Best-M Feedback in OFDM: Base-Station-Side Estimation and System Implications

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***Abstract*—Reduced feedback schemes play a critical role in orthogonal frequency division multiplexing-based cellular systems because they facilitate scheduling and rate adaptation by the base station (BS) while reducing the number of subchannels for which channel state information is fed back by the users. We address the problem of reliable transmission even on subchannels that are fed back by a few users due to feedback constraints. For the practically relevant best-M feedback scheme, in which each user reports only its M strongest subchannels to the BS, we derive a nonlinear constrained minimum mean square error estimator that enables the BS to estimate the signal-to-noise-ratios of all the subchannels of every user. We then propose two lower computational complexity approaches that incur a negligible loss in performance. The novelty of these approaches lies in their exploitation of the structure of the best-M feedback information and the correlation among subchannel gains. Applications to general channel models and to quantized feedback are also shown. In terms of system-level impact, the proposed approaches improve the cell throughput compared to several conventional approaches – without requiring any additional feedback – for uncorrelated and correlated subchannels, and for various schedulers.**

***Index Terms*—Best-M feedback, Estimation, Mean square error, OFDM, Rate adaptation, Scheduling.**

I. INTRODUCTION

TECHNIQUES such as channel-aware scheduling and rate adaptation have enabled orthogonal frequency division multiplexing (OFDM)-based cellular systems to offer high data rates to many users [1]. In these systems, several contiguous subcarriers are grouped into subchannels, with the bandwidth of a subchannel typically being less than the coherence bandwidth of the channel. For each subchannel, the base station (BS) determines the user to transmit to and the corresponding rate. For example, in the Long Term Evolution (LTE) standard, twelve subcarriers are grouped together into a physical resource block (PRB) of bandwidth 180 kHz [2], which is the basic unit of resource allocation.

In order to implement user scheduling and rate adaptation on the downlink, the BS ideally needs to know the signal-to-noise ratios (SNRs) of all the users for all the subchannels. This channel state information (CSI) needs to be fed back by the users to the BS in frequency-division duplexing systems since the uplink and downlink channels are not reciprocal. Feedback is also necessary in time-division duplexing systems when the uplink and downlink interferences or the transmit and receive radio frequency chains are not symmetric. Such a large feedback overhead, which increases as the number of users or subchannels increases, lowers the overall spectral efficiency of the system and can overwhelm the uplink.

In order to markedly reduce the feedback overhead, several schemes have been proposed in the OFDM literature. They either restrict the number of subchannel gains that are fed back by a user or the number of users that feed back CSI or both. We first briefly survey these schemes, their limitations, and how they have been addressed. We shall say that a user has *reported* a subchannel when it has fed back CSI about it.

A. Literature on Limited Feedback Schemes

In the threshold-based feedback scheme in [3], a user reports a subchannel only if the subchannel's SNR exceeds a threshold. In [4], only one bit is fed back for a subchannel to indicate whether its SNR is above the threshold or not. Another practically important scheme is the best-M scheme [5]–[9]. In it, each user reports the CSI of its M largest subchannels. Typically, M is much smaller than the number of subchannels in order to significantly reduce the feedback overhead [10], [11]. In [12], the best-M scheme is combined with a feedback scheme in which a user feeds back the CSI of a subchannel only if the throughput achievable on it exceeds a threshold. In [13], subcarriers are clustered into groups and only one bit is reported for a group if all its subcarrier SNRs exceed a pre-specified threshold. Subcarrier clustering for multiple antenna systems is studied in [14]. A contention-based random access protocol is instead proposed in [15] to limit the number of users that feed back CSI. Subcarrier clustering and a threshold-based contention scheme are considered together in [13]. In [16], the users adjust their feedback depending on the number of subchannels allocated to them in the past.

B. Shortcomings of Limited Feedback Schemes and Solutions

Reducing the feedback overhead increases the odds that some of the subchannels are not reported by any of the users. In [4]–[6], [8], [9], [13], [17], when no user reports a subchannel, the BS does not transmit data on that subchannel as it does not know the gain of that subchannel for any user. This reduces the spectral efficiency. Such a scenario can arise even in the contention-based scheme of [15] if none of the users successfully transmit during the contention period. To address this

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shortcoming, in [3], the BS requests feedback from all the users when no user reports a subchannel. However, this increases the feedback delay and entails an additional signaling overhead. This problem is partially addressed in [18] using a technique called *Data method*. Instead of no transmission, it assigns a subchannel not reported by any user to the user selected for an adjacent subchannel, but transmits at a rate lower than that for the adjacent subchannel.

The second problem is that the approaches in [4]–[6], [8], [9], [13], [17] only select a user for a subchannel from among those that reported it. This limits the ability of the BS to exploit multi-user diversity when few users report a subchannel.

C. Focus and Contributions

In this paper, we address the relatively less studied problem of transmitting reliably on subchannels regardless of the number of users that report them. Specifically, for the best-M scheme, we develop a novel minimum mean square error (MMSE) estimator that enables the BS to estimate the SNRs of the unreported subchannels. It improves the effectiveness of rate adaptation and scheduling at the BS. Our approach incorporates the CSI fed back by the best-M scheme and the correlation across subchannels, which can be significant even in multipath channels that are considered dispersive [9], to methodically estimate and transmit on unreported subchannels. This is relevant for fourth generation cellular standards such as LTE, which employ a variant of the best-M scheme [2].

We make the following specific contributions:

- *Closed-Form MMSE Estimator:* We first derive in closed-form the MMSE estimator for the SNR of an unreported subchannel, when the subchannel gains are independent. We then generalize our approach to correlated subchannel gains for the exponential correlation model [19]–[21]. The derivation of the estimator exploits the conditional independence of the subchannel SNRs, which is referred to as the *Markov property* in [21]. It enables the estimator to be written in terms of at most three of the reported SNRs instead of all M of them.
- *Lower Computational Complexity Techniques*: We develop two approaches to significantly reduce the computational complexity of the above estimator:
  1) *Windowing Approach:* This approach exploits the exponentially decaying nature of the subchannel correlation, and neglects the correlation between subchannels that are separated by at least N_w subchannels. Here, N_w is a parameter that trades off between computational complexity and accuracy. Only an $(N_w + 1)$-dimensional joint PDF, instead of an N-dimensional joint PDF, is needed to compute the estimates.
  2) *Nearest Reported Subchannel Reduction (NRSR) Approach:* In it, the estimation problem is reduced to one consisting of only two correlated subchannels, namely, the unreported subchannel and the reported subchannel nearest to it. As we show, this incurs a negligible degradation in mean square error (MSE) compared to the MMSE estimator even at high correlations.
- *Extension to Arbitrary Subchannel Correlation Models:* Obtaining the MMSE estimator for an arbitrary subchannel correlation model is, in general, intractable because the joint PDF of the subchannel SNRs involves multiple integrals [19]. However, we show that the NRSR approach is applicable to any channel power delay profile (PDP). Results are shown for the widely used typical urban (TU) and rural area (RA) channels [2].
- *System Impact and Benchmarking:* We then study the system-level cell throughput implications of the proposed estimators, which enable the BS to determine the transmit rate for unreported subchannels. This allows the scheduler at the BS to possibly assign a subchannel to a user who could not report the subchannel due to feedback constraints. For three frequency-domain schedulers, which span a wide range of the trade-off between cell throughput and fairness, the proposed approaches improve the cell throughput compared to the approaches pursued in [5], [6], [8], [18]. Notably, these gains are obtained without any additional feedback.
- *Quantized Feedback:* We also extend the NRSR approach to handle quantized feedback, which is used in practice, and show that it incurs a negligible loss in cell throughput.

D. Differences With Conventional OFDM Channel Estimation and Related Works

We note that our problem is different from the classical problem of estimating the subchannel gains in an OFDM system using pilots transmitted by the BS in some pre-specified subchannels [22]. There, the MMSE estimator of the subchannel gain is a linear function of the signals received on the pilot subchannels since the complex baseband subchannel gains are jointly Gaussian. However, in our problem, the MMSE estimator turns out to be a non-linear function of the observations. The analytical techniques that lead to it are also quite different. An approach with a motivation similar to ours is studied in [23], where the transmission parameters are optimized to maximize the throughput when temporal subsampling is used to reduce the feedback. Since the limited feedback scheme in [23] is different from ours, the modeling and analysis in it are different as well.

E. Organization and Notation

The paper is organized as follows. The system model is discussed in Section II. The MMSE estimator is derived in Section III. The system throughput implications are presented in Section IV. Our conclusions follow in Section V.

Notations: The probability of an event A is denoted by $\mathbf{P}[A]$. The PDF of a random variable (RV) X is denoted by $f_X(\cdot)$, and the conditional PDF of RV X given $Y = y$ by $f_X(\cdot|Y = y)$. Expectation with respect to X is denoted by $\mathbb{E}_X[\cdot]$ and the expectation conditioned on an event A by $\mathbb{E}_X[\cdot|A]$. The subscript is dropped when obvious from context. The notation $p(X = x, A)$ is defined

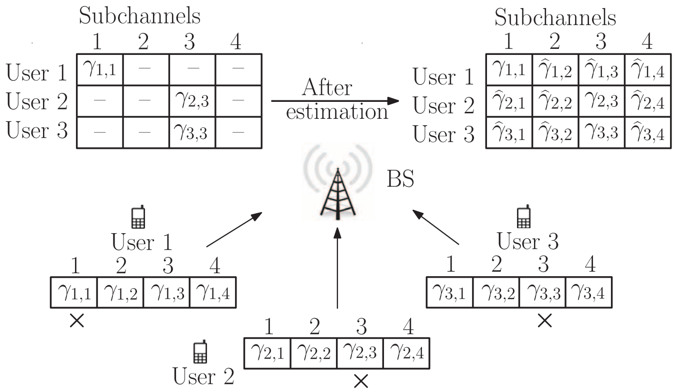

Fig. 1. Illustration of best-M feedback by $K = 3$ users and estimation at the BS ($N = 4$ and $M = 1$). Cross marks ($\times$) denote the subchannels that are fed back.

as $\lim_{\Delta x \to 0} \mathbf{P}[x < X \leq x + \Delta x, A]/\Delta x$. Further, $|c|$ and c^* denote the absolute value and the complex conjugate of c, respectively. The complement of a set $\mathcal{A}$ is denoted by $\mathcal{A}^c$.

