Optimal Design of Timer-Based, Distributed Selection with Unknown Number of Nodes

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***Abstract*—The timer-based selection scheme is a popular, simple, and distributed scheme that is used to select the best node from a set of available nodes. In it, each node sets a timer as a function of a local preference number called a *metric*, and transmits a packet when its timer expires. The scheme ensures that the timer of the best node, which has the highest metric, expires first. However, it fails to select the best node if another node transmits a packet within Δ s of the transmission by the best node. We derive the optimal timer mapping that maximizes the average success probability for the practical scenario in which the number of nodes in the system is unknown but only its probability distribution is known. We show that it has a special discrete structure, and present a recursive characterization to determine it. We benchmark its performance with ad hoc approaches proposed in the literature, and show that it delivers significant gains. New insights about the optimality of some ad hoc approaches are also developed.**

I. Introduction

Opportunistic selection finds applications in many wireless communication systems. For example, in a cooperative relaying system, relay selection exploits spatial diversity [1]–[3]; in wireless sensor networks (WSNs), sensor selection improves network lifetime [4], [5]; and in vehicular ad hoc networks (VANETs), vehicle selection speeds up information dissemination [6], [7]. Furthermore, various notions of fairness, such as proportional fairness and max-min fairness, can be formulated as a selection problem [3], [8].

In all the systems above, selection occurs as follows: Each node maintains a preference number called a *metric* that is a function of local parameters such as channel gains or sensor measurements. For example, in amplify-and-forward relaying, the metric of a relay is the harmonic mean of the source-to-relay (SR) and relay-to-destination (RD) channel gains [1]. Instead, in a cooperative system with multiple decode-and-forward (DF) relays, the metric of a DF relay is equal to its RD channel power gain in case it has decoded the source's message and is 0 otherwise. In VANETs, it is a function of the vehicle's speed and position [7]. The goal of selection is to help a common node called *sink* identify the *best* node, which has the highest metric among all the nodes. A fundamental issue common to the above systems, in which the nodes are spatially separated from each other, is that a node knows only its own metric and not the metrics of the other nodes. Therefore, a distributed selection algorithm is needed.

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The timer-based selection scheme is a popular distributed selection scheme [1], [5], [8], [9]. In it, the nodes use a common monotone non-increasing (MNI) metric-to-timer mapping $f(\cdot)$. A node i with metric μ_i sets its timer as $T_i = f(\mu_i)$ and transmits a small *timer packet* when its timer expires. The MNI property ensures that the best node's packet reaches the sink first. Given its simplicity, it has been used in a wide range of wireless systems such as cooperative relaying [1], WSNs [5], [10], wireless local area networks (WLANs) [11], and VANETs [6].

However, due to its distributed nature, the timer scheme cannot guarantee successful selection of the best node. For example, it can fail to select the best node if the timer of the best node does not expire within the stipulated selection duration of $T_{\max}$. Failure also occurs in the event of a collision, in which the timer of the second best node expires within a *vulnerability window* Δ after the expiry of the best node's timer. Δ is determined by the physical layer capabilities of the system. Typically, it is a sum of the maximum propagation delay, switching time, and maximum time synchronization error [1], [12], [13].

The probability of selecting the best node, which is a measure of how effective the selection scheme is, depends on the metric-to-timer mapping. In [9], the optimal timer mapping that maximizes the success probability is shown to be a staircase mapping, in which timers expire only at integer multiples of Δ, i.e., $0, \Delta, 2\Delta, \ldots$. However, a key assumption in [9] is that every node knows the total number of nodes in the system. As a result, the optimal mapping turns out to be a function of the total number of nodes.

The total number of nodes is often unknown in practice. For example, in a system with multiple DF relays, whether a relay decodes a source's message or not depends on the instantaneous gain of its SR channel. Therefore, the destination has to select the best relay among the set of all relays that decoded the source's message without knowing a priori the size of this set. When the SR gains are independent and identically distributed (i.i.d.), as is often assumed in the literature [1], [2], the size of the set is a binomially distributed random variable (RV). Event detection in WSNs [14], opportunistic media access control (MAC) schemes for WLANs [11], and vehicle selection in VANETs [7] are other examples where the total number of nodes is not known a priori.

A. Contributions

We consider the practically important scenario where the number of nodes is an RV and is unknown. Only the probability distribution of the number of nodes, which we shall henceforth refer to as the *prior*, is known. We characterize the optimal timer mapping that maximizes the average success probability, which is the success probability averaged over the prior.

