Fast Distributed Multiple Access Based Selection With Imperfect Parameter Knowledge

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***Abstract*—Several wireless systems that seek to exploit multi-user or spatial diversity need to opportunistically select a 'best' node from among the many available nodes. This is done on the basis of a suitability metric that is locally known at each node, and is initially not known elsewhere. While a fast, scalable, multiple access based algorithm is known today for doing this in a distributed manner, it assumes perfect knowledge of statistical parameters such as the cumulative distribution function of the suitability metric and also the total number of nodes. In this paper, we develop a novel and comprehensive analysis of the algorithm when the parameter knowledge is imperfect – as is always the case in reality due to statistical inaccuracies or the time-varying nature of the system. We show that imperfect knowledge does negatively influence the performance of the selection algorithm, and must be accurately accounted for.**

I. Introduction

Many wireless communication schemes being researched today envisage the use of a selection mechanism to discover or select the most suitable candidate, from a set of many candidates. For example, the cooperative communication systems in [1]–[4] involve the selection of the single best node by the source for relaying a message from the source to the destination. In cellular systems, the base station selects the mobile station with the highest instantaneous channel gain to the base station. In energy harvesting sensor networks, selection helps improve the energy neutral operating region [5].

More formally, in these systems, each node independently generates (or knows) its real-valued suitability metric, and the aim is to discover the node with the highest metric. Even fairness constraints, as imposed in proportional fair scheduling in cellular systems [6] or in cooperative communications [7], can be easily accommodated since the metrics can be scaled appropriately. As the channels and the metrics evolve over time, different nodes will get selected at different times.

The decentralized nature of the system implies that the knowledge of the metric is initially available only locally at the node itself. A centralized architecture for making the selection decision, such as a polling mechanism, is inefficient as the time and bandwidth resources consumed increase linearly with the number of candidate nodes. It is therefore highly desirable that the process of selection be both fast and decentralized. Recently, interesting multiple access selection (MAS) mechanisms have been proposed in [3], [8], [9] in which the nodes themselves compete based on their local information.

In [3], nodes transmits a short message when their back-off timer, which is set to be inversely proportional to the metric, expire. Therefore, the node that first transmits is the best node. While this scheme is simple, the timers can expire such that messages from different nodes overlap with each other at the sink[1] and become indecipherable. This reduces the ability of the algorithm to exploit the diversity benefits of having more nodes in the system.

An alternate selection mechanism was proposed by Qin and Berry in [8]. It uses a splitting-based time-slotted algorithm to select the best node. Note that the splitting algorithms for multiple access control, which have been studied extensively in literature (see [10] and the references therein), aim at serving all the nodes. Whereas, the selection aims at finding the single best node as fast as possible. Regardless of the number of nodes, the algorithm provably finds the best node within 2.507 slots, on average. In this algorithm, only nodes whose metrics lie between two thresholds transmit. The thresholds are a function of static parameters such as the cumulative distribution function (CDF) of the metric and the total number of nodes, and gets updated based on the outcomes of the previous steps of the algorithm.

The key question that we ask and address in this paper is the impact of imperfect parameter knowledge on the performance of this algorithm. This is an important question since even statistical information is clearly subject to inaccuracies. In this paper, we focus on the Qin-Berry algorithm given its remarkably fast selection ability, which is highly desirable given the time-varying nature of the wireless channel, and its scalability with the number of nodes. Here, the CDF of the metric and the number of nodes needs to be estimated or assumed. Estimation is subject to statistical inaccuracies. The alternate option is to assume, *i.e.*, factory preset, this information in the nodes; even this may suffer from performance degradation since the actual parameters encountered in the field may be different. While the Qin-Berry algorithm is fast with perfect knowledge, little is understood about its performance with imperfect parameter knowledge. For example, a time-varying channel model, where the metric becomes outdated while the algorithm is running was considered in [8]. But, the parameters were assumed to be perfectly known.

We develop a novel and comprehensive analysis of the splitting algorithm with imperfect parameters. For both the cases of imperfect CDF and node count knowledge, we derive

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[1] We use the generic term 'sink' to refer to the access point or base station or source, as the case may be, that needs to select the best node.

