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Minimum Error Probability MIMO-Aided Relaying: Multi-Hop, Parallel and Cognitive Designs

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***Abstract*—A design methodology based on the minimum error probability (MEP) framework is proposed for a non-regenerative multiple-input multiple-output (MIMO) relay-aided system. We consider the associated cognitive, the parallel and the multi-hop source-relay-destination (SRD) link design based on this MEP framework, including the transmit precoder, the amplify-and-forward (AF) relay matrix and the receiver equalizer matrix of our system. It has been shown in the literature that MEP based communication systems are capable of improving the error probability of other linear counterparts. Our simulation results demonstrate that the proposed scheme indeed achieves a significant BER reduction over the existing linear schemes.**

***Index Terms*—LMMSE, MEP, MC, MIMO, Relay, Cognitive.**

I. Introduction

MIMO relaying is becoming an eminent and integral part of advanced wireless communication systems [1], owing to its capability of enhancing the received signal. The joint design of the transmitter of the relay and of the destination receiver along with the MIMO benefits has attracted tremendous research namely multi-hop relays, parallel relays and a relay-aided cognitive, have been considered by numerous researchers for tackling a range of challenges, including the coverage range extension [2], [3] and the careful choice of the best links from the entire set of legitimate links [4].

Numerous design criteria, such as the mean square error (MSE), the maximization of the capacity (MC) and various others, have been used for MIMO-aided relaying in the literature. For example, multi-hop relaying, which is capable of substantially extending the cellular coverage, has been designed relying on the MSE criterion [2], [3]. On the other hand, the so-called parallel relay configuration [4], which allows the best relay link to be selected from a set of parallel relay links used the MSE criterion for designing the relaying weights. Cognitive communications, where the bandwidth is judiciously shared between the primary and secondary users, has also been extended to the family of MIMO relay-aided systems [5], [6] using the MC criterion. However, a fundamental limitation of these criteria is that they are unable to achieve the minimum-error-probability (MEP), i.e the lowest bit-error-ratio (BER) in a linear detection framework [7]. Hence, the MEP based transceiver design criterion, also known as the minimum BER (MBER) method, is a more pertinent design criterion as far as the BER performance is concerned. Although, the benefits of the MEP-based MIMO-relaying system have already been demonstrated in [8] in terms of an SNR gain of upto $3-4$ dB, in this treatise our holistic CF is conceived in the above mentioned scenarios equipped with MIMO configurations for the first time.

Against this background, we propose to invoke the MEP optimization criterion as our objective function for jointly optimizing the transmit precoder (TPC), the amplify-and-forward (AF) MIMO-weights for the relays and the equalizer weights for the destination of three different relaying topologies - namely the multi-hop, the parallel and the cognitive relaying regimes. We develop the MEP based cost function (CF) for these three network topologies based on the classic QPSK and QAM signal constellation. We opted for the projected steepest descent (PSD) [9] optimization tool for finding the minimum of the CF. Our numerical simulations demonstrate that this criterion leads to significantly lower BER than its counterparts.

II. System Model

A. Cognitive MIMO-relay model

For the cognitive MIMO-relay, we consider a single-hop relaying system consisting of a source node (SN), a relay node (RN) and a destination node (DN) having N_s, N_r, and N_d antennas, respectively, as shown in Fig. 1. Let us assume that the primary user (PU), sharing the same bandwidth and having N_p receiver antenna suffers from interference from RN [5]. Let us denote that N_x is the length of the input vector $\mathbf{x} \in \mathbb{C}^{N_x \times 1}$ before the TPC operation at the SN, where $\mathbf{A}_s \in \mathbb{C}^{N_s \times N_x}$ is the TPC matrix. We denote $\mathbf{H}_{sr} \in \mathbb{C}^{N_r \times N_s}$, $\mathbf{H}_{rd} \in \mathbb{C}^{N_d \times N_r}$ and $\mathbf{H}_{rp} \in \mathbb{C}^{N_d \times N_r}$ as the SN-RN, RN-DN and SN-PU channel gain matrices, respectively. Let us denote the i.i.d AWGN vectors at the RN and DN as $\mathbf{v}_r \in \mathbb{C}^{N_r \times 1}$ and $\mathbf{v}_d \in \mathbb{C}^{N_d \times 1}$, with the variance of σ_r^2 and σ_d^2 for each component, respectively. Thus, the vector received at the RN is given by

$$\mathbf{r}_r = \mathbf{H}_{sr}\mathbf{A}_S\,\mathbf{x} + \mathbf{v}_r, \tag{1}$$

Let us denote the AF matrix by $\mathbf{A}_F \in \mathbb{C}^{N_r \times N_r}$. The power constraint at the RN is calculated as,

$$Tr\left[\mathbf{A}_F\left(\sigma_x^2\mathbf{H}_{sr}\mathbf{A}_S\mathbf{A}_S^H\mathbf{H}_{sr}^H + \sigma_r^2\mathbf{I}_{N_r}\right)\mathbf{A}_F^H\right] \le P_r, \tag{2}$$