II. System Model

Consider a cell with K users. The system bandwidth is divided into N orthogonal subchannels. The BS and the users are equipped with a single antenna. We focus on the single antenna case since BS-side estimation for the best-M scheme is novel and challenging even for this case. The system model is illustrated in Fig. 1.

A. Channel Model

For subchannel n, let $H_{k,n}$ denote the complex baseband channel gain from the BS to user k. We assume Rayleigh fading. Thus, $H_{k,n}$ is a circularly symmetric complex Gaussian RV with zero mean and variance $2\sigma_k^2$. The subchannel SNRs are statistically identical but correlated, which follows from the uncorrelated scatterers assumption that is prevalent in most channel models [1]. Therefore, $\gamma_{k,n} = |H_{k,n}|^2$, for $1 \leq n \leq N$, are exponential RVs with mean $2\sigma_k^2$, which captures the effect of thermal noise and inter-cell interference. Let $\Gamma_k = [\gamma_{k,1}, \ldots, \gamma_{k,N}]$. The subchannel gains are assumed to be independent across the users because the users are located sufficiently far apart from each other. However, they need not be statistically identical since the users are at different distances from the BS or their noise variances are different.

We assume the exponential correlation model for the subchannel baseband gains [21]. As per this model, the covariance between $H_{k,n}$ and $H_{k,m}$ is given by $\mathbb{E}\left[H_{k,n}H_{k,m}^*\right] = 2\sigma_k^2\rho^{|n-m|}$, where ρ is the correlation coefficient. This correlation model is often used in the literature [19]–[21] because it is tractable and it captures the decrease in correlation between the subchannels as their separation increases. Subsequently, in Section III-D, we propose an approach that can accommodate any correlation model.

The joint PDF of N correlated subchannel SNRs Γ_k of a user k is given by [21]

$$
\begin{aligned}
f_{\Gamma_k}(x_1, \ldots, x_N) &= \frac{\exp\left(-\frac{1}{2\sigma_k^2(1-\rho^2)}\left[x_1 + x_N + (1+\rho^2)\sum_{i=2}^{N-1} x_i\right]\right)}{2^N \sigma_k^{2N}(1-\rho^2)^{N-1}} \\
&\quad \times \sum_{n=0}^{\infty} \frac{\delta^{2n}}{4^n} \sum_{\substack{0 \leq l_1 \leq \cdots \leq l_{N-1} \leq n \\ l_1 + l_2 + \cdots + l_{N-1} = n}} \frac{x_1^{l_1} x_2^{l_1+l_2} \ldots x_{N-1}^{l_{N-2}+l_{N-1}} x_N^{l_{N-1}}}{(l_1! l_2! \ldots l_{N-1}!)^2}, \\
&\quad \text{for } x_i \geq 0,\ i = 1, \ldots, N, \qquad (1)
\end{aligned}
$$

where $\delta = \rho/(\sigma^2(1-\rho^2))$. For $\rho = 0$, the subchannel SNRs are mutually independent. For $\rho > 0$, the subchannel SNRs exhibit the following Markov property [21], which we shall exploit in this paper. Let $\Gamma_k^{-j} = [\gamma_{k,1}, \ldots, \gamma_{k,j-1}, \gamma_{k,j+1}, \ldots, \gamma_{k,N}]$. Conditioned on $\gamma_{k,j}$, for $1 < j < N$, the joint PDF of Γ_k^{-j} factors as follows:

$$
\begin{aligned}
& f_{\Gamma_k^{-j}}(x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_N | \gamma_{k,j} = x_j) \\
&= f_{\gamma_{k,1}, \ldots, \gamma_{k,j-1}}(x_1, \ldots, x_{j-1} | \gamma_{k,j} = x_j) \\
&\quad \times f_{\gamma_{k,j+1}, \ldots, \gamma_{k,N}}(x_{j+1}, \ldots, x_N | \gamma_{k,j} = x_j). \qquad (2)
\end{aligned}
$$

The statistical parameters σ and ρ are assumed to be known at the BS. The BS can either obtain them from the users via infrequent feedback, since these are slowly varying quantities, or can learn them from the uplink channel measurements by exploiting *statistical reciprocity* because the fading distributions of the uplink and downlink channels are the same.

B. Best-M Feedback Scheme

In it, each user orders its subchannels according to their SNRs. For a user k, the ordered subchannel SNRs are denoted as $\gamma_{k,i_1} \geq \gamma_{k,i_2} \geq \cdots \geq \gamma_{k,i_N}$, where i_r indexes the subchannel with the r^{th} largest SNR. The users are assumed to know their subchannel SNRs without error [3]–[7], [13]. In LTE, for example, these are obtained using the common and dedicated pilots that are periodically transmitted by the BS. User k then feeds back its M largest subchannel SNRs, $\gamma_{k,i_1}, \ldots, \gamma_{k,i_M}$, along with their subchannel indices $i_1, \ldots, i_M$ to the BS [5], [9], [17]. Extension to quantized feedback is investigated in Section III-E.

C. Simplifications and Discussion

We note that while LTE motivates several aspects of the model studied in this paper, not all aspects of LTE are modeled. For example, acknowledgement (ACK), no ACK (NACK) feedback, hybrid automatic repeat request (HARQ), outer loop link adaptation, wideband channel quality feedback [24], and ensuring that same modulation and coding scheme (MCS) is used on all the PRBs allocated to a scheduled user are not modeled. Such simplifications are necessary in order to arrive at a tractable and analytically insightful model and have also been made in [6], [7], [9], [17], [23].

III. MMSE Estimator at BS for Best-M Feedback

In this section, we develop an MMSE estimator for the SNR of an unreported subchannel for any given user. For ease of exposition, we shall drop the user index from the subscripts in this section. Consequently, let $\mathcal{I}_M = \{i_1, i_2, \ldots, i_M\}$ denote the set of indices of the reported subchannels, and the reported SNRs are $\gamma_{i_1} = x_{i_1}, \ldots, \gamma_{i_M} = x_{i_M}$. Given these, the MMSE estimator $\widehat{\gamma}_j$ of the SNR of an unreported subchannel $j \in \mathcal{I}_M^c$ can be shown from first principles to be the following conditional mean [25]:

$$\widehat{\gamma}_j = \mathbb{E}\left[\gamma_j | \gamma_u = x_u, \forall u \in \mathcal{I}_M; \gamma_v < x_{i_M}, \forall v \in \mathcal{I}_M^c\right]. \quad (3)$$

We shall refer to it as the *constrained MMSE (CMMSE)* estimator. It incorporates the *ordering information*, which is that γ_j is less than the lowest reported SNR x_{i_M}, and the *correlation information*, which is obtained from the conditioning on the SNRs of other correlated subchannels.

Using the theorem of expectation, we get

$$\widehat{\gamma}_j = \int_0^\infty y f_{\gamma_j}\left(y | \gamma_u = x_u, \forall u \in \mathcal{I}_M; \gamma_v < x_{i_M}, \forall v \in \mathcal{I}_M^c\right) dy, \quad (4)$$

where $f_{\gamma_j}\left(y | \gamma_u = x_u, \forall u \in \mathcal{I}_M; \gamma_v < x_{i_M}, \forall v \in \mathcal{I}_M^c\right)$ is the PDF of the unreported subchannel SNR γ_j conditioned on the reported subchannel SNRs and their indices, and the fact that the SNRs of the unreported subchannels are less than x_{i_M}. Using Bayes' rule, we get

$$\widehat{\gamma}_j = \frac{\int_0^\infty y p\left(\gamma_j = y, \gamma_u = x_u, \forall u \in \mathcal{I}_M, \gamma_v < x_{i_M}, \forall v \in \mathcal{I}_M^c\right) dy}{p\left(\gamma_u = x_u, \forall u \in \mathcal{I}_M, \gamma_v < x_{i_M}, \forall v \in \mathcal{I}_M^c\right)}. \quad (5)$$

We now derive the CMMSE estimator in closed-form.

A. Independent Subchannel Gains

To build intuition, we first study the scenario in which the subchannel gains are independent.

