpha_j\right)^{K-2}$. Therefore, the probability that the second best relay lies in the r^{th} interval and successful discovery occurs is $K(K-1)\alpha_r \left(1 - \sum_{j=0}^{r} \alpha_j\right)^{K-2} \sum_{i=0}^{r-1} \alpha_i$. The desired result follows from the law of total probability by summing the above expression over all possible values of r.

C. Proof of Result 3

We provide a brief sketch of the proof here. Assume that for some i, $\alpha_i = 0$. We can always find an $\alpha_{i'}$ such that i' is the smallest integer greater than i with $\alpha_{i'} \neq 0$, or i' is the largest integer less than i with $\alpha_{i'} \neq 0$. Now, increase α_i to ϵ and decrease $\alpha_{i'}$ by ϵ, where ϵ is a small positive value. It can be shown that the value of $P_{K,N}$ increases.

Similarly if $\sum_{j=0}^{i} \alpha_j = 1$ for some i, then by decreasing α_i by ϵ, it can be shown that $P_{K,N}$ increases. It, therefore, follows that the optimal point cannot lie on the boundary of the constrained region.

D. Proof of Result 4

Taking the partial derivative with respect to t_N of $P_{K,N}$ in (5) and equating it to zero, we get

$$\left(1 - t_{N-1}^*\right)\left(t_N^*\right)^{q-1}\left(q t_{N-1}^* - (q+1)\, t_N^*\right) = 0. \quad (11)$$

From Result 3, we know that $0 < t_i^* < 1$. Therefore,

$$t_{N-1}^* = \left(1 + \frac{1}{q}\right) t_N^*. \quad (12)$$

Similarly, taking the partial derivative of $P_{K,N}$ with respect to t_0 and equating it to zero yields (7).

For $1 \leq i \leq N-1$, taking the partial derivative of $P_{K,N}$ with respect to t_i and equating it to zero, we get

$$\left(1 - t_{i-1}^*\right)\left(q\left(t_i^*\right)^{q-1}\left(t_{i-1}^* - t_i^*\right) - \left(t_i^*\right)^q\right) + \left(1 - 2t_i^* + t_{i+1}^*\right)\left(t_{i+1}^*\right)^q = 0. \quad (13)$$

The important point to observe is that the above equation is quadratic in t_{i-1}^*. Therefore, solving it yields

$$t_{i-1}^* = \frac{q + (1+q)\, t_i^*}{2q} \pm \frac{\sqrt{\left(q - (1+q)\, t_i^*\right)^2 + 4q \frac{\left(t_{i+1}^*\right)^q}{\left(t_i^*\right)^{q-1}}\left(1 - 2t_i^* + t_{i+1}^*\right)}}{2q}. \quad (14)$$

It can be shown that the negative sign must always be taken in (14) in order to ensure that $0 < t_{i-1}^* < 1$. Hence, the result follows.

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