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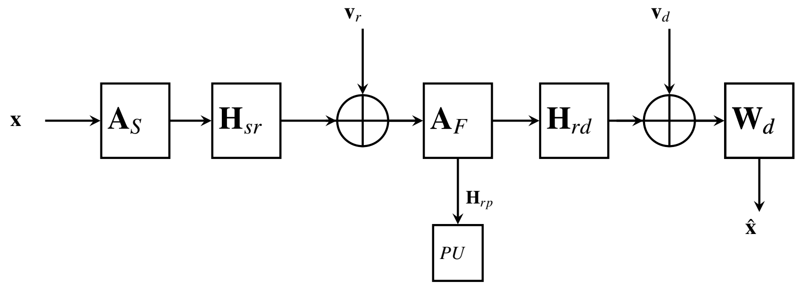

Fig. 1. Cognitive MIMO-relay system.

where P_r is the RN's transmit power and $\mathbb{E}\{\mathbf{x}\mathbf{x}^H\} = \sigma_x^2\mathbf{I}_{N_x}$. We also calculate the average interference (I_p) at the PU as

$$Tr\left[\mathbf{H}_{rp}\mathbf{A}_f\mathbf{A}_f^H\mathbf{H}_{rp}^H + \rho_1\mathbf{H}_{rp}\mathbf{A}_f\mathbf{H}_{sr}\mathbf{A}_s\mathbf{A}_s^H\mathbf{H}_{sr}^H\mathbf{A}_f^H\mathbf{H}_{rp}^H\right] \leq I_p/\sigma_r^2 \quad (3)$$

where, $\rho_1 = I_p/\sigma_r^2$. Similarly, we obtain the received signal at the DN as

$$\begin{aligned}\mathbf{r}_d &= \mathbf{H}_{rd}\mathbf{A}_F\mathbf{H}_{sr}\mathbf{A}_S\mathbf{x} + \mathbf{H}_{rd}\mathbf{A}_F\mathbf{v}_r + \mathbf{v}_d \\ &\triangleq \mathbf{H}\mathbf{x} + \mathbf{v}, \end{aligned} \quad (4)$$

where $\mathbf{H} \triangleq \mathbf{H}_{rd}\mathbf{A}_F\mathbf{H}_{sr}\mathbf{A}_S$ and $\mathbf{v} \triangleq \mathbf{H}_{rd}\mathbf{A}_F\mathbf{v}_r + \mathbf{v}_d$, while $\mathbf{v}_d$ is the noise at DN, which has a covariance matrix of $\sigma_d^2\mathbf{I}_{N_d}$. The effective noise $\mathbf{v}$ has a covariance matrix of $\mathbf{C}_v = \sigma_d^2\mathbf{I}_{N_d} + \mathbf{H}_{rd}\mathbf{A}_F\mathbf{A}_F^H\mathbf{H}_{rd}^H$. An equalizer matrix $\mathbf{W}_d \in \mathbb{C}^{N_d \times N_x}$ used at the DN would estimate the vector $\mathbf{x}$ by $\hat{\mathbf{x}} = \mathbf{W}_d^H\mathbf{r}_d$.

B. Parallel MIMO-relay model

For the parallel MIMO-relay, our final design goal is to select the *best* relay link from the set of K parallel relay links between the SN and the DN, as shown in Fig. 2. Let us assume that $\mathbf{H}_{sr}^k$, $\mathbf{H}_{rd}^k$ and $\mathbf{A}_{F,k}$ denote the SN-RN, RN-DN channels and the AF matrices (w.r.t kth. RN), respectively. The data received at the k^{th} relay after multiplication by the AF relaying-matrix is given by,

$$\mathbf{r}_{r,k} = \mathbf{A}_F\mathbf{H}_{sr,k}\mathbf{A}_S\mathbf{x} + \mathbf{A}_{F,k}\mathbf{v}_{r,k}, \quad (5)$$

with the power constraint formulated as

$$Tr\left[\mathbf{A}_{F,k}\left(\sigma_x^2\mathbf{H}_{sr,k}\mathbf{A}_S\mathbf{A}_S^H(\mathbf{H}_{sr,k})^H + \sigma_r^2\mathbf{I}_{N_r}\right)\right] \leq P_r. \quad (6)$$

We assume that each link has a maximum power budget of P_r. The data received at the DN from the k^{th} relay link is given by,

$$\mathbf{r}_{d,k} = \mathbf{H}_{rd,k}\mathbf{A}_{F,k}\mathbf{H}_{sr,k}\mathbf{A}_S\mathbf{x} + \mathbf{H}_{rd,k}\mathbf{A}_{F,k}\mathbf{v}_{r,k} + \mathbf{v}_d. \quad (7)$$