Result 1: The CMMSE estimator $\widehat{\gamma}_j$ of subchannel $j \in \mathcal{I}_M^c$, given the set of reported indices $\mathcal{I}_M$ and the corresponding SNRs $\gamma_{i_1} = x_{i_1}, \ldots, \gamma_{i_M} = x_{i_M}$, is given by

$$\widehat{\gamma}_j = 2\sigma^2 - \frac{x_{i_M}}{\exp\left(\frac{x_{i_M}}{2\sigma^2}\right) - 1}. \quad (6)$$

Proof: Since the subchannel SNRs are independent, the joint probability terms in (5) factor as

$$\widehat{\gamma}_j = \frac{\int_0^\infty y p\left(\gamma_j = y, \gamma_j < x_{i_M}\right)\left[\prod_{u \in \mathcal{I}_M} p\left(\gamma_u = x_u\right)\right] dy}{p\left(\gamma_j < x_{i_M}\right)\left[\prod_{u \in \mathcal{I}_M} p\left(\gamma_u = x_u\right)\right]} \times \frac{\prod_{v \in \mathcal{I}_M^c, v \neq j} p\left(\gamma_v < x_{i_M}\right)}{\prod_{v \in \mathcal{I}_M^c, v \neq j} p\left(\gamma_v < x_{i_M}\right)}. \quad (7)$$

Canceling common terms, we get

$$\widehat{\gamma}_j = \frac{\int_0^\infty y p\left(\gamma_j = y, \gamma_j < x_{i_M}\right) dy}{p\left(\gamma_j < x_{i_M}\right)}. \quad (8)$$

Since subchannel SNR γ_j is an exponential RV with mean $2\sigma^2$, $p\left(\gamma_j < x_{i_M}\right)$ and $\int_0^\infty y p\left(\gamma_j = y, \gamma_j < x_{i_M}\right) dy$ evaluate to $1 - \exp(-x_{i_M}/(2\sigma^2))$ and $2\sigma^2(1 - \exp(-x_{i_M}/(2\sigma^2))) - x_{i_M} \exp(-x_{i_M}/(2\sigma^2))$, respectively. Substituting these in (8) yields (6). ■

As shown in Appendix B, the MSE of the above estimator for an unreported subchannel j is given in closed-form as

$$\text{MSE} = 4\sigma^4\left(1 + \frac{N\Phi(M, N)}{N - M} - \frac{N(N-1)\Phi(M, N-1)}{(N-M)(N-M-1)}\right), \quad (9)$$

where $\Phi(i, j) = \Psi^{(1)}(i) - \Psi^{(1)}(j) + \left(\sum_{l=i}^{j-1} l^{-1}\right)^2$ and $\Psi^{(1)}(\cdot)$ is the trigamma function [26, Table 6.1].

Observations: Notice that $\hat{\gamma}_j$ depends only on the lowest reported SNR and is a non-linear function of it. Further, it does not depend on N and j. Thus, the SNR estimate is the same for all the unreported subchannels. This behavior is unlike conventional OFDM estimation in which the estimator is not a function of any other subchannel gain when the subchannel gains are independent. Further, the ratio $\text{MSE}/(4\sigma^4)$ is independent of the average subchannel SNR $2\sigma^2$.

B. Correlated Subchannel Gains

For a subchannel j, let $j_l = \operatorname{argmin}_{i \in \mathcal{I}_M, i < j}\{j - i\}$ and $j_h = \operatorname{argmin}_{i \in \mathcal{I}_M, i > j}\{i - j\}$ denote the indices of the reported subchannels that are nearest to it and are respectively lower and higher than j. If subchannel j does not have a lower reported index, then we define $j_l = 0$ and its SNR as $x_{j_l} = 0$. Similarly, if subchannel j does not have a higher reported index, then $j_h = N + 1$ and $x_{j_h} = 0$. The following is a key result for the CMMSE estimator.

Result 2: The CMMSE estimator $\hat{\gamma}_j$ of subchannel $j \in \mathcal{I}_M^c$, given the set of reported indices $\mathcal{I}_M$ and the corresponding SNRs $\gamma_{i_1} = x_{i_1}, \ldots, \gamma_{i_M} = x_{i_M}$, is given in closed-form as

$$\hat{\gamma}_j = \frac{A_j}{B_j}, \quad (10)$$

where

$$A_j = \sum_{n=0}^{\infty} \frac{\delta^{2n}}{4^n} \sum_{\substack{0 \le q_{j_l}, \ldots, q_{j_h-1} \le n \\ q_{j_l} + \ldots + q_{j_h-1} = n}} \frac{x_{j_l}^{q_{j_l}} x_{j_h}^{q_{j_h-1}} \Gamma_{\text{inc}}\left(\eta_j x_{i_M}, q_{j-1} + q_j + 2\right)}{\eta_j \left[\prod_{r=j_l}^{j_h-1} (q_r!)^2 \eta_r^{q_{r-1}+q_r+1}\right]} \times \prod_{r=j_l+1, r \neq j}^{j_h-1} \Gamma_{\text{inc}}\left(\eta_r x_{i_M}, q_{r-1} + q_r + 1\right), \quad (11)$$

$$B_j = \sum_{n=0}^{\infty} \frac{\delta^{2n}}{4^n} \sum_{\substack{0 \le q_{j_l}, \ldots, q_{j_h-1} \le n \\ q_{j_l} + \ldots + q_{j_h-1} = n}} \frac{x_{j_l}^{q_{j_l}} x_{j_h}^{q_{j_h-1}}}{\eta_j \left[\prod_{r=j_l}^{j_h-1} (q_r!)^2 \eta_r^{q_{r-1}+q_r+1}\right]} \times \prod_{r=j_l+1}^{j_h-1} \Gamma_{\text{inc}}\left(\eta_r x_{i_M}, q_{r-1} + q_r + 1\right), \quad (12)$$

$\Gamma_{\text{inc}}(x, a) = \int_0^x t^{a-1} e^{-t} dt = (a-1)!\left[1 - e^{-x} \sum_{k=0}^{a-1} x^k / k!\right]$ is the incomplete gamma function [26, Table 6.5] for integer a. Here, $\eta_r = 1/(2\sigma^2(1-\rho^2))$, for $r = 1, N$, and $\eta_r = (1+\rho^2)/(2\sigma^2(1-\rho^2))$, for $r = 2, \ldots, N-1$.

Proof: The proof is relegated to Appendix A. ■

Discussion: We note that the expressions for A_j and B_j only involve the SNRs x_{j_l}, x_{j_h}, and x_{i_M}. Also, the inner summation is only over $j_h - j_l$ variables $q_{j_l}, \ldots, q_{j_h-1}$. This is unlike (1) in which the inner summation is over $N-1$ variables. This is an outcome of our exploitation of the Markov property of the exponential correlation model. Consequently, for the n^{th} term in the infinite series, the number of terms in the inner summation decreases from $(n+N-2)!/(n!(N-2)!)$ to $(n+j_h-j_l-1)!/(n!(j_h-j_l-1)!)$. Finally, the estimator $\hat{\gamma}_j$ is a non-linear function of x_{j_l}, x_{j_h}, and x_{i_M}. The result above serves as a novel and fundamental theoretical benchmark to evaluate the efficacy of the reduced complexity approaches that we propose next.

C. Reducing Computational Complexity of CMMSE Estimator

Despite the fact that the CMMSE estimator in (10) is in closed-form and the Markov property of the subchannel SNRs helps to simplify it, it is still computationally intensive for two reasons. First, both the numerator in (11) and the denominator in (12) involve evaluating an infinite series. Further, the presence of the $(1-\rho^2)$ term in the denominator in δ and η_r results in a slower decay of the infinite series for larger ρ. Second, for the n^{th} term in the infinite series, $(n+j_h-j_l-1)!/(n!(j_h-j_l-1)!)$ terms need to be evaluated. Since this is exponential in n and (j_h-j_l), the computational complexity becomes prohibitively high for larger N. To address this, we present two approaches.

1) Windowing Approach: In it, the correlation between subchannels that are at least N_w subchannels apart is neglected. This is motivated by the fact that the correlation between the subchannels decays exponentially as their separation increases. The parameter N_w trades off between accuracy and computational complexity. The windowing approach is detailed below.

Consider an unreported subchannel j. As before, let j_l and j_h denote the reported subchannels that are respectively lower and higher than j and nearest to j. Their SNRs are denoted by x_{j_l} and x_{j_h}, respectively. The following four different cases can occur:

i) $j_h - j_l \le N_w + 1$: This implies that $j - j_l \le N_w$ and $j_h - j \le N_w$. In this case, the SNR estimate is exactly given by (10) because of the Markov property. Since $j_h - j_l \le N_w + 1$, the estimate is not computationally intensive.

ii) $j - j_l \le N_w$ and $j_h - j > N_w$: Here, the CSI of all the reported subchannels with indices j_h and above is neglected in computing $\hat{\gamma}_j$. This is equivalent to the scenario where there are no reported subchannels with indices higher than j and there are N_w subchannels in total. The SNR estimate is then computed using (10) with j_h replaced by $j'_h = j_l + N_w + 1$ and x_{j_h} replaced by $x'_{j_h} = 0$. Doing so reduces the number of variables to be summed over in the inner summations in (11) and (12) to $N_w + 1$.

iii) $j_h - j \le N_w$ and $j - j_l > N_w$: This case is similar to the previous one except that the SNR estimate is computed with $j'_l = j_h - N_w - 1$ and $x'_{j_l} = 0$ instead of j_l and x_{j_l}, respectively. Doing so significantly reduces the complexity just as in the previous case.

iv) $j - j_l > N_w$ and $j_h - j > N_w$: Here, the SNR estimate in (6) is used because subchannel j does not lie within N_w subchannels of either j_l or j_h. It corresponds to the independent subchannel gains scenario, for which the CMMSE estimator takes a simple form.[1]

2) NRSR Approach: In this approach, the estimator uses the ordering information of the best-M scheme, but ignores the correlation with subchannels other than the nearest reported subchannel. Doing so reduces the N subchannel problem to a two subchannel problem consisting of an unreported subchannel and its nearest reported subchannel.

Let j_n and x_{j_n} denote the index of the reported subchannel nearest to subchannel j and its SNR, respectively. Formally, $j_n = \operatorname{argmin}_{i \in \mathcal{I}_M} \{|i-j|\}$. Further, let $\omega_j = \rho^{|j-j_n|}$ denote the correlation coefficient between subchannels j and j_n. Then, the SNR estimate $\tilde{\gamma}_j$ using the NRSR approach is given by

$$\tilde{\gamma}_j = \mathbb{E}\left[\gamma_j | \gamma_{j_n} = x_{j_n}, \gamma_j < x_{i_M}\right], \tag{13}$$

and has the following closed-form expression.