We prove that the optimal timer mapping again has a discrete, staircase structure, in which a node's timer expires only at $0, \Delta, 2\Delta, \ldots, N\Delta$ or not at all. Here, $N = \left\lfloor \frac{T_{\max}}{\Delta} \right\rfloor$, where $\lfloor \cdot \rfloor$ denotes the floor function. We then focus on the scenario where the number of nodes is a binomial RV, and show that the optimal mapping is given in the form of a simple recursion in N. The choice of the binomial prior is motivated by the cooperative DF relaying example discussed above. We also benchmark the optimal mapping with ad hoc approaches that have been proposed in the literature. We note that the above results can be generalized to include other priors and also the scenario where even the prior distribution is not known [15].

The paper is organized as follows. In Section II, we describe our system model. The optimal timer mapping is developed in Section III. We present numerical results in Section IV and conclude in Section V.

II. System Model and Timer Scheme

Consider a system with k nodes and a sink. Each node i maintains a metric $\mu_i \in \mathbb{R}^+$, which is not known to any other node, where $\mathbb{R}^+$ denotes the set of positive real numbers. The goal is for the sink to find the best node i^*, where

$$i^* = \underset{i \in \{1,2,\ldots,k\}}{\operatorname{argmax}} \ \mu_i. \tag{1}$$

We assume that the value of k is not known to the nodes and the sink, but its prior distribution is known to all. The metrics are assumed to be i.i.d., as has often been assumed in the literature [1], [5], [11], [16]. Further, without loss of generality, we assume that the metrics are uniformly distributed in the interval $[0, 1]$.[1]

Timer Scheme: A node i sets its timer T_i as a function of its metric μ_i as $T_i = f(\mu_i)$, where $f : [0, 1] \to [0, \infty)$ is a MNI function. When the timer of a node expires, it transmits a *timer packet*. Furthermore, nodes whose timers expire after $T_{\max}$ do not transmit. The timer packet contains the identity of the node to enable the sink to identify which node transmitted.

MAC interaction model: If two or more nodes transmit within a time window of Δ, a collision occurs and the sink cannot decode any of the transmissions. However, if only one node transmits, the receiver can decode the timer packet successfully. This assumption is often made in the multiple access literature [18, Chap. 4], [11], [12], [16]. It is justified because, given the low payload of the timer packet, its packet error rate can be made small by using a sufficiently large fading margin in the link budget calculations.

[1]This is because node i can generate a new metric $y_i = F(\mu_i)$ that is uniformly distributed over $[0, 1]$, where F is the cumulative distribution function (CDF) of the metrics. Since F is a monotone non-decreasing function, the node with the highest y_i is the same as the node with the highest μ_i. Thus, ordering of nodes is also preserved. We assume that F is known as it varies at a time scale that is several orders of magnitude slower than that of the metric, and can be estimated accurately [17].

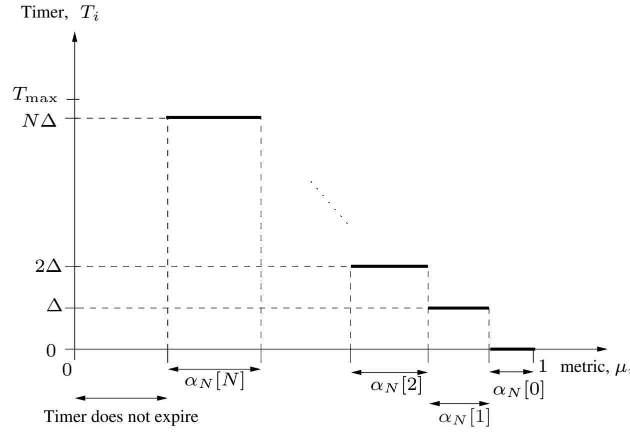

Fig. 1. Illustration of a staircase metric-to-timer mapping for a maximum selection duration of $T_{\max}$ and a vulnerability window Δ.

III. Optimal Timer Mapping

Our goal is to maximize the average success probability, which is defined as the success probability averaged over the distribution of the number of nodes. We first show a general result that holds for any prior. We show that a timer mapping, of the form shown in Figure 1, is optimal for our problem in which the number of nodes is not known to all the nodes.