C. Multi-hop MIMO-relay model

For the multi-hop MIMO-relay scenario, we assume that there are K recursive single relays, as shown in Fig. 3. For simplicity, we assume having a single source and a destination node. The matrices $\mathbf{H}_{r,k} \in \mathbf{C}^{N_r \times N_r}$ and $\mathbf{A}_{F,k} \in \mathbf{C}^{N_r \times N_r}$ represent the $(k-1)^{th}$ to k^{th} relay link and the AF relaying-matrix of the k^{th} RN, respectively. We impose the power constraint of $P_{r,k}$ at the k^{th} RN. Hence, the signal received at the k^{th} RN after multiplication by the AF relaying-matrix becomes [2], [3]

$$\mathbf{r}_{f,k} = \prod_{i=1}^{k}(\mathbf{H}_{r,i}\mathbf{A}_{F,i})\mathbf{A}_S\mathbf{x} + \sum_{j=2}^{k}\left[\prod_{i=1}^{k}(\mathbf{H}_{r,i}\mathbf{A}_{F,i})\mathbf{v}_{r,j-1}\right] + \mathbf{v}_{r,k} \quad (8)$$

Similarly, the signal received at the DN is given by

$$\begin{aligned}\mathbf{r}_d &= \mathbf{H}_{rd}\mathbf{A}_{F,K}\prod_{k=1}^{K-1}(\mathbf{H}_{r,i}\mathbf{A}_{F,k})\mathbf{A}_S\mathbf{x} + \\ &\mathbf{H}_{rd,K-1}\mathbf{A}_{F,K-1} \times \left[\sum_{j=2}^{K-1}\left[\prod_{i=1}^{K-1}(\mathbf{H}_{r,i}\mathbf{A}_{F,i})\mathbf{v}_{r,j-1}\right] + \mathbf{v}_{r,K-1}\right] + \mathbf{v}_d. \\ &\triangleq \mathbf{H}\mathbf{x} + \mathbf{v}, \end{aligned} \quad (9)$$

where $\mathbf{H}$ and $\mathbf{v}$ are defined as follows

$$\mathbf{H} \triangleq \mathbf{H}_{rd}\mathbf{A}_{F,K}\prod_{k=1}^{K-1}(\mathbf{H}_{r,i}\mathbf{A}_{F,i})\mathbf{A}_S. \quad (10)$$

$$\mathbf{v} \triangleq \mathbf{H}_{rd,K-1}\mathbf{A}_{F,K-1} \times \left[\sum_{j=2}^{K-1}\left[\prod_{i=1}^{K-1}(\mathbf{H}_{r,i}\mathbf{A}_{F,i})\mathbf{v}_{r,j-1}\right] + \mathbf{v}_{r,K-1}\right] + \mathbf{v}_d.$$

The overall covariance matrix is then defined as

$$\mathbf{C}_v = \sum_{k=2}^{K}\sigma_k^2\left(\prod_{i=k}^{K}\mathbf{H}_{r,i}\mathbf{A}_{F,i}\right)\left(\prod_{i=k}^{K}\mathbf{H}_{r,i}\mathbf{A}_{F,i}\right)^H + \sigma_d^2\mathbf{I}_{N_d}. \quad (11)$$

We assume that the channel state information (CSI) required at various nodes as depicted in Table I. We assume that DN and the primary user send the CSI to the RN through feedback channel.

TABLE I
REQUIREMENT OF CSI AT VARIOUS NODES FOR MEP CRITERION BASED RELAY DESIGN.

Link	*SN*	*RN*	*DN*
SN-RN-DN		$\mathbf{H}_{sr}$, $\mathbf{H}_{rd}$, $\mathbf{H}_{rp}$	$\mathbf{H}_{rd}$

III. MEP COST FUNCTION (CF)

In the current context, the MEP CF directly minimizes the BER of the system at the DN. We start the formulation of the MEP CF with QPSK constellation and then extend it to the QAM case. Let us denote the symbol error ratio (SER) by $P_{e,i}$, when detecting x_i (the i^{th} component of $\mathbf{x}$) at the DN. With a slight 'abuse' of notation, we consider the SER here instead of BER, since the BER and SER are

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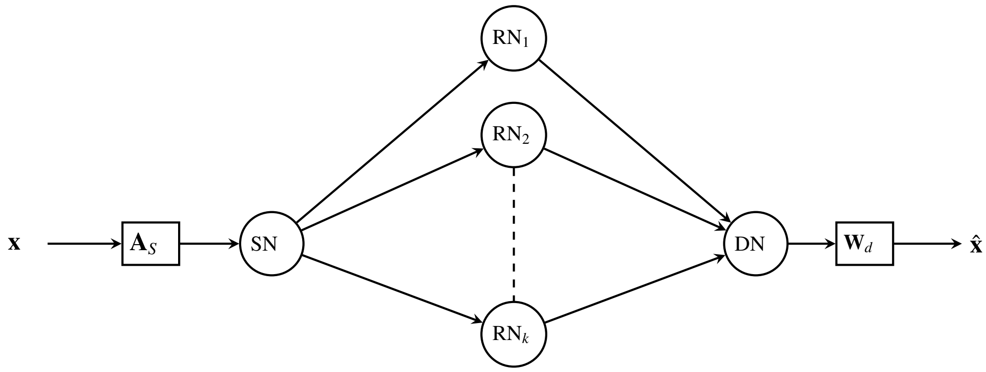

Fig. 2. Parallel MIMO-relay system.