Result 3: The SNR estimate $\tilde{\gamma}_j$ of subchannel j in the NRSR approach, given the set of reported indices $\mathcal{I}_M$ and the reported SNRs $x_{i_1}, \ldots, x_{i_M}$, is given by

$$\tilde{\gamma}_j = \frac{\sum_{l=0}^{\infty} \frac{\lambda_j^{2l} x_{j_n}^l}{4^l (l!)^2 \mu_j^{l+2}} \Gamma_{\text{inc}}\left(\mu_j x_{i_M}, l+2\right)}{\sum_{l=0}^{\infty} \frac{\lambda_j^{2l} x_{j_n}^l}{4^l (l!)^2 \mu_j^{l+1}} \Gamma_{\text{inc}}\left(\mu_j x_{i_M}, l+1\right)}, \tag{14}$$

where $\lambda_j = \omega_j/(\sigma^2(1-\omega_j^2))$ and $\mu_j = 1/(2\sigma^2(1-\omega_j^2))$.

Proof: The proof is relegated to Appendix C. ■

Discussion: Note that the estimator above still accounts for the fact that γ_j is lower than the lowest reported SNR x_{i_M}. For the n^{th} term in the infinite series in the numerator and denominator of (14), only one term needs to be computed unlike $(n+j_h-j_l-1)!/(n!(j_h-j_l-1)!)$ terms in the CMMSE estimator in (10). Since the incomplete gamma function can be written in a closed-form that involves elementary functions such as exponentials and polynomials, it can be readily evaluated. Also, it satisfies the recursion: $\Gamma_{\text{inc}}(x,a) = (a-1)\Gamma_{\text{inc}}(x,a-1) - e^{-x}x^{a-1}$. It can be used to reduce complexity since $\Gamma_{\text{inc}}(\mu_j x_{i_M}, l+1)$ in the denominator of the l^{th} term in the series can be used to compute $\Gamma_{\text{inc}}(\mu_j x_{i_M}, l+2)$ in the numerator and the denominator of the l^{th} and $(l+1)^{\text{th}}$ terms, respectively.

D. Extension to General Subchannel Correlations

The NRSR approach can be extended to any other channel model as shown below. Let (τ_l, P_l), for $l = 1, 2, \ldots, L$, denote

[1] Cases ii and iii reduce to Case iv for $j_l = 0$ and $j_h = N+1$, respectively.

the PDP of an L-tap channel, where τ_l and P_l are the delay and power of the l^{th} tap, respectively [1]. Then, the absolute value $\theta_{m,n}$ of the correlation coefficient between subchannels m and n is given by [1]

$$\theta_{m,n} = \frac{\left|\sum_{l=1}^{L} P_l \exp\left(-\frac{2\pi i \tau_l F_s N_c (m-n)}{N_{\text{fft}}}\right)\right|}{\sum_{l=1}^{L} P_l}, \tag{15}$$

where F_s is the sampling frequency, N_{fft} is the order of the fast Fourier transform, N_c is the number of contiguous subcarriers in a subchannel, and $i = \sqrt{-1}$.

For an unreported subchannel j, the correlation coefficient with respect to its nearest reported subchannel j_n can then be computed using (15) with $m = j$ and $n = j_n$. Thus, $\omega_j = \theta_{j,j_n}$. Given ω_j, the SNR estimate $\tilde{\gamma}_j$ for subchannel j can be computed using (14).

E. Extension to Quantized Feedback

We now study the practical scenario in which the SNRs fed back by the best-M scheme are quantized. In such a case, the maximum rate MCSs that can be reliably supported on each of the subchannels with the M largest SNRs and their subchannel indices are reported to the BS [6], [8]. The MCS is selected from a pre-specified set of S MCSs with rates $0 = R_1 < R_2 < \cdots < R_S$. An MCS s with rate R_s has an associated rate adaptation threshold, denoted by T_s, such that MCS s is selected on a subchannel n if $\gamma_n \in [T_s, T_{s+1})$ [1]. Here, $T_1 = 0$ and $T_{S+1} = \infty$.

Let Z_n denote the rate of the MCS reported on subchannel n. Then, the MMSE estimator $\hat{\gamma}_j$ for the SNR of an unreported subchannel j, given $Z_{i_1} = R_{s_1}, \ldots, Z_{i_M} = R_{s_M}$, is

$$\begin{aligned}\hat{\gamma}_j &= \mathbb{E}\left[\gamma_j | Z_u = R_{s_u}, \forall u \in \mathcal{I}_M; \gamma_v < \gamma_{i_M}, \forall v \in \mathcal{I}_M^c\right],\\ &= \mathbb{E}\left[\gamma_j | T_{s_u} \le \gamma_u < T_{s_u+1}, \forall u \in \mathcal{I}_M; \gamma_v < \gamma_{i_M}, \forall v \in \mathcal{I}_M^c\right].\end{aligned} \tag{16}$$

Computing (16) is even more challenging than computing (3). Therefore, we extend the NRSR approach to handle this scenario. As in Section III-C2, the SNR estimate $\tilde{\gamma}_j$ using the NRSR approach with quantized feedback is given by

$$\begin{aligned}\tilde{\gamma}_j &= \mathbb{E}\left[\gamma_j \,\middle|\, T_{s_{j_n}} \le \gamma_{j_n} < T_{s_{j_n}+1},\right.\\ &\qquad\qquad \left. T_{s_{i_M}} \le \gamma_{i_M} < T_{s_{i_M}+1}, \gamma_j < \gamma_{i_M}\right],\\ &\approx \mathbb{E}\left[\gamma_j | T_{s_{j_n}} \le \gamma_{j_n} < T_{s_{j_n}}, \gamma_j < T_{s_{i_M}}\right], \qquad (17)\\ &= \frac{\sum_{l=0}^{\infty} \frac{\lambda_j^{2l} x_{j_n}^l}{4^l (l!)^2 \mu_j^{l+2}} \Gamma_{\text{inc}}\left(\mu_j T_{s_{i_M}}, l+2\right)}{\sum_{l=0}^{\infty} \frac{\lambda_j^{2l} x_{j_n}^l}{4^l (l!)^2 \mu_j^{l+1}} \Gamma_{\text{inc}}\left(\mu_j T_{s_{i_M}}, l+1\right)}. \qquad (18)\end{aligned}$$

The approximation in (17) conservatively replaces γ_{i_M} with the lower threshold $T_{s_{i_M}}$. Doing so ensures that only the joint distribution of γ_j and γ_{j_n} is needed to evaluate (17). The steps involved in arriving at (18) are similar to those given in Appendix C, except that x_{i_M} is replaced by $T_{s_{i_M}}$.

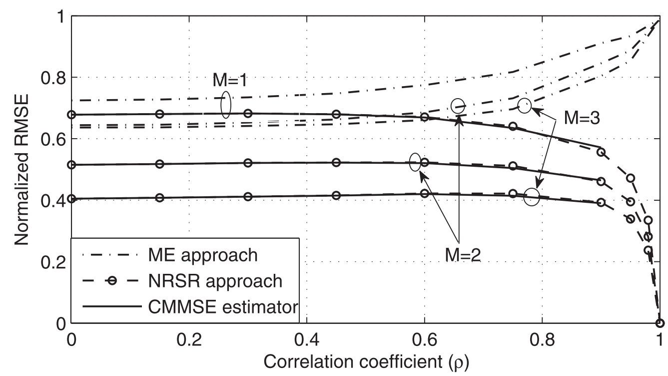

Fig. 2. Normalized RMSE (RMSE/$(2\sigma^2)$) as a function of correlation coefficient ρ for different approaches ($N = 10$ and $\sigma^2 = 4.5$ dB).

IV. MSE, System Impact, and Benchmarking

We now present Monte Carlo simulation results to evaluate the performance of the proposed BS-side estimators. Since the CMMSE estimator approach need not be throughput-optimal, in addition to MSE, we also evaluate system-level performance measures such as cell throughput to present a more comprehensive evaluation. This incorporates important aspects such as the frequency-domain scheduler, discrete rate adaptation, and the impact of estimation errors. We employ the windowing approach described in Section III-C1 with $N_w = 3$. Increasing N_w further makes a negligible difference for $\rho < 0.9$. We have found that 25 terms of the infinite series in (11) and (12) for $\rho = 0.5$ and 65 terms for $\rho = 0.75$ are sufficient to ensure numerical accuracy.

A. MSE

Fig. 2 plots the square root MSE (RMSE) divided by $2\sigma^2$, which we shall refer to as the *normalized* RMSE, for the unreported subchannels as a function of ρ for the CMMSE estimator and the NRSR approach. The division by $2\sigma^2$ ensures that the normalized RMSE is not a function of $2\sigma^2$ (for example, see (9)). To better understand the role of the CSI fed back in improving the accuracy of the estimate, we also show the normalized RMSE for the mean-as-estimate (ME) approach, which ignores the fed back CSI and, thus, sets the SNR estimate of an unreported subchannel as $\mathbb{E}\left[\gamma_j\right] = 2\sigma^2$.

Even when the subchannel gains are uncorrelated ($\rho = 0$), we see that the CMMSE estimator lowers the RMSE by 6.3%, 20%, and 36.4% compared to the ME approach for $M = 1$, 2, and 3, respectively. Note that the CMMSE estimator estimates $N - M$ subchannel SNRs from just M reported subchannel SNRs. For example, for $M = 1$, it estimates 9 subchannel SNRs from just 1 reported subchannel SNR. The RMSE of the CMMSE estimator decreases rapidly as M increases and decreases towards zero as ρ increases. On the other hand, the normalized RMSE of the ME approach increases to unity as ρ increases. This demonstrates the benefits of taking the CSI fed back by the best-M scheme into account. The RMSE of the NRSR approach is within 2% of that of the CMMSE estimator, which verifies its efficacy. The results for the CMMSE estimator are not shown for $\rho > 0.9$ because the number of terms required to compute it accurately becomes prohibitively large. However, the NRSR approach does not face such a limitation. Note that the RMSE of the CMMSE estimator marginally exceeds that of the NRSR approach at $\rho = 0.9$ because using $N_w = 3$ makes the CMMSE estimate sub-optimal at such a large correlation.