Theorem *1:* There exists a MNI timer mapping that maximizes the average success probability in which the timers expire only at $0, \Delta, \ldots, N\Delta$, or not at all, where $N = \left\lfloor \frac{T_{\max}}{\Delta} \right\rfloor$. When the metric μ lies in the interval $[1 - \alpha_N[0], 1)$, then the timer expires immediately at time 0. When μ lies in the interval $[1 - \alpha_N[0] - \alpha_N[1], 1 - \alpha_N[0])$, the timer expires at time Δ. In general, when $\mu \in \left[1 - \sum_{j=0}^{i} \alpha_N[j], 1 - \sum_{j=0}^{i-1} \alpha_N[j]\right)$, the timer expires at $i\Delta$, for $i = 0, 1, \ldots, N$. Timers of nodes whose metrics lie in the interval $\left[0, 1 - \sum_{j=0}^{N} \alpha_N[j]\right)$ do not expire at all.

Proof: The proof is presented in Appendix A. ■

The optimal mapping, thus, looks like a staircase with the height of each stair being Δ and the length of the j^{th} stair given by $\alpha_N[j]$. We, therefore, call it a *staircase mapping* henceforth. Consequently, we shall refer to $\alpha_N[j]$ as the *j^{th} stair length* and N as the *number of timer levels*. Note that N is completely determined by the system parameters $T_{\max}$ and Δ. Thus, the problem reduces to optimizing the $N + 1$ stair lengths $\alpha_N[0], \alpha_N[1], \ldots, \alpha_N[N]$. This generalizes the result in [9], which only proved that the staircase mapping is optimal when the number of nodes is known.

Intuitively, this result of Theorem 1 is similar to the well-known result in the MAC literature that slotted Aloha has a higher throughput than unslotted Aloha because it allows nodes to transmit only at discrete time slots.

A. Optimal Timer Mapping for Binomial Prior

Let $\boldsymbol{\alpha}_N = (\alpha_N[0], \alpha_N[1], \ldots, \alpha_N[N])$. For the binomial prior, the probability that the number of nodes is r is

$$\Pr(k = r) = \binom{K}{r} p^r (1 - p)^{K-r}, \quad \text{for } 0 \le r \le K, \tag{2}$$

where the maximum possible number of nodes is K and $p \in [0,1]$ is called the *participation probability*.

When k nodes participate in a timer scheme, the probability that the best node is selected is with the staircase mapping is given by $\sum_{i=0}^{N} k\alpha_N[i]\left(1-\sum_{j=0}^{i}\alpha_N[j]\right)^{k-1}$. Averaging this over the binomial distribution on k, the average success probability $P_N(\boldsymbol{\alpha}_N)$ can be shown to be

$$P_N(\boldsymbol{\alpha}_N) = Kp\sum_{i=0}^{N}\alpha_N[i]\left(1-p\sum_{j=0}^{i}\alpha_N[j]\right)^{K-1}. \quad (3)$$

Therefore, the optimization problem can be stated as follows:

$$\mathcal{OB}: \quad \underset{\boldsymbol{\alpha}_N}{\text{maximize}} \quad P_N(\boldsymbol{\alpha}_N) \quad (4)$$

$$\text{subject to} \quad \sum_{j=0}^{N}\alpha_N[j] \leq 1, \quad (5)$$

$$\alpha_N[i] \geq 0, \text{ for } 0 \leq i \leq N. \quad (6)$$

We now present the complete solution to the problem $\mathcal{OB}$. Let $\alpha_N^*[i]$ denote the optimal length of the i^{th} interval.

Theorem *2:* Let $\boldsymbol{\beta}_N = (\beta_N[0], \beta_N[1], \ldots, \beta_N[N])$ be generated as follows:

$$\beta_N[i] = \begin{cases} \frac{1}{p}\left(\frac{1-P_{N-1}(\beta_{N-1}[0],\ldots,\beta_{N-1}[N-1])}{K-P_{N-1}(\beta_{N-1}[0],\ldots,\beta_{N-1}[N-1])}\right), & \text{if } i=0 \\ (1-p\beta_N[0])\beta_{N-1}[i-1], & \text{if } 1 \leq i \leq N \end{cases}, \quad (7)$$

where $\beta_0[0] = \frac{1}{Kp}$ and $P_N(\cdot)$ is given by (3). If $\sum_{j=0}^{N}\beta_N[j] \leq 1$ then $\boldsymbol{\alpha}_N^* = \boldsymbol{\beta}_N$. Otherwise,