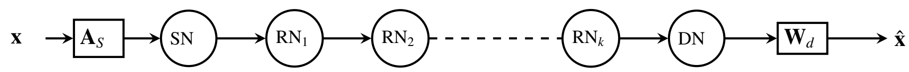

Fig. 3. Multi-hop MIMO-relay system.

approximately related to each other as SER $\approx \log_2(M)\times$BER in conjunction with grey coding. If every x_i is detected independently, the average probability of a symbol error associated with detecting the complete vector $\mathbf{x}$ is given by

$$P_e = \frac{1}{N_x}\sum_{i=1}^{N_x} P_{e,i}. \tag{12}$$

Let us denote $\mathbf{w}_i$ as the i^{th} column of the DN's equalizer matrix $\mathbf{W}_d$. Assume that $L = 2^{N_x}$ represents the total number of unique realizations of $\mathbf{x}$, while $\mathbf{x}_j$ is the j^{th} such realization of $\mathbf{x}$. For the Gaussian $Q(x)$-function, we use an approximation, which works well for a good range of x. This is given as [10]

$$Q(x) = K_c \exp\left(-\frac{m_c x^2}{2}\right). \tag{13}$$

where m_c is chosen from $1 \le x \le 2$ and K_c is function of m_c as defined in [10]. If $\hat{x}_i$ is the estimate of x_i for the QPSK constellation, we arrive at the expression of $P_{e,i}$ in (14) [8].

A. CF with M-QAM constellation

Here the CF formulation is extended to the general M-QAM constellation. Let us assume that $2a$ (for any $a > 0$) denotes the distance between two adjacent constellation points along either the real or the imaginary axis. The M-QAM constellation can be interpreted as two orthogonal PAM sequences of length $\sqrt{M}$. Therefore, the SER can be obtained as,

$$P_{e,i}^{QAM} = 1 - P_{c,i}^{R} \cdot P_{c,i}^{I}, \tag{15}$$

where $P_{c,i}^{R}$, $P_{c,i}^{I}$ represent the probability of correct decision along the real and imaginary axes, respectively. For computational simplicity, we assume that the decision region of each point along either the real or imaginary axis is bounded by $2a$, although this can be exceeded with a small probability. Let us define $L_1 = M^{\left(\frac{N_s-1}{2}\right)}$. Now, $P_{c,i}^{R}$, $P_{c,i}^{I}$ are derived in equations (16) and (18), respectively.

B. Optimization problem

We now have to obtain the optimal TPC weights as well as the AF and equalizer matrices by optimizing the CF. Hence, for the cognitive case, the optimization problem can be stated as

$$\begin{aligned}
\mathbf{A}_S^{mep}, \mathbf{A}_F^{mep}, \mathbf{W}_d^{mep} &= \underset{\mathbf{A}_S, \mathbf{A}_F, \mathbf{W}_d}{\arg\min}\; P_e(\mathbf{A}_S, \mathbf{A}_F, \mathbf{W}_d) \\
s.t\ (1)\ & Tr\left[\mathbf{A}_F\left(\sigma_x^2 \mathbf{C}_r^{-1}\mathbf{H}_{sr}\mathbf{A}_S\mathbf{A}_S^H\mathbf{H}_{sr}^H(\mathbf{C}_r^H)^{-1} + \mathbf{I}_{N_r}\right)\mathbf{A}_F^H\right] \le P_r \\
(2)\ & \sigma_x^2 Tr\{\mathbf{A}_S^H\mathbf{A}_S\} \le P_t, \\
(3) Tr & \left[\mathbf{H}_{rp}\mathbf{A}_f\mathbf{A}_f^H\mathbf{H}_{rp}^H + \rho_1\mathbf{H}_{rp}\mathbf{A}_f\mathbf{H}_{sr}\mathbf{A}_s\mathbf{A}_s^H\mathbf{H}_{sr}^H\mathbf{A}_f^H\mathbf{H}_{rp}^H\right] \le I_p/\sigma_r^2.
\end{aligned} \tag{20}$$

For the parallel relaying case, this is a two-step process. In the first step, we optimize each parallel link independently as per equation similar to (20) and then during the second step, we choose the specific link having the lowest value of the CF, i.e. the lowest P_e. For the multi-hop relaying case, the optimization problem is stated as follows

$$\begin{aligned}
\mathbf{A}_S^{mep}, \mathbf{A}_{F,k}^{mep}, \mathbf{W}_d^{mep} &= \underset{\mathbf{A}_S, \mathbf{A}_{F,k}, \mathbf{W}_d}{\arg\min}\; P_e(\mathbf{A}_S, \mathbf{A}_{F,k}\mathbf{W}_d) \\
s.t\ (1)\ & Tr\{\mathbf{A}_F\left(\sigma_x^2\mathbf{H}_{sr}\mathbf{H}_{sr}^H + \sigma_r^2\mathbf{I}_{N_r}\right)\mathbf{A}_F^H\} \le P_{r,k}, \\
(2)\ & \sigma_x^2 Tr\{\mathbf{A}_S^H\mathbf{A}_S\} \le P_t,\ (\text{for k } = 1, 2, \ldots, K.)
\end{aligned} \tag{21}$$