B. System Impact Evaluation and Benchmarking

We now evaluate the impact of using the proposed CMMSE estimator on the cell throughput, which accounts for both rate adaptation and scheduling based on the estimates obtained. It also captures the impact of imperfect estimates. Now, for rate adaptation and scheduling, the BS can use not just the reported subchannel SNRs but also the estimates of the SNRs of the unreported subchannels for each user.

1) Rate Adaptation: We investigate discrete rate adaptation, since it is always used in standards such as LTE [2]. As mentioned in Section III-E, there are S available MCSs and $S+1$ rate adaptation thresholds. Let $r_{k,n}$ denote the rate assigned to subchannel n of user k. With perfect CSI, the rate adaptation scheme assigns rate $r_{k,n} = R_s$ if $\gamma_{k,n} \in [T_s, T_{s+1})$ [1].

Due to estimation errors, the SNR estimate $\hat{\gamma}_{k,n}$ for an unreported subchannel n of user k can be different from the actual SNR $\gamma_{k,n}$. If $\hat{\gamma}_{k,n} > \gamma_{k,n}$, then the transmit rate on subchannel n can be higher than the rate that can be supported reliably. In such a case, the user cannot decode the transmitted packet and the throughput is zero. On the other hand, if $\hat{\gamma}_{k,n} < \gamma_{k,n}$, then the rate chosen can be reliably decoded.

Rate Backoff Technique [27]: To address the above mismatch in the throughput penalty for underestimating and overestimating the SNR, a rate backoff technique is used. In it, a rate lower than that indicated by the rate adaptation scheme is assigned to an unreported subchannel. The decrement in rate is determined by the rate backoff. For example, with one-rate backoff, a one level lower MCS $s-1$ is assigned to subchannel n if $\hat{\gamma}_{k,n} \in [T_s, T_{s+1})$, i.e., $r_{k,n} = R_{s-1}$.

2) Frequency-Domain Scheduling: The user assigned to a subchannel depends on the scheduler employed by the BS. We consider the following three frequency-domain schedulers, which cover a wide range of the trade-off between cell throughput and fairness [3], [5], [6], [28], [29]:

- *Greedy Scheduler:* On subchannel n, it schedules the user with the largest assigned rate for that subchannel. Let k_n^* denote the user scheduled on subchannel n. Then,

$$k_n^* = \underset{1 \le k \le K}{\operatorname{argmax}} \; r_{k,n}. \tag{19}$$

  If multiple users have the same highest rate, then one among them is chosen with uniform probability.
- *Round Robin (RR) Scheduler:* It schedules users in a predetermined, periodic manner for any subchannel. While it is time-fair, it does not exploit multi-user diversity.
- *Proportional Fair (PF) Scheduler [28], [29]:* On subchannel n, it schedules the user with the highest PF metric for that subchannel. The PF metric of a user for any subchannel is defined as the ratio of the assigned rate to the throughput received by the user so far on that subchannel. Let k_n^* denote the user scheduled on subchannel n. Then, at time t,

$$k_n^* = \underset{1 \le k \le K}{\operatorname{argmax}} \; \frac{r_{k,n}}{\overline{R}_{k,n}(t-1)}, \tag{20}$$

  where $\overline{R}_{k,n}(t-1)$ is the throughput received by user k on subchannel n until time $t-1$. This scheduler achieves a trade-off between cell throughput and user fairness.

3) Data Reception Model: We consider an outage based model in which an outage can occur in two ways. First, a *transmission outage* occurs when the BS does not transmit any data due to the lack of CSI.[2] Second, a *channel outage* occurs when the subchannel SNR is below the threshold T_s if MCS s is chosen for transmission to the scheduled user. In such a case, this transmission does not contribute to the cell throughput. This model is physically justified because of the waterfall nature of the packet error rate curves, and is often used in system-level performance analyses [6], [9], [17]. Channel outages capture the impact of estimation errors on the cell throughput.

4) Performance Benchmarking: We benchmark the proposed approaches against the following:

- *Conventional Approach [5], [6], [8], [13], [17]:* In this widely used approach, the BS does not transmit on subchannels that were not reported by any user. Here, transmission outages can occur but not channel outages.
- *ME Approach:* As discussed in Section IV-A, this estimator ignores the CSI fed back by the best-M scheme. Therefore, the unreported subchannel SNRs are set to be equal to their mean value $\mathbb{E}\left[\gamma_{k,n}\right] = 2\sigma_k^2$. Here, only channel outages can occur.
- *Data Method [18]:* A subchannel that is not reported by any user is assigned to the user selected for its adjacent subchannel. Its MCS is one level lower than that assigned to the adjacent subchannel. If the adjacent subchannels are unassigned, then a transmission outage occurs. Channel outages can also occur on this subchannel.
- *Full CSI:* The BS is assumed to know the SNRs of all subchannels for all users. While unrealistic, it provides an upper limit on the achievable cell throughput.

The Monte Carlo simulation results are obtained by averaging over 5,000 channel realizations. The $S = 16$ rates are as specified in LTE [2, Table 10.1]. These range from $R_2 = 0.15$ bits/symbol to $R_{16} = 5.55$ bits/symbol. The rate adaptation thresholds are calculated using the formula [30]: $T_l = (2^{R_l} - 1)/\zeta$, where $\zeta = 0.398$ accounts for the coding loss of a practical code. We set $K = N = 10$. We focus on the single-cell scenario, as has been done in [5]–[8], [13], [17]. To gain insights, we first consider the case in which the users see statistically identical channels. The PF and greedy schedulers are identical in this case. Thereafter, we show results when the users see statistically non-identical channels.

Fig. 3 plots the cell throughputs of the various approaches for the greedy scheduler as a function of ρ for $M = 1$. We see that one-rate backoff maximizes the cell throughput for $\rho \le 0.9$,

[2]Instances when the BS decides not to transmit data because the scheduled user cannot support any non-zero-rate MCS are not considered as outages.

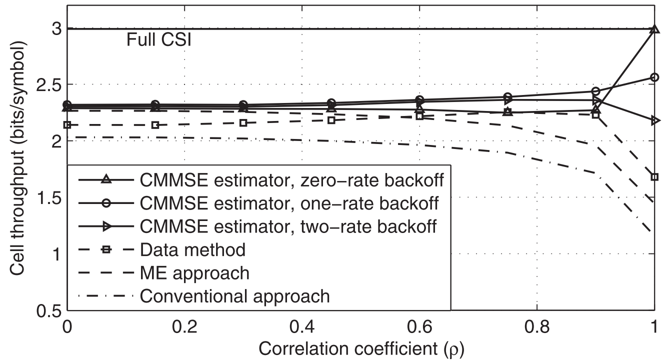

Fig. 3. Greedy scheduler for $M = 1$ and statistically identical users: Zoomed-in view of cell throughput as a function of correlation coefficient ρ for different approaches ($K = 10$, $N = 10$, and $\sigma^2 = 4.5$ dB).

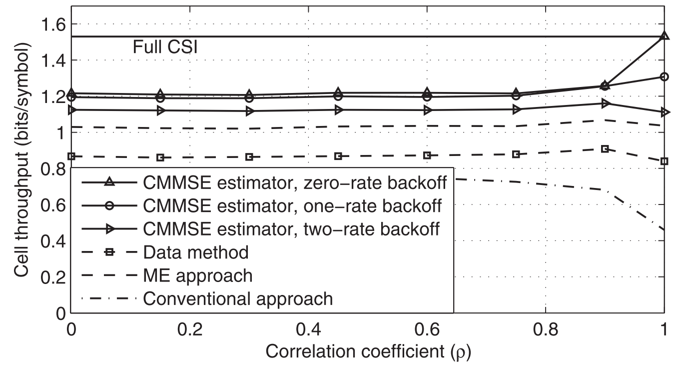

Fig. 5. RR scheduler for $M = 3$ with statistically identical users: Cell throughput as a function of correlation coefficient ρ for different approaches ($K = 10$, $N = 10$, and $\sigma^2 = 4.5$ dB).

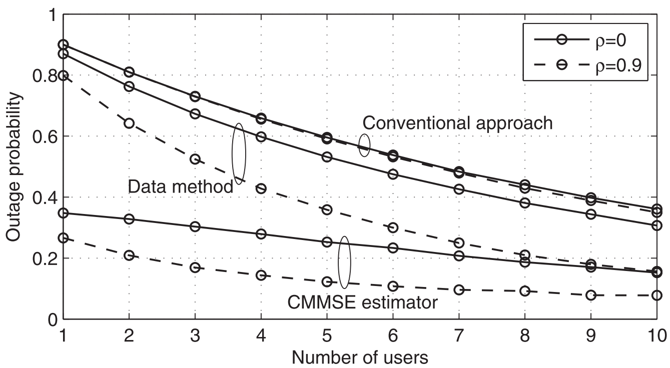

Fig. 4. Outage probability as a function of the number of users for the different approaches (greedy scheduler, statistically identical users, $M = 1$, $N = 10$, and $\sigma^2 = 4.5$ dB).