$$\alpha_N^*[i] = \begin{cases} \frac{1}{p}\left(\frac{1-L_{N-1}^{\eta}(\boldsymbol{\alpha}_{N-1}^*)}{K-L_{N-1}^{\eta}(\boldsymbol{\alpha}_{N-1}^*)}\right), & \text{if } i=0 \\ (1-p\alpha_N^*[0])\alpha_{N-1}^*[i-1], & \text{if } 1 \leq i \leq N \end{cases}, \quad (8)$$

where

$$L_N^{\eta}(\boldsymbol{\alpha}_N^*) = P_N(\boldsymbol{\alpha}_N^*) + \eta\left(1-p\sum_{j=0}^{N}\alpha_N^*[j]\right)^{K}, \quad (9)$$

and $\alpha_0^*[0] = \frac{1}{p}\left(\frac{1-\eta}{K-\eta}\right)$. Here, $0 < \eta < 1$ is chosen such that $\sum_{j=0}^{N}\alpha_N^*[j] = 1$, and such a choice of η always exists.

Proof: The proof is given in Appendix B. ■

Intuitively, the above recursion arises because in the case of an idle, we know that the metrics of the nodes are once again uniformly distributed and i.i.d., and the problem reduces to designing an optimal timer scheme that maximizes the average success probability with one less timer level.

Notice that the recursion in (8) reduces to that in (7) when $\eta = 0$. The parameter η is found numerically. This is typical of several constrained optimization problems that arise in wireless systems, e.g., water-filling [19] and rate adaptation [20]. Both η and the optimal stair lengths have to be computed only once at the beginning, and not every time selection takes place.

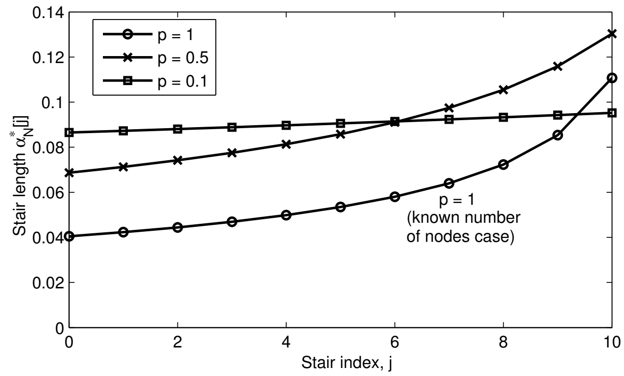

Fig. 2. Optimal stair lengths as a function of j and participation probability p ($K = 4$ and $N = 10$).

IV. Numerical Results

We now present numerical results to better understand the optimal mapping and also to benchmark its performance against various approaches used in the literature. Figure 2 plots the optimal stair lengths for several values of p for $K = 4$ and $N = 10$. It also plots the case when $p = 1$, which is the case when the number of nodes is known. Note that the stair length $\alpha_N^*[j]$ increases as j increases. Thus, the optimal mapping becomes more aggressive in making the nodes transmit as time progresses. However, for small p, the increase in the optimal stair lengths is marginal. In this case, the mapping becomes similar to the ad hoc equal stair length mapping used in [11].

To better understand the performance of the optimal mapping we benchmark it against the following four designs: (i) *Design for K nodes:* Here, the stair lengths are designed assuming K nodes are always present in the system, i.e., the stair lengths are obtained by setting $p = 1$ in Theorem 2. (ii) *Design for average node count:* Here, the stair lengths are designed assuming that there are always $\lceil Kp \rceil$ nodes in the system, where $\lceil\cdot\rceil$ denotes the ceil function. (iii) *Equal stair length mapping [11]:* Here, the stair lengths are equal, and are, therefore, set to $\frac{1}{N+1}$. (iv) *Inverse mapping [1]:* Here, the mapping is given by $f(\mu) = c/\mu$, where $c > 0$ is a constant and is determined numerically to maximize the average success probability. We also compare against the probability that at least one node is present in the system, which is given by $1-(1-p)^K$. Clearly, this probability is an upper bound on the selection probability because any selection is bound to fail if there are no nodes in the system.