We have opted for the projected steepest descent (PSD) [9] for solving our constrained optimization problem, because it was found beneficial in [8]. The initial condition for all of them are chosen to be the LMMSE solution except for the cognitive case, where an MC based initial solution is chosen. This is because unless the matrices involved are strongly rank-deficient and hence non-invertible, it is reasonable to assume that the MEP solution will be in this neighborhood [8]. For the case of multi-hop relaying, even the simplest LMMSE solution has no closed-form expression. Hence, in that case, we opted for using a random initial condition for the LMMSE case and invoked the LMMSE solution for the MEP based one.

$$P_{e,i} = \frac{1}{2}\mathbb{E}_{\mathbf{x}}\left[Q\left(\frac{\Re\left[(\mathbf{w}_i)^H\mathbf{H}\mathbf{x}\right]\Re\{x_i\}}{\sqrt{\frac{1}{2}(\mathbf{w}_i)^H\mathbf{C}_v\mathbf{w}_i}}\right)Q\left(\frac{\Im\left[(\mathbf{w}_i)^H\mathbf{H}\mathbf{x}\right]\Im\{x_i\}}{\sqrt{\frac{1}{2}(\mathbf{w}_i)^H\mathbf{C}_v\mathbf{w}_i}}\right)\right] = \frac{1}{L}\left[\sum_{j=1}^{L}Q\left(\frac{\Re\left[(\mathbf{w}_i)^H\mathbf{H}\mathbf{x}_j\right]\Re\{x_i\}}{\sqrt{\frac{1}{2}(\mathbf{w}_i)^H\mathbf{C}_v\mathbf{w}_i}}\right)+Q\left(\frac{\Im\left[(\mathbf{w}_i)^H\mathbf{H}\mathbf{x}_j\right]\Im\{x_i\}}{\sqrt{\frac{1}{2}(\mathbf{w}_i)^H\mathbf{C}_v\mathbf{w}_i}}\right)\right],$$
$$\approx \frac{K_c}{L}\sum_{j=1}^{L}\exp\left(-\frac{m_c a_{1,j}^2}{2}\right)+\frac{1}{L}\sum_{j=1}^{L}\exp\left(-\frac{m_c a_{2,j}^2}{2}\right), \quad \text{where } a_{1,j} = \frac{\Re\left[(\mathbf{w}_i)^H\mathbf{H}\mathbf{x}_j\right]\Re\{x_i\}}{\sqrt{\frac{1}{2}(\mathbf{w}_i)^H\mathbf{C}_v\mathbf{w}_i}} \text{ and } a_{2,j} = \frac{\Im\left[(\mathbf{w}_i)^H\mathbf{H}\mathbf{x}_j\right]\Im\{x_i\}}{\sqrt{\frac{1}{2}(\mathbf{w}_i)^H\mathbf{C}_v\mathbf{w}_i}}. \tag{14}$$

$$P_{c,i}^R = \sum_{m=-(\sqrt{M}-1),\ odd}^{\sqrt{M}-1} Pr\{ma-a<y_r<ma+a\} = \frac{1}{L_1}\sum_{j=1}^{L_1}\sum_{m=-(\sqrt{M}-1)}^{\sqrt{M}-1}\left[Q\left(\frac{ma-a-\Re\left[(\mathbf{w}_i)^H\mathbf{H}\mathbf{x}_j\right]}{\sqrt{\frac{1}{2}(\mathbf{w}_i)^H\mathbf{C}_n\mathbf{w}_i}}\right)-Q\left(\frac{ma+a-\Re\left[(\mathbf{w}_i)^H\mathbf{H}\mathbf{x}_j\right]}{\sqrt{\frac{1}{2}(\mathbf{w}_i)^H\mathbf{C}_n\mathbf{w}_i}}\right)\right]. \tag{16}$$

$$\approx \frac{1}{L_1}\sum_{j=1}^{L_1}\sum_{m=-(\sqrt{M}-1)}^{\sqrt{M}-1}\left[\exp\left(\frac{-m_c a_{j,m}^2}{2}\right)-\exp\left(\frac{-m_c b_{j,m}^2}{2}\right)\right], \quad a_{j,m} = \frac{ma-a-\Re\left[(\mathbf{w}_i)^H\mathbf{H}\mathbf{x}_j\right]}{\sqrt{\frac{1}{2}(\mathbf{w}_i)^H\mathbf{C}_n\mathbf{w}_i}}, b_{j,m} = \frac{ma+a-\Re\left[(\mathbf{w}_i)^H\mathbf{H}\mathbf{x}_j\right]}{\sqrt{\frac{1}{2}(\mathbf{w}_i)^H\mathbf{C}_n\mathbf{w}_i}}. \tag{17}$$