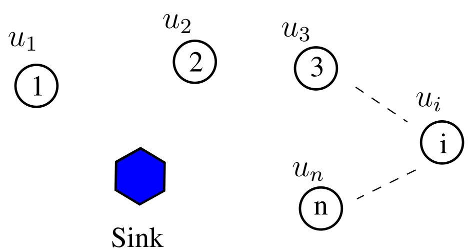

Fig. 1. A relay selection system consisting of a sink and n nodes, with a node i possessing a metric u_i.

expressions for the probability distribution function and the average of total number of slots required by the algorithm

The paper is organized as follows. The system model and the selection algorithm is briefly described in Sec. II. The analysis is developed in Sec. III. The results are studied and verified using simulations in Sec. IV. Our conclusions follow in Sec. V. Proofs are relegated to the appendix.

II. System Model

Consider a time-slotted system with n active nodes and a sink, as shown in Fig. 1. Each node i has a suitability metric u_i, which is known only to that specific node. The goal is to select the node with the highest metric. The metrics are assumed to be i.i.d. with continuous complementary CDF (CCDF) $F_c(u) = \Pr(u_i > u)$. A node does not change its metric during the process of selection.

In the Qin-Berry selection algorithm, at the beginning of each slot, each node determines whether to transmit or not in a distributed manner as detailed below. At the end of each slot, the sink broadcasts one of three outcomes: (i) 0 if the slot was idle (when no node transmitted), (ii) 1 if the outcome was a success (when exactly one node transmitted), and (iii) e if the outcome was a collision (when multiple nodes transmitted).

The algorithm runs independently in each of the contending nodes. It basically determines two thresholds, $H_L(k)$ and $H_H(k)$, for each time slot k. Only those nodes whose metric u satisfies $H_L(k) < u < H_H(k)$ transmit in slot k. It also specifies another variable $H_{\min}(k)$, which is the largest value of the metric known up to slot k above which the best metric surely lies. The algorithm terminates when the outcome is a success (1).

Formally, the algorithm can be defined as follows. Let $\text{split}\,(a,b) = F_c^{-1}\left(\frac{F_c(a)+F_c(b)}{2}\right)$. (The split function makes sure that the nodes involved in the last collision transmit with probability 0.5.) In the first slot, the parameters are initialized as follows: $H_L(1) = F_c^{-1}(1/n)$, $H_H(1) = \infty$, and $H_{\min}(1) = 0$. Then, the algorithm in the $(k+1)^{\text{th}}$ slot proceeds as follows [8]:

1) If feedback (of the k^{th} slot) is an idle (0) and there have been no collisions thus far, then set $H_H(k+1) = H_L(k)$ and $H_L(k+1) = F_c^{-1}(\frac{i+1}{n} + O(\frac{1}{n^2}))$.[2]
2) If feedback is a collision (e), then set $H_{\min}(k+1) = H_L(k)$ and $H_L(k+1) = \text{split}\,(H_L(k), H_H(k))$.
3) If feedback is an idle (0) and a collision has occurred in the past, then set $H_H(k+1) = H_L(k)$ and $H_L(k+1) = \text{split}\,(H_{\min}(k), H_H(k))$.

We shall refer to the phase of the algorithm before the first non-idle slot as the *idle phase*. Subsequent slots constitute the *collision phase*.

As can be seen, the algorithm requires each node to know the following two parameters: (i) the CCDF of the metric (or, equivalently, its CDF), and (ii) the number of nodes, n. We now analyze the case where the CCDF or the number of nodes is imperfectly known.[3]

III. Analysis

First, we consider a generalization of the Qin-Berry algorithm in which during the idle phase, the minimum metric threshold for transmission at slot k is set as $H_L(k) = F_c^{-1}(kp_e/n)$. It is easy to see that the Qin-Berry algorithm, described in section II, corresponds to $p_e = 1$. p_e will help us understand better the performance of the algorithm with imperfect knowledge of the number of nodes. Also, setting $p_e = 1$ as done in Qin-Berry algorithm only greedily maximizes probability of success in each slot of the idle phase, which need not be optimal overall.

We first derive for the perfect case, the CDF of the number of slots required to select the best node. It must be noted that even these results for the perfect case are novel to the best of our knowledge; only the average number of slots has been considered in [8]. The average number of slots can be derived from the CDF using following Lemma. Let $F_T(t) = \Pr(T \leq t)$ denote the CDF of the number of slots, T, required to select the best node. (T is an integer-valued positive random variable (RV).)

