Joint Evaluation of Channel Feedback Schemes, Rate Adaptation, and Scheduling in OFDMA Downlinks With Feedback Delays

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***Abstract*—Orthogonal frequency-division multiple access (OFDMA) systems divide the available bandwidth into orthogonal subchannels and exploit multiuser diversity and frequency selectivity to achieve high spectral efficiencies. However, they require a significant amount of channel state feedback for scheduling and rate adaptation and are sensitive to feedback delays. We develop a comprehensive analysis for OFDMA system throughput in the presence of feedback delays as a function of the feedback scheme, frequency-domain scheduler, and rate adaptation rule. Also derived are expressions for the outage probability, which captures the inability of a subchannel to successfully carry data due to the feedback scheme or feedback delays. Our model encompasses the popular best-n and threshold-based feedback schemes and the greedy, proportional fair, and round-robin schedulers that cover a wide range of throughput versus fairness tradeoff. It helps quantify the different robustness of the schedulers to feedback overhead and delays. Even at low vehicular speeds, it shows that small feedback delays markedly degrade the throughput and increase the outage probability. Further, given the feedback delay, the throughput degradation depends primarily on the feedback overhead and not on the feedback scheme itself. We also show how to optimize the rate adaptation thresholds as a function of feedback delay.**

***Index Terms*—Channel quality feedback, feedback delays, frequency-domain scheduling, orthogonal frequency-division multiple access (OFDMA), rate adaptation.**

I. Introduction

ORTHOGONAL frequency-division multiple access (OFDMA), in which the bandwidth is divided into several orthogonal subchannels, is the physical layer access technology of choice in next-generation wireless cellular downlinks [1]. The frequency-domain scheduler at the base station (BS) can assign different users to different subchannels on the basis of their instantaneous subchannel gains. This coupled with rate adaptation enables the BS to achieve high downlink spectral efficiencies. However, for both scheduling and rate adaptation, the scheduler ideally must acquire downlink channel state information (CSI) of each subchannel for each user it serves. In the frequency-division-duplex mode of operation, the uplink and downlink channels are not reciprocal. Therefore, each user needs to feed back its downlink CSI for each subchannel to the BS, which is practically infeasible. Such feedback is also needed in the time-division-duplex mode when uplink and downlink interferences are asymmetric.

Consequently, numerous feedback reduction schemes have been studied for OFDMA systems [2]–[9]. These schemes reduce the feedback overhead but at the expense of a reduction in system throughput. In addition to the feedback scheme, the scheduler and the feedback delay both significantly affect system throughput. Feedback delays lead to outdated channel estimates at the time of transmission. If the BS underestimates the subchannel gain, then the selected user is served at a rate lower than the rate its assigned subchannel can support. On the other hand, if the BS overestimates the rate, then the data may not be decoded correctly, resulting in an outage in that subchannel. Outdated estimates can also lead to a suboptimal assignment of subchannels to users by the scheduler.

A. Related Literature

Several schemes have been proposed in the literature to reduce the feedback overhead. In [2], a user feeds back CSI only for the subchannels whose channel power gains exceed a certain threshold. Increasing the threshold reduces the feedback overhead but degrades system throughput. In [3], at most 1 bit per subchannel is fed back. Thresholding is combined with a multiple-access protocol in [4] and with subcarrier grouping in [5]. In [6] and [7], for every subchannel, users with higher subchannel gains, which are likely to be assigned resources by the BS, send their feedback earlier so as to reduce the overall feedback overhead. An altogether different best-n feedback scheme is considered in [8] and [9]. In it, each user only feeds back the indices and subchannel gains of its n subchannels that have the highest gains among all subchannels. Reducing the subset size reduces the feedback overhead but degrades the system throughput.

Considerable work has also been done on the performance analysis of the feedback schemes. In [10], the throughput of the proportional fair (PF) scheduler with the best-n and threshold-based feedback schemes was analyzed, but without feedback delays. Further, the data rate was assumed to be linearly proportional to the subchannel power gain. Similarly, in [11] and [12], comprehensive models that considered rate

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TABLE I
COMPARISON OF MODELS USED IN THE LITERATURE FOR ANALYZING THE PERFORMANCE OF OFDMA SYSTEMS