$$P_{c,i}^I = \frac{1}{L_1}\sum_{j=1}^{L_1}\sum_{m=-(\sqrt{M}-1),\ m\ odd}^{\sqrt{M}-1}\left[Q\left(\frac{ma-a-\Im\left[(\mathbf{w}_i)^H\mathbf{H}\mathbf{x}_j\right]}{\sqrt{\frac{1}{2}(\mathbf{w}_i)^H\mathbf{C}_n\mathbf{w}_i}}\right)-Q\left(\frac{ma+a-\Im\left[(\mathbf{w}_i)^H\mathbf{H}\mathbf{x}_j\right]}{\sqrt{\frac{1}{2}(\mathbf{w}_i)^H\mathbf{C}_n\mathbf{w}_i}}\right)\right]. \tag{18}$$

$$\approx \frac{1}{L_1}\sum_{j=1}^{L_1}\sum_{m=-(\sqrt{M}-1),odd}^{\sqrt{M}-1}\left[\exp\left(\frac{-m_c c_{j,m}^2}{2}\right)-Q\left(\frac{-m_c d_{j,m}^2}{2}\right)\right], c_{j,m} = \frac{ma-a-\Im\left[(\mathbf{w}_i)^H\mathbf{H}\mathbf{x}_j\right]}{\sqrt{\frac{1}{2}(\mathbf{w}_i)^H\mathbf{C}_n\mathbf{w}_i}}, \quad d_{j,m} = \frac{ma+a-\Im\left[(\mathbf{w}_i)^H\mathbf{H}\mathbf{x}_j\right]}{\sqrt{\frac{1}{2}(\mathbf{w}_i)^H\mathbf{C}_n\mathbf{w}_i}}. \tag{19}$$

C. Computational complexity

Let us now approximate the computational complexity of the relay link designs using the MEP CF. We characterize it in terms of the number of operations, which can be additions, subtractions and multiplications. The results have been extrapolated from [8]. For the case of parallel relaying, the results remain similar to [8], except we need to incur an additional cost of $O\log K$ for searching best link. Hence, we present the complexity results only for the cognitive and for the multi-hop relaying.

Let us assume that N_Q represents the approximate number of operations required for computing the $Q(\cdot)$-function, which can be accurately approximated as Taylor series. The computational complexity of the LMMSE solution conceived for the multi-hop scenario has not been analyzed in the literature. We approximate it as $N_{itn}(K(8N_s-2)N_s^2 + 29N_s + 3 + K(8N_r-2)N_r^2 + 2N_r + (8N_s-2)N_rN_s + (32N_s^3 + 60N_s^2 - 14N_s)/3 + (8N_s-2)N_dN_s + (8N_d-2)N_sN_d + 2N_sN_d + 4N_d^2 + (32N_d^3 + 60N_d^2 - 14N_d)/3 + 3\min(N_d, N_r, N_s)2N_dN_s + K(8N_r-2)N_rN_d + N_d)$, where N_{itn} is the average number of iterations used by our optimization method. Note that even the LMMSE solution has no closed-form expression for the multi-hop scenario. Finally, the complexity is presented in Table II. A typical comparison curve is presented in Fig. 4 for the multi-hop relay design varying N_d.

TABLE II
COMPUTATION COMPLEXITY OF THE PROPOSED MEP METHODS (MULTI-HOP AND COGNITIVE) WITH M-QAM CONSTELLATION.

Type of Relay	Approximate complexity number
Cognitive	$N_{itn}(3\min(N_d, N_r, N_s)$ $+2N_dN_s + (22N_r-2)N_rN_dNM^{N_x} + M^{N_x}N_d^2$ $+M^{N_x}N_s(8N_d^2 + 17N_d) + 4N_dN_sN_s$ $+6N_sM^{N_x}N_Q + 18N_r + N_s + 12 + N_d)$
Multi-hop	$N_{itn}(M^{N_x}K(14N_r^2 + N_sN_d)$ $+4N_dN_sN_x + 4N_sN_d^2 + 2N_sN_d$ $+M^{N_x}(32KN_d^3 + 60KN_d^2 - 14N_d)/3$ $+M^{N_x}(8N_s-2)N_dN_s + (8N_d-2)N_sN_d$

IV. Numerical results

Let us now study the BER performance of the proposed method against LMMSE/MC methods for all the above-mentioned MIMO-relay configurations. We consider a non-dispersive Rayleigh fading i.i.d channel with unit variance for each complex element of the channel matrix of the various links. We have used perfect channel for our simulation. The RN's SNR is defined as $\text{SNR}_1 = 10\log_{10}\left(\frac{\sigma_x^2}{\sigma_1^2}\right)$ dB, where σ_x^2 is the power of each x_i, which is set to $\frac{P_t}{N_x}$. The DN's SNR is defined as $\text{SNR}_2 = 10\log_{10}\left(\frac{P_r}{N_r\sigma_2^2}\right)$ dB. The SNR_1 is kept at 20,5 dB. $I_p/\sigma_r^2 = 1$ dB. Our simulation results are averaged over 1000 channel realizations per SNR value. We summarize the simulation parameters in Table III. In this work, we have designed only the SN-RN-DN link of the various configurations.