Figure 3 plots the average success probability of the various timer mappings as a function of the number of timer levels N. We see that the average success probability of the optimal mapping increases with N, and is very close to the upper bound, which is the probability that at least one node is present in the system. For example, for $N = 20$ and $p = 0.01$, it is 99% of the upper bound. This illustrates the effectiveness of the optimal timer-based selection algorithm. Furthermore, for $p = 0.5$, the design for average node count performs almost as well as the optimal mapping. However, when p is small, e.g., $p = 0.01$, the average success probability of the optimal

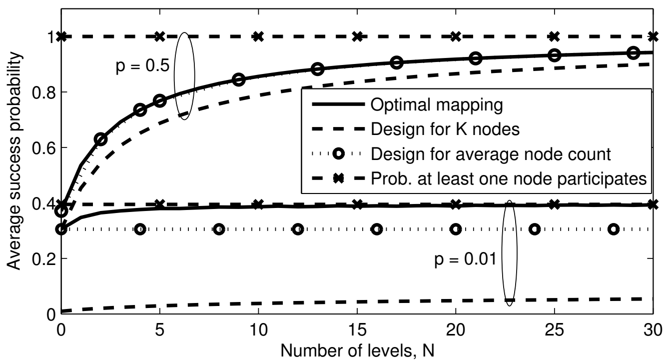

Fig. 3. Comparison of average success probability vs. the number of timer levels ($K = 50$).

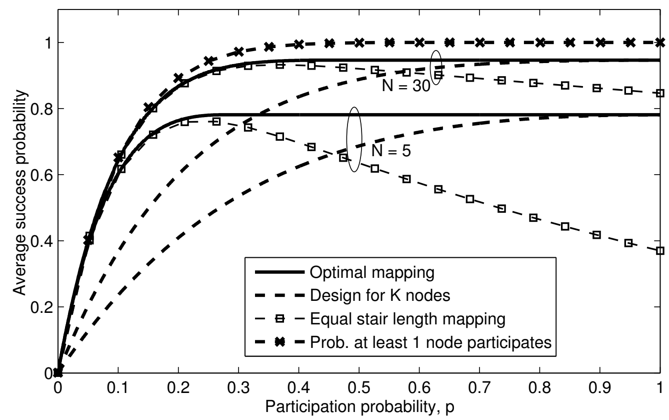

Fig. 4. Comparison of average success probability vs. the participation probability p for different values of N ($K = 10$).

mapping is 29% better than that of the design for average node count. When $N = 20$, the average success probability of the optimal mapping is 6% more than the design for K nodes for $p = 0.5$. For $p = 0.01$ this increases markedly to 715%.

Figure 4 plots the average success probability as a function of the participation probability p for $N = 5$ and $N = 30$. For the optimal mapping, we observe that the average success probability is very close to its upper bound when p is small and is, thus, limited by the absence of nodes to select form. While the equal stair length mapping is close to optimal for small p, this is not so as p increases. For example, for $p = 0.5$, the average success probability of the equal stair length mapping is 44% lower than that of the optimal mapping. The average success probability of the inverse mapping is not plotted in order to avoid clutter. Its performance turns out to be the worst among all the schemes. For example, when $N = 20$ and $p = 0.5$, its average success probability is 0.37, while that of the optimal mapping is 0.92.

V. Conclusions

We developed the optimal timer mapping that maximizes the average success probability for the practical scenario in which the number of nodes in the system is not known and is a binomial random variable. We proved that it is a staircase mapping in which the timers expire at $0, \Delta, \dots, N\Delta$ or not at all. We then showed that the optimal stair lengths are given in the form of a recursion in N. The equal stair length mapping was close to optimal only when the participation probability is small. When Np was large, the design for average node count turned out to be close-to-optimal. Given its staircase nature, the optimal mapping is easily implementable and can be stored in a node as a one-dimensional lookup table with $N + 1$ entries.

Appendix

A. Proof of Theorem 1

Let $f : [0, 1] \to [0, \infty)$ be an MNI mapping. Define a new mapping g as follows:

$$g(\mu) = \begin{cases} \left\lfloor \frac{f(\mu)}{\Delta} \right\rfloor \Delta, & \text{if } f(\mu) \le T_{\max} \\ T_{\max}^{+}, & \text{if } f(\mu) > T_{\max} \end{cases}, \qquad (10)$$

where $T_{\max}^{+}$ indicates that the timer does not expire within the selection duration $T_{\max}$. This new mapping g can be shown to be an MNI mapping because f is MNI.