Lemma *1:* The average number of slots required to select the best node is given by

$$\mathbf{E}\left[T\right] = 1 + \sum_{t=1}^{\infty}(1 - F_T(t)). \quad (1)$$

Proof: The proof follows from the fact that for any non-negative integer-valued RV, X, $\mathbf{E}\left[X\right] = \sum_{i=0}^{\infty} \Pr(X > i)$. ■

A. Perfect Parameter Knowledge Case

We now find the probability distribution of T when $F_c(.)$ and n are perfectly known. Let $p(a,b)$ be the probability of *exactly* b slots being required to resolve a collision involving a nodes. Then, $p(a,b)$ is given by the following Lemma.

Lemma *2:* The probability that exactly b slots are required to resolve a collision involving a nodes is

$$p(a,b) = \frac{1}{2^a}\left(p(a,b-1) + \sum_{i=2}^{a}\binom{a}{i}p(a,b-1)\right), b > 1, \quad (2)$$

[2]We shall henceforth ignore the $O(1/n^2)$ term that arises in setting the thresholds as its effect is known to be quite negligible even when the number of nodes is as small as 7.

[3]To ensure that the best node still gets selected, all nodes must possess the same imperfect knowledge. This can be easily ensured by having the access point/sink broadcast this slowly-varying information occasionally. Imperfect node counts can still occur when nodes go to sleep or move away.

where the recursion is initialized by $p(a,1) = a/2^a$.

Proof: The proof is given in Appendix A. ■

Let $q = \left\lceil \frac{n}{p_e} \right\rceil - 1$, where $\lceil . \rceil$ denotes the ceil function. The main result for the perfect knowledge case now follows.

Theorem *1:* With perfect knowledge of $F_c(.)$ and n, the CDF of the number of slots, $F_P(t,n,p_e)$, required to select the best node is:

$$\begin{aligned} F_P(t,n,p_e) &= \sum_{i=1}^{\min(t,q)} p_e \left(1 - \frac{ip_e}{n}\right)^{n-1} \\ &+ \sum_{i=1}^{\min(t,q)-1} \sum_{k=2}^{n} \binom{n}{k} \left(\frac{p_e}{n}\right)^k \left(1 - \frac{ip_e}{n}\right)^{n-k} \sum_{j=1}^{t-i} p(k,j) \\ &+ \mathrm{I}_{\{t>q+1\}} \left(1 - \frac{qp_e}{n}\right)^n \sum_{j=1}^{t-q-1} p(n,j), \quad (3) \end{aligned}$$

where $\mathrm{I}_{\{X\}} = 1$, if X is true, else $\mathrm{I}_{\{X\}} = 0$.

Proof: The proof is given in Appendix B. ■

Finally, the average number of slots, $m_P(n,p_e)$, required to find the best node is given by the following theorem.

Theorem *2:* With perfect knowledge of $F_c(.)$ and n,

$$\begin{aligned} m_P(n,p_e) &= \left(1 - \frac{qp_e}{n}\right)^n (EX_n + q + 1) \\ &+ \sum_{i=1}^{q} \sum_{k=1}^{n} \binom{n}{k} \left(\frac{p_e}{n}\right)^k \left(1 - \frac{ip_e}{n}\right)^{n-k} (EX_k + i) \quad (4) \end{aligned}$$

where $EX_k = \frac{2^k + \sum_{l=2}^{k-1} \binom{k}{l} EX_l}{2^k - 2}$, $k>2$, $EX_1 = 1$, and $EX_2 = 2$.

Proof: The proof is given in Appendix C. ■

We now analyze the imperfect knowledge case.

B. Imperfect CDF

Let the CCDF assumed by all the nodes be $F_{c,\text{ass}}(.)$, while it is actually $F_{c,\text{act}}(.)$. (The number of nodes, n, is assumed to be perfectly known.)

Let $p(a,b,\ell,h)$ be the probability that exactly b slots are required to resolve a collision involving a nodes with $H_L = F_{c,\text{ass}}^{-1}(\ell)$ and $H_H = F_{c,\text{ass}}^{-1}(h)$, since $F_{c,\text{ass}}(.)$ is the CCDF assumed. First, we derive $p(a,b,l,h)$ in the following Lemma.