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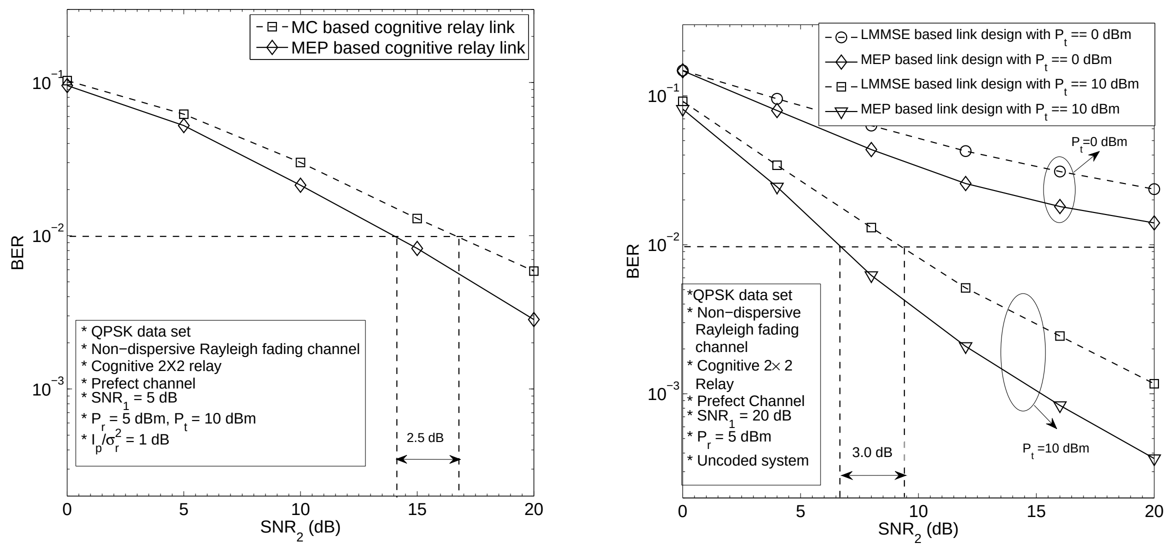

(a) BER performance with $P_t = 10$ dBm and $\text{SNR}_1 = 5$ dB. (b) BER performance with $P_t = 0, 10$ dBm.

Fig. 5. BER vs. SNR_2 performance of the SRD link design for a cognitive MIMO-relay based on the MEP method along with the MC method [5] over a flat Rayleigh fading channel without the channel estimation. $N_s, N_r, N_d, N_p = 2$, P_r is constrained to 5 dBm as shown in Table-III

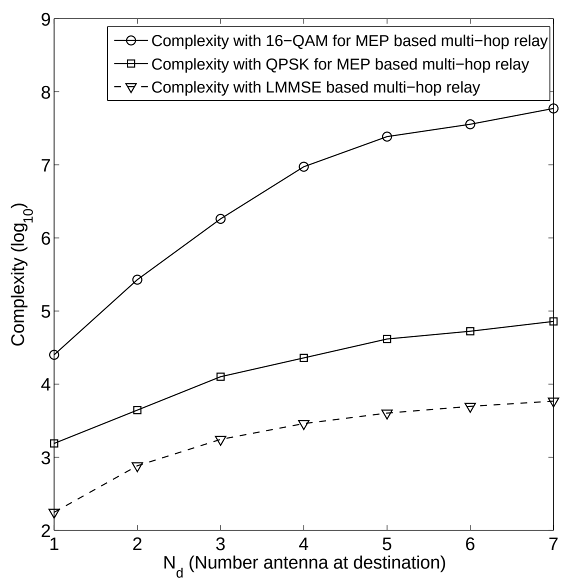

Fig. 4. A typical complexity comparison between the LMMSE and MEP methods for multi-hop relay design, varying the N_d only.

Cognitive relay: This characterizes our cognitive relay link design based on the BER performance of the proposed MEP method against that of the MC benchmark [5]. It can be observed in Fig. 5a ($\text{SNR}_1 = 5$ dB) that the MEP method achieves a BER of 10^{-2} at the SNR of ≈ 14.2 dB, whereas its MC counterpart achieves the same BER at the SNR of ≈ 16.7 dB. Hence, the MEP based relay design attains an overall SNR gain of about 2.5 dB at the BER of 10^{-2}. This gain is further increased for higher SNRs. As expected, the BER performance is poorer for $P_t = 0$ dBm, as observed

TABLE III
SIMULATION PARAMETERS.

Parameter Name	Value
N_x, N_s, N_r, N_d, N_p	2
P_t	0 dBm,10 dBm
P_r (Each relay link)	5 dBm
Constellation	QPSK, QAM
SNR_1 (Each Relay link)	$20, 5$ dB
K	4(Parallel), 2(Multi-hop)

in Fig. 5b. Fig. 6 shows a capacity comparison. We observe that the capacity of the MEP method is poorer as expected.

Parallel relay: This solution relies on finding the best link from the set of parallel relay links using $K = 4$. For each link, we have kept the total relay power at 5 dBm. It can be observed in Fig. 7a that the MEP method attains the BER of 10^{-3} at the SNR of about 10.2 dB, whereas its LMMSE counterpart achieves the same BER at the SNR of ≈ 13 dB. Hence, the MEP based relay design attains an overall SNR gain of about ≈ 2.8 dB at the BER of 10^{-3}.