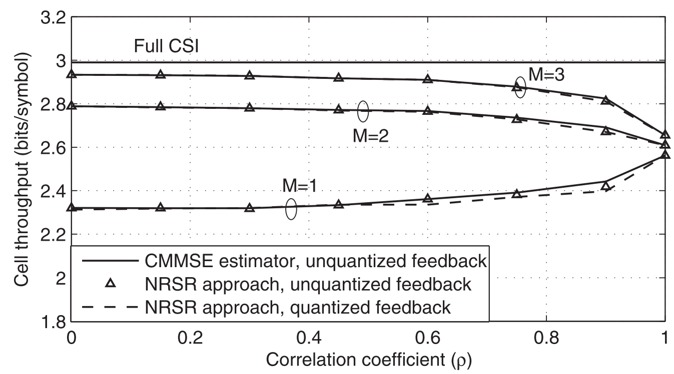

Fig. 6. Greedy scheduler with statistically identical users: Zoomed-in view of cell throughput as a function of ρ for unquantized SNR feedback and quantized feedback from users ($K = 10$, $N = 10$, and $\sigma^2 = 4.5$ dB).

while zero-rate backoff is optimal for $\rho > 0.9$. This is because the estimation accuracy increases as ρ increases, thus, avoiding the need for rate backoff. Two-rate backoff is suboptimal for all ρ because of its overly conservative choice of MCS. Notice that the CMMSE estimator approach achieves a higher cell throughput compared to the other approaches for all ρ. At $\rho = 0$, the CMMSE estimator approach with one-rate backoff improves the cell throughput by 2.3%, 8.3%, and 14.2% compared to the ME approach, Data method, and conventional approach, respectively. Thus, exploiting the ordering information given by the best-M scheme itself yields gains. At $\rho = 0.75$, the corresponding gains are 11.8%, 5.9%, and 26.1%. In general, the gains increase as the average SNR increases; the figure for this is not shown to save space.

We see that the cell throughputs of the ME and conventional approaches decrease as ρ increases. This is because of the decrease in frequency diversity since the subchannel gains are likely to be more alike. For the Data method, the choice of rate based on the adjacent subchannel becomes more accurate as subchannel correlation increases. Hence, its cell throughput increases as ρ increases from 0 to 0.9. However, for lower ρ, its cell throughput is lower than even the simpler ME approach.

Fig. 4 plots the outage probability of the various approaches as a function of the number of users for $M = 1$. We see that the CMMSE estimator approach with one-rate backoff has a lower outage probability than the conventional approach and the Data method for $\rho = 0$ and 0.9. Further, the outage probability is lower for $\rho = 0.9$ compared to $\rho = 0$. This is because the SNR estimates become more accurate as ρ increases, which lowers the odds that an incorrect MCS gets chosen for transmission.

Fig. 5 plots the cell throughput for the RR scheduler and $M = 3$. Results for $M = 1$ and 2 are not shown to avoid clutter. We see that zero-rate backoff is optimal for all values of ρ. Here again, we see that the CMMSE estimator approach achieves a higher cell throughput compared to the other approaches. At $\rho = 0$, the gains over the ME approach, Data method, and conventional approach are 18.2%, 40.3%, and 58.0%. Further, the ME approach outperforms the Data method for all values of ρ.

We now comment on the special case of $\rho = 1$. Here, the subchannel gains are equal with probability 1. Hence, the CMMSE estimates are perfect and the MSE is zero. Therefore, zero-rate backoff is optimal. Non-zero-rate backoff results in a loss in cell throughput, as we saw in Figs. 3 and 5.

Efficacy of NRSR Approach: Fig. 6 plots the cell throughputs of the CMMSE estimator approach and the NRSR approach with unquantized and quantized feedback for different values of ρ and M. The results are shown for one-rate backoff as it was seen to be optimal for $\rho \le 0.9$. The results for zero-rate backoff are skipped to avoid clutter. With unquantized feedback, the cell throughput of the NRSR approach is indistinguishable from that of the CMMSE estimator approach for $\rho \le 0.75$. Even at $\rho = 0.9$, the loss in cell throughput is just 1%, 0.8%, and 0.5% for $M = 1$, 2, and 3, respectively. The loss at high correlations occurs because ignoring the correlations among multiple subchannels is sub-optimal. Even with quantized feedback, the cell

TABLE I
CELL THROUGHPUT (BITS/SYMBOL) OF DIFFERENT APPROACHES FOR THE GREEDY SCHEDULER

	Full CSI	Conventional approach	ME approach	Data method	NRSR approach
TU channel					
$M = 1$	2.99	1.83	2.06	2.41	2.46
$M = 2$	2.99	2.56	2.62	2.69	2.72
$M = 3$	2.99	2.82	2.84	2.86	2.87
RA channel					
$M = 1$	2.96	1.20	1.58	1.94	2.43
$M = 2$	2.96	1.95	2.11	2.30	2.58
$M = 3$	2.96	2.40	2.46	2.54	2.70

throughput achieved by the NRSR approach is very close to that of the CMMSE estimator approach. For example, at $\rho = 0.9$, the loss in throughput is just 1.8%, 0.8%, and 0.5% for $M = 1$, 2, and 3, respectively.

For the CMMSE estimator approach, the cell throughput increases as ρ increases for $M = 1$, while it decreases as ρ increases for $M \geq 2$. This behavior is a consequence of the following two effects, which counteract each other. First, as ρ increases to 1, the frequency diversity gain due to ordering of the subchannel SNRs is lost, which lowers the cell throughput. Second, as ρ increases, the outage probability of transmissions on unreported subchannels decreases, which improves the cell throughput. The behavior of the cell throughput with ρ and M depends on which of these two effects dominates.

5) Application to General Channel Models: We now evaluate the effectiveness of the NRSR approach for general channel models, which need not follow the exponential correlation structure. We set $N_{\text{fft}} = 512$, $F_s = 7.68$ MHz, and $N_c = 12$. The cell throughputs of the various approaches for the TU and RA channels [2] are compared in Table I. We see that the NRSR approach with one-rate backoff achieves a higher cell throughput than all the benchmark schemes for all M. The cell throughput with the RA channel is lower than that of the TU channel. However, the relative gains over the benchmark schemes are higher for the less dispersive RA channel.

6) Users With Statistically Non–identical Channels: To model the scenario in which the subchannel gains of different users are statistically non-identical, we set the average SNR of user k as $2\sigma_k^2 = 2\sigma^2\alpha^{k-1}$, for $k = 1, \ldots, K$, where $\alpha > 1$ [17]. Hence, the subchannel gains of different users become more statistically non-identical as α increases. Fig. 7 plots the cell throughput results for the PF scheduler for $M = 1$. Results for $M = 2$ and 3, and for the other schedulers are not shown to avoid clutter. We see that two-rate backoff is marginally better than one-rate backoff for $\rho \leq 0.9$ and zero-rate backoff is optimal for $\rho > 0.9$. As before, the CMMSE estimator approach with a suitably chosen rate backoff improves the cell throughput compared to the other approaches. For example, at $\rho = 0.75$, the gains over the ME approach, Data method, and conventional approach are 21.5%, 7.8%, and 34.9%, respectively. The other trends, such as the behavior of the cell throughput as a function of ρ and M, are similar to those for the greedy scheduler. Among the three schedulers, the greedy scheduler, as expected,

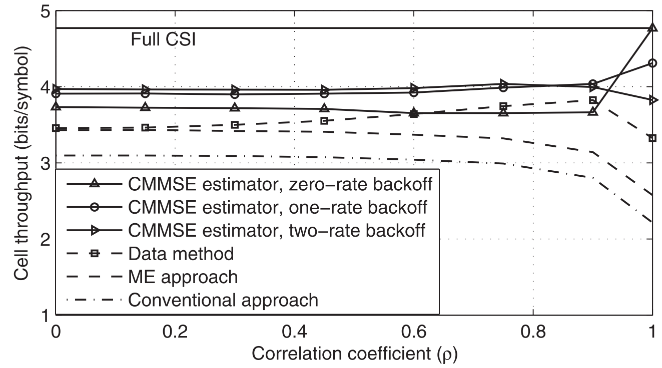

Fig. 7. PF scheduler for $M = 1$ with statistically non-identical users: Zoomed-in view of cell throughput as a function of correlation coefficient ρ for different approaches ($K = 10$, $N = 10$, $\alpha = 1.4$, and $\sigma^2 = 4.5$ dB).

is the most unfair while the RR scheduler is the most fair. The fairness results are not shown due to space constraints.

V. CONCLUSIONS

We proposed a novel constrained estimator in which the BS used the ordered feedback generated by the best-M scheme and the subchannel correlation to estimate the SNRs of the unreported subchannels for each user. The BS then utilized these constrained MMSE estimates for scheduling and rate adaptation. Compared to conventional channel estimation based on pilots, the CMMSE estimator was non-linear and its derivation, which was based on order statistics, quite different. We also proposed the windowing and NRSR approaches, which reduced the computational complexity while incurring a negligible performance loss. Another advantage of the NRSR approach was its applicability to general channels models and to quantized feedback. Unlike the ME approach, whose RMSE increased as the subchannel correlation increased, the RMSE of the proposed estimators decreased to zero. In terms of system impact, we saw that the proposed estimators improved the cell throughput for the greedy, RR, and PF schedulers and for both uncorrelated and correlated subchannels – without any additional feedback. An avenue for future work is to obtain BS-side estimators incorporating wideband CSI as well.

APPENDIX

A. Derivation of CMMSE Estimator

Let $\mathcal{J}_j = \{j_l + 1, \ldots, j_h - 1\}$ denote the set of unreported subchannels lying between the reported subchannels j_l and j_h. Thus, $j \in \mathcal{J}_j$. Using the Markov property, the conditional expectation in (3) reduces to

$$\hat{\gamma}_j = \begin{cases} \mathbb{E}\left[\gamma_j | \gamma_{j_l} = x_{j_l}, \gamma_{j_h} = x_{j_h}, \gamma_r < x_{i_M}, \forall r \in \mathcal{J}_j\right], \\ \qquad\qquad \text{if } j_l \geq 1,\ j_h \leq N, & \text{(21a)} \\ \mathbb{E}\left[\gamma_j | \gamma_{j_h} = x_{j_h}, \gamma_r < x_{i_M}, \forall r \in \mathcal{J}_j\right], \\ \qquad\qquad \text{if } j_l = 0,\ j_h \leq N, & \text{(21b)} \\ \mathbb{E}\left[\gamma_j | \gamma_{j_l} = x_{j_l}, \gamma_r < x_{i_M}, \forall r \in \mathcal{J}_j\right], \\ \qquad\qquad \text{if } j_l \geq 1,\ j_h = N + 1. & \text{(21c)} \end{cases}$$

We consider the three cases one by one.