Paper	**Feedback Scheme**	**Scheduler**	**Rate Adaptation**
	Without feedback delay		
Choi, Bahk [10]	Threshold-based, best-n, and hybrid	PF and hybrid	Continuous
Donthi, Mehta [11]	Best-n	Greedy, PF, RR	Discrete
Choi, Rangarajan [13]	Best-n	Hybrid: RR with greedy/PF	Discrete with rate index feedback
Leinonen, Hamalainen, Juntti [15]	Best-n	RR	None
Torabi, Haccoun, Ajib [12]	Threshold-based	PF	Continuous, discrete
	With feedback delay		
Kuhne, Klein [16]	Complete feedback	Greedy (i.i.d. users)	Discrete
Ma, Tepedelenlioglu [17]	Complete feedback	Greedy, PF	Discrete
Falahati et al. [18]	Complete feedback	None (1 user)	Discrete

adaptation, multiple antenna techniques, threshold-based and best-n feedback schemes, and different schedulers were analyzed, but feedback delays were not considered. In [13], the best-n scheme with quantized feedback and a hybrid scheduler that partitioned users into groups was analyzed, but without feedback delays. In [14], simulations were used to characterize the performance of frequency-domain scheduling with limited feedback without feedback delays. In [15], the performance of the round-robin (RR) scheduler with the best-n feedback scheme was analyzed, but rate adaptation and feedback delays were not modeled.

The throughput of the PF scheduler with both noisy and outdated CSI was analyzed in [16], but complete feedback was assumed, and the channel gains of different users were assumed to be independent identically distributed (i.i.d.). The impact of feedback delay on the performance of the PF and greedy schedulers was also studied in [17], but complete feedback was assumed. In [18], the performance of rate adaptation with outdated channel knowledge was studied, but frequency-selective channels and reduced feedback schemes were not considered, and only one user was assumed. In [19], simulations were used to compare the downlink system throughput and uplink feedback overhead of the threshold-based and best-n feedback schemes.

A concise summary and comparison of the models used in the foregoing papers is given in Table I. We, thus, see that a thorough performance analysis and optimization that takes into account the interactions between frequency-domain schedulers, feedback schemes, rate adaptation, and feedback delays remains to be done. This will be the focus of this paper.

B. Contributions

We develop a comprehensive analysis that leads to novel expressions for the OFDMA system throughput as a function of feedback delay, feedback scheme, frequency-domain scheduler, and the rate adaptation rule. Our model encompasses the greedy, PF, and RR schedulers, given that they are commonly used and cover a wide range of throughput versus fairness tradeoff. It also encompasses the popular threshold-based and best-n channel feedback schemes. The threshold-based scheme is relevant because it has been extensively studied in the literature [2], [3], [10] and because it is sum-rate-optimal for an asymptotically large number of users [3]. The best-n scheme is equally relevant because it has been adopted in next-generation OFDMA systems such as long term evolution (LTE) [1]. We focus on discrete rate adaptation given that practical systems always choose from a finite set of modulation and coding schemes [20, Ch. 9].

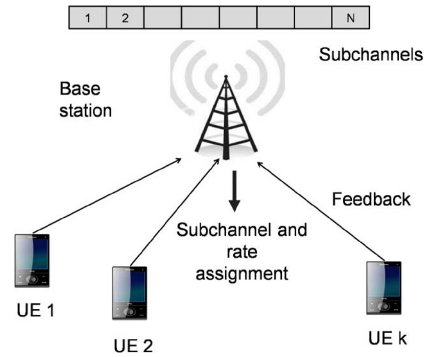

Fig. 1. System model with BS and k users (UEs). The users feed back CSI to the BS, which then assigns subchannels to users and determines the transmission rate for each subchannel.

We also derive corresponding expressions for the outage probability of a subchannel, which is the probability that no data get successfully transmitted in a subchannel due to no or outdated feedback. It is an important performance metric because it affects the behavior of higher layer mechanisms such as retransmissions and brings out the combined impact of feedback delays and the feedback scheme.

We also show how the rate adaptation thresholds, which drive discrete rate adaptation, should be optimized as a function of feedback delay. To this end, we show that a computationally feasible single parameter-based threshold scaling technique offers close-to-optimal performance and improves throughput significantly.

This paper is organized as follows. Section II presents the system model. Section III analyzes the case where the users see statistically identical channels. Section IV analyzes the general case where the channel gains of different users are not statistically identical. Section V provides simulation results and is followed by our conclusions in Section VI.

II. SYSTEM MODEL

As shown in Fig. 1, the system bandwidth is divided into N orthogonal subchannels. Let G_{ui} denote the channel power gain of subchannel i of user u at the time instant at which the channel gains are estimated and fed back for rate and subchannel assignment. The corresponding subchannel power gain at the time of transmission, which happens τ s later, is G_{ui}^d, where d denotes delay. For brevity, we will henceforth refer to the subchannel power gain as subchannel gain and use the terms user and user equipment (UE) interchangeably.