Multi-hop relay: Let us now embark on characterizing a multi-hop MIMO relay link with 16-QAM constellation. We opted for $N_r = 2$ for all the intermediate RNs. We have chosen $K = 2$, i.e two serial relay links. For each link, we have kept the total relay power at 5 dBm. It can be observed in Fig. 7b that the MEP method attains the BER of $2{\times}10^{-2}$ at the SNR of about 17.5 dB, whereas its LMMSE counterpart achieves the same BER at the SNR of ≈ 20 dB. Hence, the MEP based relay design attains an overall SNR gain of almost 2.6 dB at the BER of 2×10^{-2}.

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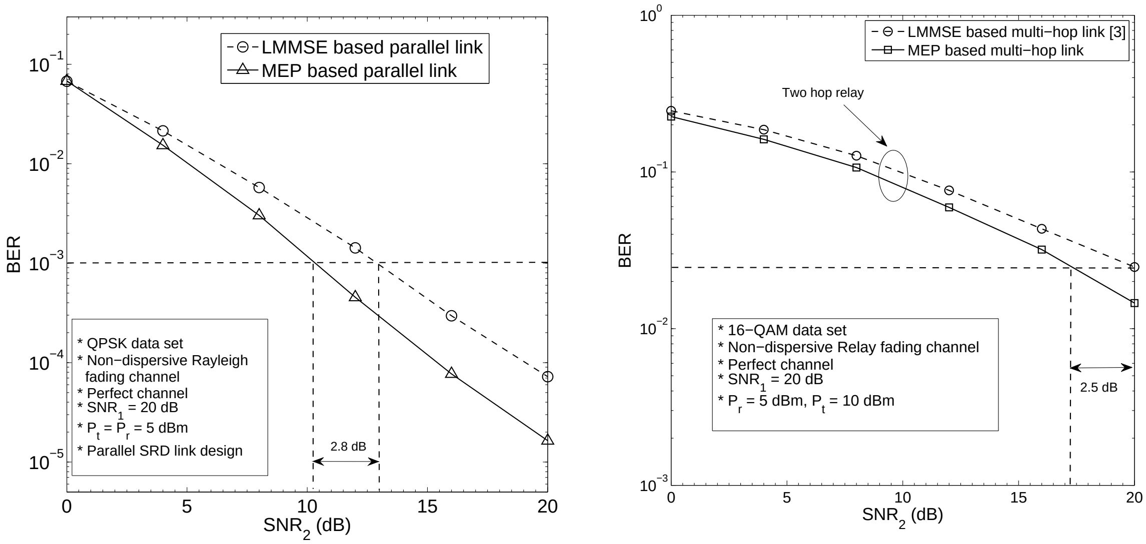

(a) BER vs. SNR_2 performance of the SRD link design for a 4-parallel MIMO-relay based on the MEP method along with the LMMSE method over a flat Rayleigh fading channel. $N_s, N_r, N_d = 2$, P_r at each RN is constrained to 5 dBm and SNR_1 is 20 dB as shown in Table-III.

(b) BER vs. SNR_2 performance of the SRD link design for a multi-hop MIMO-Relay link based on the MEP method along with 16-QAM constellation over a flat Rayleigh fading channel. $N_s, N_r, N_d = 2$, P_r at each RN is constrained to 5 dBm and SNR_1 is 20 dB as shown in Table-III.

Fig. 7. BER vs. SNR_2 performance of the SRD link design for a parallel and a multi-hop relay systems. The parameters are defined in Table-III.

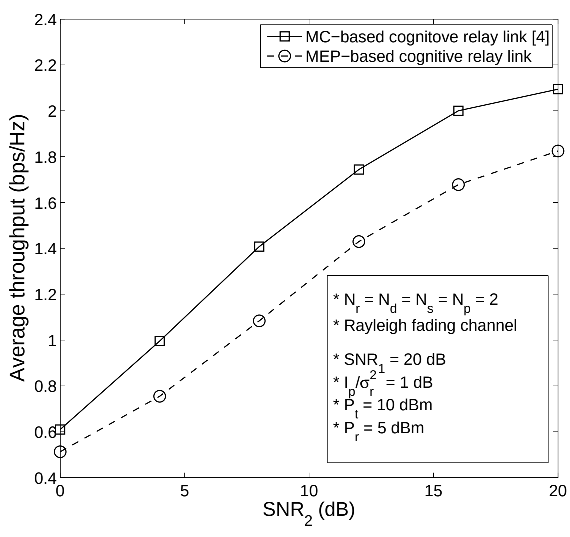

Fig. 6. Capacity comparison for MEP and MC based cognitive system with $SNR_1 = 20$ dB.

V. Conclusions

In this correspondence, we have extended the MEP based framework to the design of various types of relaying configurations. We have considered cognitive, parallel and multi-hop relaying. Cost functions have been developed and optimization frameworks have been conceived. Numerical simulations have shown considerable BER performance improvements in all these cases.

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