Lemma *3:*

$$\begin{aligned} p(a,b,\ell,h) &= p\left(a, b-1, \ell, \frac{h+\ell}{2}\right)(1-\beta(\ell,h))^a \\ &+ \sum_{i=2}^{a} \binom{a}{i} \beta^i(\ell,h)(1-\beta(\ell,h))^{a-i} p\left(i, b-1, \frac{h+\ell}{2}, h\right), \quad (5) \end{aligned}$$

where $p(a,1,\ell,h) = a\beta(\ell,h)(1-\beta(\ell,h))^{a-1}$ and $\beta(\ell,h) = \frac{F_{\text{eq}}(h) - F_{\text{eq}}(\frac{\ell+h}{2})}{F_{\text{eq}}(h) - F_{\text{eq}}(\ell)}$. Here, $F_{\text{eq}}(.)$ is a composite function given by $F_{\text{eq}}(x) = F_{c,\text{act}}(F_{c,\text{ass}}^{-1}(x))$.

Proof: The proof is given in Appendix D. ■

Theorem *3:* The CDF of the total number of slots required by the algorithm when the CCDF assumed by all the nodes is $F_{c,\text{ass}}(.)$ and the actual CCDF of the metric is $F_{c,\text{act}}(.)$ is

$$\begin{aligned} &F_{\text{Ic}}(t,n,p_e) = \\ &\sum_{i=1}^{t'-1} \sum_{k=2}^{n} \left[\binom{n}{k} \left(F_{\text{eq}}\left(\frac{ip_e}{n}\right) - F_{\text{eq}}\left(\frac{(i-1)p_e}{n}\right)\right)^k \right. \\ &\left. \times \left(1 - F_{\text{eq}}\left(\frac{ip_e}{n}\right)\right)^{n-k} \sum_{j=1}^{T-i} p\left(k, j, \frac{ip_e}{n}, \frac{(i-1)p_e}{n}\right) \right] \\ &+ \mathrm{I}_{\{t>q+1\}} \left(1 - F_{\text{eq}}\left(\frac{qp_e}{n}\right)\right)^n \sum_{j=1}^{t-q-1} p\left(n, j, 1, \frac{qp_e}{n}\right) \\ &+ \sum_{i=1}^{t'} n\left(1 - F_{\text{eq}}\left(\frac{ip_e}{n}\right)\right)^{n-1} \left(F_{\text{eq}}\left(\frac{ip_e}{n}\right) - F_{\text{eq}}\left(\frac{(i-1)p_e}{n}\right)\right), \quad (6) \end{aligned}$$

where $t' = \min(t,q)$.

Proof: The proof is given in Appendix E. ■

The average number of slots follows by substituting the above result in Lemma 1.

C. Imperfect knowledge of n

We now derive an expression for the CDF of the total number of slots required, $F_{\text{In}}(t,n,\hat{n},p_e)$, and its average, $m_{\text{In}}(n,\hat{n},p_e)$, when the number of nodes contending is assumed (by all) to be $\hat{n}$ when the actual value is n.

Theorem *4:* The CDF of the average number of slots required when the number of nodes is assumed to be $\hat{n}$ and the actual value is n is

$$F_{\text{In}}(t,n,\hat{n},p_e) = F_P\left(t, n, \frac{np_e}{\hat{n}}\right). \quad (7)$$

Proof: The proof is given in Appendix F. ■

Theorem *5:* The average number of slots required when the number of nodes assumed is $\hat{n}$ and the actual value is n is

$$m_{\text{In}}(n,\hat{n},p_e) = m_P\left(n, \frac{np_e}{\hat{n}}\right). \quad (8)$$

Proof: The proof is omitted due to space constraints. ■

The above expressions give nice insights about the performance of the algorithm with imperfect knowledge of n. They show that the performance depends only on the ratio $\frac{\hat{n}}{n}$ and the value of p_e chosen.

IV. Simulations

We now study graphically the analytical results derived in Section III. We also compare our analytical results with Monte Carlo simulations (with 10^5 samples generated for each point in the plot). The figures are plotted for $p_e = 1$.