We assume a wide sense stationary Rayleigh fading process. Therefore, G_{ui} and G_{ui}^d are correlated exponential random variables (RVs) both with mean Ω_u. The joint probability density function (pdf) f_{G_{ui},G_{ui}^d} of G_{ui} and G_{ui}^d is given by [23, Ch. 6]

$$f_{G_{ui},G_{ui}^d}(x,y) = \frac{e^{-\frac{x+y}{\Omega_u(1-\rho)}}}{\Omega_u^2(1-\rho)} I_0\left(\frac{2\sqrt{\rho}\sqrt{xy}}{\Omega_u(1-\rho)}\right), \quad x,y \geq 0 \tag{1}$$

where $I_0(\cdot)$ is the 0th-order modified Bessel function of the first kind [22, Ch. 9]. As per Jakes' fading model [21], the correlation coefficient ρ is $\rho = J_0^2(2\pi\phi_d\tau)$, where ϕ_d is the maximum Doppler spread, and $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind [22, Ch. 9].

A. Frequency-Domain Schedulers

The scheduler at the BS uses the CSI fed back by the users to decide which user is to be served by each subchannel. We consider the following three schedulers:

Greedy scheduler: For a subchannel, among the UEs that fed back their subchannel gains for it, the greedy scheduler selects the UE with the highest gain. It exploits multiuser diversity but is unfair.

RR scheduler: For each subchannel, the RR scheduler determines which UE to serve in a sequential manner without considering the subchannel gains of the UEs. It ensures fairness but does not exploit multiuser diversity.

PF scheduler: For each subchannel, among the UEs that fed back their gains for this subchannel, the PF scheduler selects the UE with the largest *normalized subchannel gain*, which is the ratio of the subchannel gain to the mean subchannel gain [10]–[12], [17], [24].[1] It ensures fairness while also exploiting multiuser diversity.

B. Reduced Feedback Schemes

Threshold-based feedback: For every subchannel, an UE feeds back its gain only if it exceeds a threshold λ for the greedy scheduler and only if its normalized subchannel gain exceeds λ for the PF scheduler. The threshold λ is chosen so that an average fraction β of the users send feedback.

Best-n feedback: Each UE feeds back the subchannel gains and indices of only its n best subchannels, which are defined as those that have the highest gains among the N subchannels. Thus, the scheme reduces the feedback overhead of each user to a fraction $\beta = n/N$ of complete (per-subchannel) feedback.

We say that a user reports subchannel i when it feeds back its gain for this subchannel. We will call β the feedback overhead.

C. Discrete Rate Adaptation

For all the schedulers, the BS still adapts the transmit rate of each subchannel as a function of the subchannel gain reported by the selected user. The subchannel gain range is divided into $M-1$ intervals by a set of rate adaptation thresholds $T_1, T_2, \ldots, T_M$, where $0 = T_1 < T_2 < \cdots < T_M = \infty$. The set of available rates is $\{r_1, \ldots, r_{M-1}\}$, where $0 = r_1 < r_2 < \cdots < r_{M-1}$. For subchannel i, let the user selected be denoted by S_i. If $G_{S_i i} \in [T_j, T_{j+1})$, then the selected user is served with rate r_j in subchannel i. Further, if $G_{S_i i}^d < T_j$, then transmission over the subchannel fails. Thus, the selected user successfully receives at rate r_j only if $T_j \leq G_{S_i i} < T_{j+1}$ and $G_{S_i i}^d \geq T_j$.

D. Assumptions

To develop a tractable model that captures the interactions in a wideband system of the scheduler, rate adaptation scheme, and feedback scheme, we make the following assumptions. As explained below, these assumptions are also employed often in the related literature.

A) The gains of the different subchannels of a user are assumed to be i.i.d. This assumption is valid when the coherence bandwidth of the channel is close to the subchannel bandwidth and has also been assumed in [3], [5], [11], [15], and [16]. This assumption is not required for analyzing the threshold-based feedback scheme. However, it makes the analysis for the best-n feedback scheme tractable.

B) The users know the subchannel gains without error [9]–[11], [15] and feed back the gains in accordance with the feedback scheme. The analysis can be generalized to handle the more practical scenario where only the index of the rate that the subchannel can support is fed back [1], [13]. However, the details are omitted due to space constraints.[2]

C) We focus on a single-cell system, as has been done in [5], [11], [12], [15], [16], and [24]. While [26] explicitly accounted for fading of the cochannel interfering links, it focused on frequency-flat fading. In [27], a simulation-based approach was used to incorporate cochannel interferers. However, rate adaptation and scheduling were not considered.