Key proof idea: We show below that for any $k \in \mathbb{N}$ and for any realization of the metrics $\mu_1, \dots, \mu_k$, using g as the timer mapping results in a success if using f results in a success. This proves that the average success probability of g is greater than or equal to that of f. Since g is MNI, from (10) it follows that it must have the form given in the theorem statement.

We consider the following three cases: *Case* 1 *($k = 0$):* Here, the success probability is 0 for both g and f since there are no nodes in the system.

Case 2 *($k = 1$):* Let the node's metric be μ_1. Then from (10), $g(\mu_1) \le T_{\max}$ if $f(\mu_1) \le T_{\max}$. Thus, g will result in a success if f results in a success since only one node's timer expires in both mappings.

Case 3 *($k \ge 2$):* Let $\mu_1, \mu_2, \dots, \mu_k$ be a particular realization of the nodes' metrics. Using standard order statistics notation, let $\mu_{[i]}$ denote the i^{th} largest metric among the k metric values. f succeeds in selecting the best node only in the following two cases: (i) *When* $f(\mu_{[2]}) > T_{\max}$ *and* $f(\mu_{[1]}) \le T_{\max}$*:* In this case, from (10), $g(\mu_{[1]}) \le T_{\max}$ and $g(\mu_{[j]}) = T_{\max}^{+}$, for all $j \ne 1$. Thus, only the best node transmits its timer packet even with g and a success will occur. (ii) *When* $f(\mu_{[2]}) \le T_{\max}$ *and* $|f(\mu_{[1]}) - f(\mu_{[2]})| > \Delta$*:* In this case, from (10), we get $g(\mu_{[2]}) \le T_{\max}$. Furthermore, from (10) and the definition of the floor function, $|g(\mu_{[1]}) - g(\mu_{[2]})| > \Delta$. Thus, even in this case, a success will occur and g will select the best node.

B. Brief Proof of Theorem 2

Define an auxiliary function $L_N^{\eta}(\boldsymbol{\alpha}_N)$ as

$$L_N^{\eta}(\boldsymbol{\alpha}_N) = P_N(\boldsymbol{\alpha}_N) + \eta \left(1 - p \sum_{j=0}^{N} \alpha_N[j]\right)^K, \quad \text{for } \eta \ge 0. \qquad (11)$$

The proof consists of two parts. In the first part, we derive the optimal $\tilde{\boldsymbol{\alpha}}_N$ that maximizes $L_N^{\eta}(\boldsymbol{\alpha}_N)$. In the second part, we show that $\tilde{\boldsymbol{\alpha}}_N$ also solves $\mathcal{OB}$ for an appropriate choice of η.

1) Derivation of $\tilde{\boldsymbol{\alpha}}_N$: After algebraic manipulations, $L_N^\eta(\boldsymbol{\alpha}_N)$ can be rewritten as follows:

$$L_N^\eta(\boldsymbol{\alpha}_N) = Kp\alpha_N[0]\left(1 - p\alpha_N[0]\right)^{K-1} + \left(1 - p\alpha_N[0]\right)^K \times L_{N-1}^\eta\left(\frac{\alpha_N[i]}{1 - p\alpha_N[0]}, \ldots, \frac{\alpha_N[N]}{1 - p\alpha_N[0]}\right). \quad (12)$$

Thus,

$$L_N^\eta(\boldsymbol{\alpha}_N) \leq Kp\alpha_N[0]\left(1 - p\alpha_N[0]\right)^{K-1} + \left(1 - p\alpha_N[0]\right)^K L_{N-1}^\eta(\tilde{\boldsymbol{\alpha}}_{N-1}), \quad (13)$$

where $\tilde{\boldsymbol{\alpha}}_{N-1} = (\tilde{\alpha}_{N-1}[0], \ldots, \tilde{\alpha}_{N-1}[N-1])$ maximizes L_{N-1}^η. Furthermore, from (12) and (13), given any $\alpha_N[0] \in (0,1)$, the upper bound in (13) can indeed be achieved by setting

$$\alpha_N[j] = \left(1 - p\alpha_N[0]\right)\tilde{\alpha}_{N-1}[j-1], \quad \text{for } 1 \leq j \leq N. \quad (14)$$

Hence, we have

$$\max_{\boldsymbol{\alpha}_N} L_N^\eta(\boldsymbol{\alpha}_N) = \max_{\alpha_N[0] \in (0,1)} \Big\{ Kp\alpha_N[0]\left(1 - p\alpha_N[0]\right)^{K-1} + \left(1 - p\alpha_N[0]\right)^K L_{N-1}^\eta(\tilde{\boldsymbol{\alpha}}_{N-1}) \Big\}. \quad (15)$$