We now consider the imperfect CCDF case. We take the metric to have an exponential distribution, as is the case for multiuser diversity over frequency-flat fading channels. Without loss of generality, we set its mean power to unity. For this the (actual) CCDF is $F_{c,\text{act}}(t) = e^{-t}$. The assumed CCDF is taken to be $F_{c,\text{ass}}(t) = e^{-\lambda_{\text{est}} t}$. Figure 2 plots the CDF of the number of slots required (T) to select the best node, for $n = 8$ nodes. It shows that the CDF of T shifts downwards as compared to the perfect knowledge case

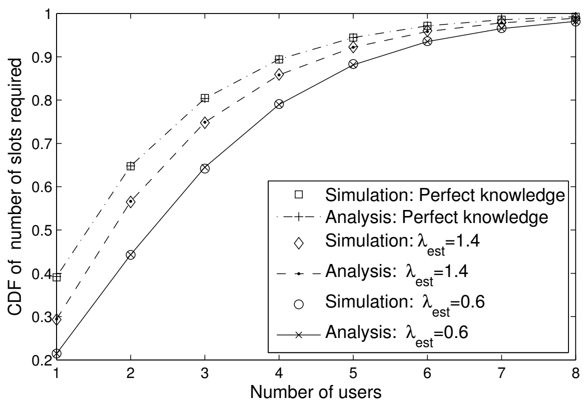

Fig. 2. CDF of the total number of slots with imperfect CCDF knowledge ($p_e = 1$ and $n = 8$).

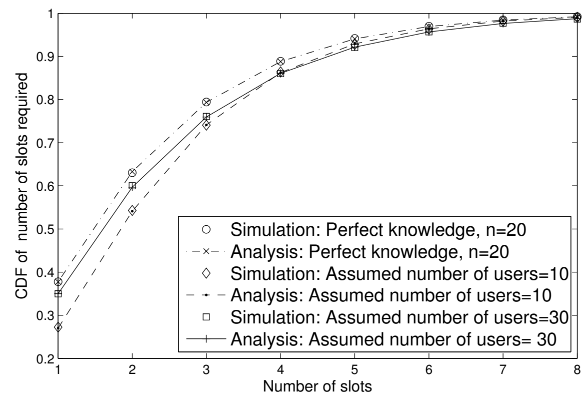

Fig. 4. CDF of the total number of slots with imperfect knowledge of number of nodes ($p_e = 1$, $n = 20$).

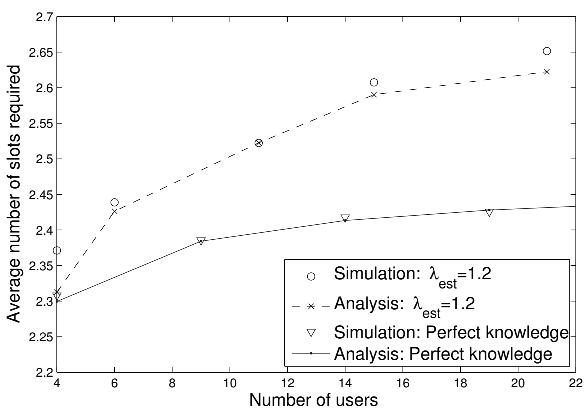

Fig. 3. Average number of slots with imperfect CCDF knowledge ($p_e = 1$).

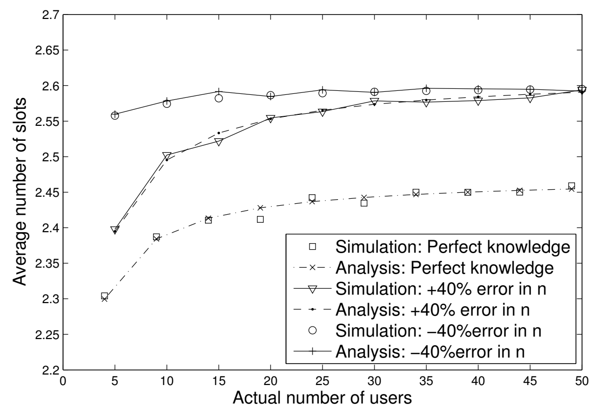

Fig. 5. Average number of slots required with imperfect knowledge of number of contending nodes ($p_e = 1$).

which implies degradation in performance. For example, the median increases by 24% when $\lambda_{\text{est}} = 1.4$ and by 60% when $\lambda_{\text{est}} = 0.6$. The average number of slots is plotted in Figure 3, as a function of number of nodes in the system for $\lambda_{\text{est}} = 1.2$. Also, it can be seen that the upper bound of 2.507 slots for perfect knowledge case [8] is exceeded when $n \geq 11$ when the knowledge is imperfect.