1) When $j_l \geq 1$ and $j_h \leq N$: The conditional mean in (21a) can be written as

$$\hat{\gamma}_j = \frac{\int_0^{x_{i_M}} \cdots \int_0^{x_{i_M}} z_j f_{\gamma_{j_l},\ldots,\gamma_{j_h}}\left(x_{j_l}, \mathbf{z}, x_{j_h}\right) d\mathbf{z}}{\int_0^{x_{i_M}} f_{\gamma_{j_l},\ldots,\gamma_{j_h}}\left(x_{j_l}, \mathbf{z}, x_{j_h}\right) d\mathbf{z}}, \quad (22)$$

where $\mathbf{z} = [z_{j_l+1}, \ldots, z_{j_h-1}]$. Since $\gamma_{j_l}, \ldots, \gamma_{j_h}$ are jointly exponentially correlated RVs, their joint PDF is given by [21]

$$f_{\gamma_{j_l},\ldots,\gamma_{j_h}}\left(x_{j_l}, \mathbf{z}, x_{n_h}\right) = \frac{e^{-\left(\eta_{j_l} x_{j_l} + \eta_{j_h} x_{j_h} + \sum_{r=j_l+1}^{j_h-1} \eta_r z_r\right)}}{2^{j_h-j_l+1}\sigma^{2(j_h-j_l+1)}(1-\rho^2)^{j_h-j_l}} \times \sum_{n=0}^{\infty} \frac{\delta^{2n}}{4^n} \sum_{\substack{0 \leq q_{j_l} \leq \cdots \leq q_{j_h-1} \leq n \\ q_{j_l}+\cdots+q_{j_h-1}=n}} \frac{x_{j_l}^{q_{j_l}} x_{j_h}^{q_{j_h-1}} \prod_{r=j_l+1}^{j_h-1} z_r^{q_{r-1}+q_r}}{\prod_{r=j_l}^{j_h-1} (q_r!)^2}, \quad (23)$$

where $\eta_r = \left(1+\rho^2\right) / \left(2\sigma^2(1-\rho^2)\right)$, for $r \in \mathcal{J}_j$, and $\eta_r = 1/\left(2\sigma^2(1-\rho^2)\right)$, for $r \in \{j_l, j_h\}$. Substituting (23) in (22) and pooling together the terms with the same variable of integration, we eventually get

$$\hat{\gamma}_j = \frac{A_j}{B_j}, \quad (24)$$

where

$$A_j = \sum_{n=0}^{\infty} \frac{\delta^{2n}}{4^n} \sum_{\substack{0 \leq q_{j_l}, \cdots, q_{j_h-1} \leq n \\ q_{j_l}+\cdots+q_{j_h-1}=n}} \frac{x_{j_l}^{q_{j_l}} x_{j_h}^{q_{j_h-1}} \int_0^{x_{i_M}} z_j^{q_{j-1}+q_j+1} e^{-\eta_j z_j} dz_j}{\prod_{r=j_l}^{j_h-1} (q_r!)^2} \times \prod_{r=j_l+1, r\neq j}^{j_h-1} \int_0^{x_{i_M}} z_r^{q_{r-1}+q_r} e^{-\eta_r z_r} dz_r, \quad (25)$$

$$B_j = \sum_{n=0}^{\infty} \frac{\delta^{2n}}{4^n} \sum_{\substack{0 \leq q_{j_l}, \cdots, q_{j_h-1} \leq n \\ q_{j_l}+\cdots+q_{j_h-1}=n}} \frac{x_{j_l}^{q_{j_l}} x_{j_h}^{q_{j_h-1}}}{\prod_{r=j_l}^{j_h-1} (q_r!)^2} \times \prod_{r=j_l+1}^{j_h-1} \int_0^{x_{i_M}} z_r^{q_{r-1}+q_r} e^{-\eta_r z_r} dz_r, \quad (26)$$

and $\eta_r = \left(1+\rho^2\right) / \left(2\sigma^2(1-\rho^2)\right)$, for $r = j_l+1, \ldots, j_h-1$.

2) When $j_l = 0$ and $j_h \leq N$: We proceed along the same lines as above, except that we use the joint PDF of the exponentially correlated RVs $\gamma_{j_l+1}, \ldots, \gamma_{j_h}$ to evaluate (21b). This yields

$$\hat{\gamma}_j = \frac{C_j}{D_j}, \quad (27)$$

where

$$C_j = \sum_{n=0}^{\infty} \frac{\delta^{2n}}{4^n} \sum_{\substack{0 \leq q_{j_l+1}, \cdots, q_{j_h-1} \leq n \\ q_{j_l+1}+\cdots+q_{j_h-1}=n}} \frac{x_{j_h}^{q_{j_h}-1} \int_0^{x_{i_M}} z_j^{q_{j-1}+q_j+1} e^{-\eta_j z_j} dz_j}{\prod_{r=j_l+1}^{j_h-1} (q_r!)^2} \times \prod_{r=j_l+2, r\neq j}^{j_h-1} \int_0^{x_{i_M}} z_r^{q_{r-1}+q_r} e^{-\eta_r z_r} dz_r, \quad (28)$$

$$D_j = \sum_{n=0}^{\infty} \frac{\delta^{2n}}{4^n} \sum_{\substack{0 \leq q_{j_l+1}, \cdots, q_{j_h-1} \leq n \\ q_{j_l+1}+\cdots+q_{j_h-1}=n}} \frac{x_{j_h}^{q_{j_h-1}}}{\prod_{r=j_l+1}^{j_h-1} (q_r!)^2} \times \prod_{r=j_l+2}^{j_h-1} \int_0^{x_{i_M}} z_r^{q_{r-1}+q_r} e^{-\eta_r z_r} dz_r, \quad (29)$$

$\eta_1 = 1/\left(2\sigma^2(1-\rho^2)\right)$, $\eta_r = \left(1+\rho^2\right) / \left(2\sigma^2(1-\rho^2)\right)$, for $r = 2, \ldots, j_h-1$, and $q_{j_l} = 0$. Equivalently, substituting $j_l = 0$, $x_{j_l} = 0$, and $\eta_{j_l+1} = \eta_1 = 1/\left(2\sigma^2(1-\rho^2)\right)$ in A_j and B_j reduces them to C_j and D_j, respectively.

3) When $j_l \geq 1$ and $j_h = N+1$: Here, we evaluate $\hat{\gamma}_j$, which is given by (21c), using the joint PDF of the exponentially correlated RVs $\gamma_{j_l}, \ldots, \gamma_{j_h-1}$. It can be shown, as before, that the resulting expression for $\hat{\gamma}_j$ can be equivalently obtained from (24), where A_j and B_j are computed with $j_h = N+1$, $x_{j_h} = 0$, and $\eta_{j_h-1} = \eta_N = 1/\left(2\sigma^2(1-\rho^2)\right)$.

Thus, $\hat{\gamma}_j$ in (24) compactly represents all the cases. Finally, writing (25) and (26) in terms of the incomplete gamma function yields (11) and (12), respectively.

B. Brief Derivation of Expression for MSE

The MSE for an unreported subchannel j is given by

$$\text{MSE} = \mathbb{E}_{\gamma_{i_1},\ldots,\gamma_{i_M},\mathcal{I}_M}\left[\mathbb{E}_{\gamma_j}\left[\left(\gamma_j - \hat{\gamma}_j\right)^2 \middle| j \in \mathcal{I}_M^c; \gamma_{i_1}, \ldots, \gamma_{i_M}; \mathcal{I}_M\right]\right]. \quad (30)$$

Substituting the expression for $\hat{\gamma}_j$ from (6) in (30) and using the fact that $\hat{\gamma}_j$ depends only on γ_{i_M}, we get

$$\text{MSE} = \mathbb{E}_{\gamma_j,\gamma_{i_M}}\left[\left(\gamma_j - 2\sigma^2 + \frac{\gamma_{i_M}}{e^{\frac{\gamma_{i_M}}{2\sigma^2}}-1}\right)^2 \middle| j \in \mathcal{I}_M^c\right],$$
$$= \int_0^{\infty}\int_0^{\infty}\left(y - 2\sigma^2 + \frac{x}{e^{\frac{x}{2\sigma^2}}-1}\right)^2 \times p\left(\gamma_j = y, \gamma_{i_M} = x \middle| j \in \mathcal{I}_M^c\right) dydx. \quad (31)$$

We now evaluate $p\left(\gamma_j = y, \gamma_{i_M} = x \middle| j \in \mathcal{I}_M^c\right)$. This probability is zero for $y > x$ since the SNR of an unreported subchannel cannot exceed γ_{i_M}. For $y \leq x$, we proceed as follows. Since the subchannel SNRs are statistically identical, any subchannel, e.g., subchannel n, is equally likely to be i_M. Combining this with Bayes' rule yields

$$p\left(\gamma_j = y, \gamma_{i_M} = x \middle| j \in \mathcal{I}_M^c\right) = \frac{(N-1)p\left(\gamma_j = y, \gamma_n = x, i_M = n, j \in \mathcal{I}_M^c\right)}{p\left(j \in \mathcal{I}_M^c\right)}, \quad (32)$$
$$= \frac{(N-1)p\left(\gamma_j = y, \gamma_n = x, i_M = n\right)}{1-(M/N)}. \quad (33)$$

The second step follows because $p\left(j \in \mathcal{I}_M^c\right) = 1-(M/N)$ and since $y < x$ implies that $j \in \mathcal{I}_M^c$. The probability that a subchannel SNR exceeds x is $e^{-x/(2\sigma^2)}$. Using the theory of order statistics, we get

$$p\left(\gamma_j = y, \gamma_n = x, i_M = n\right) = \binom{N-2}{M-1} \frac{e^{-\frac{y+Mx}{2\sigma^2}} \left(1 - e^{-\frac{x}{2\sigma^2}}\right)^{N-M-1}}{4\sigma^4}. \quad (34)$$

Substituting (34) in (33) yields $p\left(\gamma_j = y, \gamma_{i_M} = x \middle| j \in \mathcal{I}_M^c\right)$. Upon substituting this in (31) and integrating over y, we get

$$\mathrm{MSE} = \frac{N}{2\sigma^2}\binom{N-1}{M-1}\int_0^\infty e^{-\frac{Mx}{2\sigma^2}}\left(1 - e^{-\frac{x}{2\sigma^2}}\right)^{N-M-1} \times \left(4\sigma^4\left(1 - e^{-\frac{x}{2\sigma^2}}\right) + x^2 - \frac{x^2}{1 - e^{-\frac{x}{2\sigma^2}}}\right) dx. \quad (35)$$

Using the definition of the trigamma function $\Psi^{(1)}(z) = \int_0^\infty t e^{-zt} / \left(1 - e^{-t}\right) dt$ and simplifying yields (9).