Using the first order condition, it can be shown that this maximum is achieved when $\alpha_N[0] = \frac{1}{p}\left(\frac{1 - L_{N-1}^\eta(\tilde{\boldsymbol{\alpha}}_{N-1})}{K - L_{N-1}^\eta(\tilde{\boldsymbol{\alpha}}_{N-1})}\right)$. Thus, using (14), we see that $\tilde{\boldsymbol{\alpha}}_N$ is given by

$$\tilde{\alpha}_N[i] = \begin{cases} \frac{1}{p}\left(\frac{1 - L_{N-1}^\eta(\tilde{\boldsymbol{\alpha}}_{N-1})}{K - L_{N-1}^\eta(\tilde{\boldsymbol{\alpha}}_{N-1})}\right), & \text{if } i = 0 \\ \left(1 - p\tilde{\alpha}_N[0]\right)\tilde{\alpha}_{N-1}[j-1], & \text{if } 1 \leq i \leq N \end{cases}. \quad (16)$$

For $N = 0$, it can be easily shown that $\tilde{\alpha}_0[0] = \frac{1}{p}\left(\frac{1-\eta}{K-\eta}\right)$ maximizes $L_0^\eta(\alpha_0[0])$. Further, notice that $\tilde{\boldsymbol{\alpha}}_N = \tilde{\boldsymbol{\beta}}_N$ for $\eta = 0$.

2) Optimality of $\tilde{\boldsymbol{\alpha}}_N$: We shall say that an $(N+1)$-tuple of stair lengths $\boldsymbol{\alpha}_N$ is feasible if it satisfies the constraints in (5) and (6). If $\sum_{j=0}^N \beta_N[j] \leq 1$, then $\boldsymbol{\beta}_N$ is feasible. Furthermore, it solves problem $\mathcal{OB}$. Clearly, $\boldsymbol{\alpha}_N^* = \boldsymbol{\beta}_N$ in this case.

Consider now the case where $\sum_{j=0}^N \beta_N[j] > 1$. Thus, for $\eta = 0$, we are given that $\sum_{j=0}^N \tilde{\alpha}_N[j] = \sum_{j=0}^N \beta_N[j] > 1$. Furthermore, for $\eta = 1$, it can be shown that (11) is maximized when $\sum_{j=0}^N \tilde{\alpha}_N[j] = 0$. Using the intermediate value theorem [21], it then follows that there exists an $\eta \in (0,1)$ such that $\sum_{j=0}^N \tilde{\alpha}_N[j] = 1$.[2] Clearly, such an $\tilde{\boldsymbol{\alpha}}_N$ is also feasible.

Furthermore, for this choice of η, the auxiliary function is given by

$$L_N^\eta(\tilde{\boldsymbol{\alpha}}_N) = P_N(\tilde{\boldsymbol{\alpha}}_N) + \eta\left(1 - p\right)^K. \quad (17)$$

By definition, for any feasible $\boldsymbol{\alpha}_N$, we have $L_N^\eta(\tilde{\boldsymbol{\alpha}}_N) \geq L_N^\eta(\boldsymbol{\alpha}_N)$. From (11), this implies that

$$P_N(\tilde{\boldsymbol{\alpha}}_N) \geq P_N(\boldsymbol{\alpha}_N) + \eta\left[\left(1 - p\sum_{j=0}^N \alpha_N[j]\right)^K - (1-p)^K\right]. \quad (18)$$

[2]To apply the intermediate value theorem, we need to show that $\tilde{\alpha}_N[j]$, for $j = 0, 1, \ldots, N$, are continuous functions in η. This can be proved using induction. The steps are now shown due to space constraints.

Note that $\eta\left[\left(1 - p\sum_{j=0}^N \alpha_N[j]\right)^K - (1-p)^K\right] \geq 0$ since $\sum_{j=0}^N \alpha_N[j] \leq 1$ and $\eta \geq 0$. Therefore,

$$P_N(\tilde{\boldsymbol{\alpha}}_N) \geq P_N(\boldsymbol{\alpha}_N), \quad (19)$$

for every feasible $\boldsymbol{\alpha}_N$. Hence, $\boldsymbol{\alpha}_N^* = \tilde{\boldsymbol{\alpha}}_N$.

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