Figure 4 plots CDF of T when the number of nodes is known inaccurately. The actual number of nodes is taken to be 20. (The CDF of the metric, which is perfectly known, is $F_c(t) = e^{-t}$.) Again, one can see that the CDF shifts downwards, which implies a degradation in the performance of the algorithm. For example, the median increases by 10% and 26% when $\hat{n} = 30$ and $\hat{n} = 10$, respectively. Figure 5 plots the average number of slots to select the best node as a function of n, for $\hat{n} = (1 \pm 0.4)n$. We see that the average number of slots increases. However, it saturates for larger n.

V. Conclusions

We analyzed and studied the performance of the Qin-Berry algorithm used to select the node with highest metric, for the practical scenario where the CDF of the metric and the total number of nodes in the system is not known perfectly. Our analysis for the CDF and the average of the number of slots required show that the degradation in the performance due to imperfect knowledge is not catastrophic. In fact, the algorithm scales well even in the presence of a large inaccuracy in the estimate of the number of nodes; the average number of slots required, while higher, did saturate. Given the important role of selection, we believe our results are widely applicable in the areas such as co-operative relay selection, cellular systems exploiting multiuser diversity and multi-hop sensor networks. An interesting avenue for future work is developing mechanisms to make the algorithm more robust.

Appendix

A. *Proof of Lemma 2*

If a nodes are involved in a collision, then the distribution of the number of nodes transmitting in the next slot is a binomial distribution with a trials and probability of success of $1/2$, which we denote by $\text{Bi}(a, 1/2)$. Thus, $p(a, 1)$, which is the probability that one node transmits in the next slot, equals

$a/2^a$. For $b > 1$, the following three cases arise in calculating $p(a,b)$:

1) The next slot is idle: This case happens with probability $\binom{a}{0}/2^a$. Now, the collision involving a nodes needs to be resolved in $b-1$ slots.
2) The next slot is a collision among i nodes: This collision, which happens with probability $\binom{a}{i}/2^a$, needs to be resolved in exactly $b-1$ slots.
3) The next slot is a success: This implies that the collision is resolved in fewer than b slots. Consequently, it is impossible to resolve it in *exactly* b slots.

B. Proof of Theorem 1

By definition, $p(a,b)$ is the probability that exactly b slots are required to resolve a collision among a nodes. Also, the idle phase can consist of a maximum of q slots. Given that the first non-idle slot is the i^{th} slot and k nodes are involved, the probability that the collision is resolved in the remaining $t-i$ slots is $\sum_{j=1}^{t-i} p(k,j)$. The probability that the first non-idle slot is the i^{th} slot and k nodes are involved is $\binom{n}{k}\left(\frac{p_e}{n}\right)^k\left(1-\frac{ip_e}{n}\right)^{n-k}$, for $i \le q$. The probability that the $(q+1)^{\text{th}}$ slot is the first non-idle slot is $(1-\frac{qp_e}{n})^n$ since all the $k=n$ nodes must necessarily be involved. (Other values of k are impossible.) Hence, the result follows.

C. Proof of Theorem 2

EX_k is the average number of slots required to resolve a collision among k nodes. (EX_k is given in equation (6) in [8].) Given that the first non-idle slot is the i^{th} slot and k nodes are involved, the average number of slots required is $EX_k + i$. The probability that this happens is $\binom{n}{k}\left(\frac{p_e}{n}\right)^k\left(1-\frac{ip_e}{n}\right)^{n-k}$, for $i \le q$. For the $(q+1)^{\text{th}}$ slot, the probability that it is the first non-idle slot and k nodes are involved is $(1-\frac{qp_e}{n})^n$, for $k=n$, and is 0 otherwise.

D. Proof of Lemma 3

The a priori probability of a metric falling in an interval (c,d) is $F_{c,\text{act}}(d) - F_{c,\text{act}}(c)$. The split function makes sure that, among all the nodes involved in a collision, only those whose metrics lie in the interval $\left(F_{c,\text{ass}}^{-1}(h), F_{c,\text{ass}}^{-1}(\frac{h+\ell}{2})\right)$ transmit. Therefore, the probability that a node i transmits in the next slot, given that it was just involved in a collision, equals $\frac{Pr\left(u_i \in \left(F_{c,\text{ass}}^{-1}(h), F_{c,\text{ass}}^{-1}(\frac{h+\ell}{2})\right)\right)}{Pr\left(u_i \in \left(F_{c,\text{ass}}^{-1}(h), F_{c,\text{ass}}^{-1}(\ell)\right)\right)}$. This further simplifies to $\frac{F_{c,\text{act}}(F_{c,\text{ass}}^{-1}(h)) - F_{c,\text{act}}(F_{c,\text{ass}}^{-1}(\frac{\ell+h}{2}))}{F_{c,\text{act}}(F_{c,\text{ass}}^{-1}(h)) - F_{c,\text{act}}(F_{c,\text{ass}}^{-1}(\ell))} = \beta(\ell,h)$. Hence, if a nodes are involved in a collision, the distribution of the number of nodes transmitting in the next slot is $\text{Bi}(a, \beta(\ell,h))$.