C. Derivation of SNR Estimate for NRSR Approach

The conditional expectation in (13) can be written as

$$\tilde{\gamma}_j = \mathbb{E}\left[\gamma_j | \gamma_{j_n} = x_{j_n}, \gamma_j < x_{i_M}\right], = \frac{\int_0^{x_{i_M}} y f_{\gamma_j,\gamma_{j_n}}(y, x_{j_n}) dy}{\int_0^{x_{i_M}} f_{\gamma_j,\gamma_{j_n}}(y, x_{j_n}) dy}. \quad (36)$$

The joint PDF $f_{\gamma_j,\gamma_{j_n}}(u,v)$ of γ_j and γ_{j_n} is given by [21]

$$f_{\gamma_j,\gamma_{j_n}}(u,v) = \sum_{n=0}^{\infty} \frac{e^{-\frac{u+v}{2\sigma^2(1-\omega_j^2)}} \omega_j^{2n} (uv)^n}{\left(4\sigma^4\right)^{n+1} \left(1 - \omega_j^2\right)^{2n+1} (n!)^2}, \; u, v \geq 0. \quad (37)$$

Substituting (37) in (36), we get

$$\tilde{\gamma}_j = \frac{\sum_{l=0}^{\infty} \frac{(\lambda_j)^{2l} (x_{j_n})^l}{4^l (l!)^2} \int_0^{x_{i_M}} y^{l+1} e^{-\mu_j y} dy}{\sum_{l=0}^{\infty} \frac{(\lambda_j)^{2l} (x_{j_n})^l}{4^l (l!)^2} \int_0^{x_{i_M}} y^l e^{-\mu_j y} dy}, \quad (38)$$

where λ_j and μ_j are defined in Result 3's statement. Expressing the integrals in the numerator and denominator of (38) in terms of the incomplete gamma function yields (14).

References

[1] A. J. Goldsmith, *Wireless Communications*, 1st ed. Cambridge, U.K.: Cambridge Univ. Press, 2005.

[2] S. Sesia, M. Baker, and I. Toufik, *LTE - The UMTS Long Term Evolution: From Theory to Practice*, 2nd ed. Hoboken, NJ, USA: Wiley, 2011.

[3] D. Gesbert and M.-S. Alouini, "How much feedback is multi-user diversity really worth?" in *Proc. Int. Conf. Commun. (ICC)*, Jun. 2004, pp. 234–238.

[4] S. Sanayei and A. Nosratinia, "Opportunistic downlink transmission with limited feedback," *IEEE Trans. Inf. Theory*, vol. 53, no. 11, pp. 4363–4372, Nov. 2007.

[5] P. Svedman, S. K. Wilson, L. J. Cimini, and B. Ottersten, "A simplified opportunistic feedback and scheduling scheme for OFDM," in *Proc. Veh. Technol. Conf. (VTC-Spring)*, May 2004, pp. 1878–1882.

[6] Y.-J. Choi and S. Rangarajan, "Analysis of best channel feedback and its adaptive algorithms for multicarrier wireless data systems," *IEEE Trans. Mobile Comput.*, vol. 10, no. 8, pp. 1071–1082, Aug. 2011.

[7] J. Leinonen, J. Hamalainen, and M. Juntti, "Performance analysis of downlink OFDMA resource allocation with limited feedback," *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 2927–2937, Jun. 2009.

[8] M. Kang and K. S. Kim, "Performance analysis and optimization of best-M feedback for OFDMA systems," *IEEE Commun. Lett.*, vol. 16, no. 10, pp. 1648–1651, Oct. 2012.

[9] S. N. Ananya and N. B. Mehta, "Performance of OFDM systems with best-M feedback, scheduling, and delays for uniformly correlated subchannels," *IEEE Trans. Wireless Commun.*, vol. 14, no. 4, pp. 1983–1993, Apr. 2015.

[10] K. Pedersen *et al.*, "Frequency domain scheduling for OFDMA with limited and noisy channel feedback," in *Proc. Veh. Technol. Conf. (VTC-Fall)*, Sept. 2007, pp. 1792–1796.

[11] N. Varanese, J. Vicario, and U. Spagnolini, "On the asymptotic throughput of OFDMA systems with best-M CQI feedback," *IEEE Commun. Lett.*, vol. 1, no. 3, pp. 145–148, Jun. 2012.

[12] A. Nguyen, Y. Huang, and B. Rao, "Novel partial feedback schemes and their evaluation in an OFDMA system with CDF based scheduling," in *Proc. ASILOMAR Conf.*, Nov. 2013, pp. 1589–1593.

[13] J. Chen, R. A. Berry, and M. L. Honig, "Limited feedback schemes for downlink OFDMA based on sub-channel groups," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 8, pp. 1451–1461, Oct. 2008.

[14] S. Schwarz and M. Rupp, "Evaluation of distributed multi-user MIMO-OFDM with limited feedback," *IEEE Trans. Wireless Commun.*, vol. 13, no. 11, pp. 6081–6094, Nov. 2014.

[15] S. Y. Park, D. Park, and D. J. Love, "On scheduling for multiple-antenna wireless networks using contention-based feedback," *IEEE Trans. Commun.*, vol. 55, no. 5, pp. 1089–1089, May 2007.

[16] J. Jeon, K. Son, H.-W. Lee, and S. Chong, "Feedback reduction for multiuser OFDM systems," *IEEE Trans. Veh. Technol.*, vol. 59, no. 1, pp. 160–169, Jan. 2010.

[17] S. Guharoy and N. B. Mehta, "Joint evaluation of channel feedback schemes, rate adaptation, and scheduling in OFDMA downlinks with feedback delays," *IEEE Trans. Veh. Technol.*, vol. 62, no. 4, pp. 1719–1731, May 2013.

[18] P. Svedman, L. J. Cimini, and B. Ottersten, "Using unclaimed sub-carriers in opportunistic OFDMA systems," in *Proc. Veh. Technol. Conf. (VTC-Fall)*, Sep. 2006, pp. 1–5.

[19] M. Elkashlan, T. Khattab, C. Leung, and R. Schober, "Statistics of general order selection in correlated Nakagami fading channels," *IEEE Trans. Commun.*, vol. 56, no. 3, pp. 344–346, Mar. 2008.

[20] Y. Chen and C. Tellambura, "Distribution functions of selection combiner output in equally correlated Rayleigh, Rician, and Nakagami-M fading channels," *IEEE Trans. Commun.*, vol. 52, no. 11, pp. 1948–1956, Nov. 2004.

[21] R. K. Mallik, "On multivariate Rayleigh and exponential distributions," *IEEE Trans. Inf. Theory*, vol. 49, no. 6, pp. 1499–1515, Jun. 2003.

[22] M. Ozdemir and H. Arslan, "Channel estimation for wireless OFDM systems," *IEEE Commun. Surv. Tuts.*, vol. 9, no. 2, pp. 18–48, Jul. 2007.

[23] B. Makki and T. Eriksson, "Feedback subsampling in temporally-correlated slowly-fading channels using quantized CSI," *IEEE Trans. Commun.*, vol. 61, no. 6, pp. 2282–2294, Jun. 2013.

[24] T. Cui, F. Lu, V. Sethuraman, A. Goteti, S. Rao, and P. Subrahmanya, "Throughput optimization in high speed downlink packet access (HSDPA)," *IEEE Trans. Wireless Commun.*, vol. 10, no. 2, pp. 474–483, Feb. 2011.

[25] H. V. Poor, *An Introduction to Signal Detection and Estimation*. New York, NY, USA: Springer, 1994.

[26] M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, 9th ed. New York, NY, USA: Dover, 1972.

[27] Y. Hu and A. Ribeiro, "Optimal wireless communications with imperfect channel state information," *IEEE Trans. Signal Process.*, vol. 61, no. 11, pp. 2751–2766, Jun. 2013.

[28] A. Jalali, R. Padovani, and R. Pankaj, "Data throughput of CDMA-HDR a high efficiency-high data rate personal communication wireless system," in *Proc. Veh. Technol. Conf. (VTC-Spring)*, May 2000, pp. 1854–1858.

[29] D. Parruca, M. Grysla, S. Gortzen, and J. Gross, "Analytical model of proportional fair scheduling in interference-limited OFDMA/LTE networks," in *Proc. Veh. Technol. Conf. (VTC-Fall)*, Sep. 2013, pp. 1–7.

[30] K. L. Baum, T. A. Kostas, P. J. Sartori, and B. K. Classon, "Performance characteristics of cellular systems with different link adaptation strategies," *IEEE Trans. Veh. Technol.*, vol. 52, no. 6, pp. 1497–1507, Nov. 2003.

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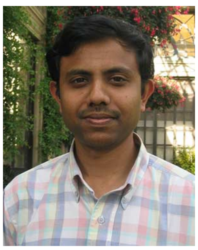

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