Thus, $p(a,1,\ell,h)$, which is the probability that one node transmits in the next slot, equals $a\beta(\ell,h)(1-\beta(\ell,h))^{a-1}$. Also, the probability that i nodes transmit in the next slot is $a\beta(\ell,h)(1-\beta(\ell,h))^{a-1}$.

For $b > 1$, if $i > 2$ nodes are in the interval $(F_c^{-1}(\frac{h+\ell}{2}), F_c^{-1}(h))$, then there is a collision which needs to be resolved in exactly $b-1$ slots. If all the a nodes belong to the lower half interval $(F_c^{-1}(\ell), F_c^{-1}(\frac{h+\ell}{2}))$, then there is an idle which needs to be resolved in exactly $b-1$ slots. If only one node transmits in next slot, then the collision is already resolved in strictly less than b slots.

E. Proof of Theorem 3

For $i \le q$, the probability that the first non-idle slot is the i^{th} slot and $k \ge 1$ nodes are involved is equal to the probability that k nodes lie in the interval $\left(F_{c,\text{ass}}^{-1}(\frac{(i-1)p_e}{n}), F_{c,\text{ass}}^{-1}(\frac{ip_e}{n})\right)$ and the rest lie below $F_{c,\text{ass}}^{-1}(\frac{ip_e}{n})$. Therefore, it equals

$$\binom{n}{k}\left(F_{\text{eq}}\left(\frac{ip_e}{n}\right) - F_{\text{eq}}\left(\frac{(i-1)\,p_e}{n}\right)\right)^k \times \left(1 - F_{\text{eq}}\left(\frac{ip_e}{n}\right)\right)^{n-k},$$

for $i \le q$. The probability that after the i^{th} non-idle slot, the collision is resolved in the remaining $t-i$ slots is $\sum_{j=1}^{t-i} p\left(k, j, \frac{ip_e}{n}, \frac{(i-1)p_e}{n}\right)$. Also, the probability that the first non-idle slot is the $(q+1)^{\text{th}}$ slot is the probability that all the nodes are in the interval $\left(F_{\text{eq}}\left(\frac{qp_e}{n}\right), 1\right)$, which equals $\left(1 - F_{\text{eq}}\left(\frac{qp_e}{n}\right)\right)^n$. We have $t-(q+1)$ slots to resolve this collision among n nodes, which happens with probability $\sum_{j=1}^{t-q-1} p\left(n, j, 1, \frac{qp_e}{n}\right)$. Hence, the result follows.

F. Proof of Theorem 4

During the idle phase, the lower thresholds $H_L(.)$ for the first i idle slots are $F_c^{-1}(p_e/\hat{n}), F_c^{-1}(2p_e/\hat{n}), \ldots, F_c^{-1}(ip_e/\hat{n})$. Given that the first non-idle slot is the i^{th} slot and k nodes are involved, the probability that the collision is resolved in the remaining $t-i$ slots is $\sum_{j=1}^{t-i} p(k,j)$. The probability that the first non-idle slot is the i^{th} slot and k nodes are involved is $\binom{n}{k}\left(\frac{p_e}{\hat{n}}\right)^k\left(1-\frac{ip_e}{\hat{n}}\right)^{n-k}$, for $i \le q$. Here, $q = \left\lceil \frac{p_e}{\hat{n}} \right\rceil - 1$. The probability that the first non-idle slot is the $(q+1)^{\text{th}}$ slot and k nodes are involved is $(1-\frac{qp_e}{\hat{n}})^n$, for $k=n$, and is 0 otherwise. Hence, the expression is same as that given by Theorem 2, with p_e replaced by $\frac{np_e}{\hat{n}}$.

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