Hybrid automatic repeat request (HARQ) is beyond the scope of this paper.

E. Notation

We will denote the throughputs per subchannel for the greedy, PF, and RR schedulers by η_G, η_{PF}, and η_{RR}, respectively. Moreover, P_G^{out}, $P_{\mathrm{PF}}^{\mathrm{out}}$, and $P_{\mathrm{RR}}^{\mathrm{out}}$ will denote the corresponding outage probabilities per subchannel. $\Pr(A)$ denotes the probability of an event A, and $\Pr(A|B)$ denotes the

[1] In practice, an alternate version of the PF scheduler is used. It involves selecting users on the basis of the ratio of the instantaneous rate and a moving window average-based mean rate of each user [25]. This shares similar characteristics, such as allotting almost the same amount of time to each user as our formulation, which is also used in the literature. We assume that the BS knows Ω_u, which can be acquired by a moving window average.

[2] Our investigations indicate that subchannel gain feedback overestimates the system throughput by 0–9% for up to 35 users/cell compared to rate index feedback and therefore provides a good reference benchmark. Note that in the absence of feedback delays, this distinction does not affect system throughput and is the scenario that has been modeled in [13].

conditional probability of A given B. Further, $\{X_i\}_{i=a}^{b}$ denotes the sequence $X_a, X_{a+1}, \ldots, X_b$. The notation $\{X_i\}_{i=a}^{b} \geq \lambda$ will denote $X_a \geq \lambda, X_{a+1} \geq \lambda, \ldots, X_b \geq \lambda$. The notation $\{X_i\}_{i=a}^{b} < \lambda$ is defined in a similar manner.

III. Performance Analysis: Independent and Identically Distributed Users Scenario

Since the subchannel gains are statistically identical for each user, it is sufficient to focus on a given subchannel, for example, subchannel i. To gain intuition, the analysis is first developed for the case where the users see statistically identical channels. Thus, in this section, $\Omega_u = \Omega$ for all users $1 \leq u \leq k$. For i.i.d. users, the greedy and PF schedulers are the same. We therefore focus on the greedy and RR schedulers in this section.

A. Threshold-Based Feedback Scheme

Let w denote the index such that $T_w \leq \lambda < T_{w+1}$.

1) Greedy Scheduler:

Result 1: The throughput per subchannel for threshold-based feedback with the greedy scheduler for i.i.d. users is

$$\eta_G = \sum_{l=1}^{k} l \binom{k}{l} e^{-\frac{\lambda}{\Omega}(l-1)} \left(1 - e^{-\frac{\lambda}{\Omega}}\right)^{k-l} \sum_{j=w}^{M-1} r_j \int_{\max(T_j,\lambda)}^{T_{j+1}} \times \int_{T_j}^{\infty} \left(1 - e^{-\frac{x-\lambda}{\Omega}}\right)^{l-1} \frac{e^{-\frac{x+y}{\Omega(1-\rho)}} I_0\left(\frac{2\sqrt{\rho}\sqrt{xy}}{\Omega(1-\rho)}\right)}{\Omega^2(1-\rho)} dy dx. \tag{2}$$

$$\geq \frac{1}{\Omega^2(1-\rho)} \sum_{l=1}^{k} l \binom{k}{l} e^{-\frac{\lambda(l-1)}{\Omega}} \left(1 - e^{-\frac{\lambda}{\Omega}}\right)^{k-l} \sum_{q=0}^{l-1} (-1)^q \times \binom{l-1}{q} e^{\frac{q\lambda}{\Omega}} \sum_{j=w}^{M-1} r_j \xi_q^{[\Omega]}(\max(T_j,\lambda), T_{j+1}; T_j) \tag{3}$$

where

$$\xi_q^{[\Omega]}(a,b;c) \triangleq \Omega^2(1-\rho)^2 \sum_{p=0}^{L} \frac{\rho^p \Gamma\left(p+1, \frac{c}{\Omega(1-\rho)}\right)}{(p!)^2 (q-\rho q+1)^{p+1}} \times \left[\gamma\left(p+1, \frac{b(q-\rho q+1)}{\Omega(1-\rho)}\right) - \gamma\left(p+1, \frac{a(q-\rho q+1)}{\Omega(1-\rho)}\right)\right] \tag{4}$$

and $\gamma(a,b) \triangleq \int_0^b e^{-x} x^{a-1}\, dx$ and $\Gamma(a,b) \triangleq \int_b^\infty e^{-x} x^{a-1}\, dx$ are the lower and upper incomplete gamma functions, respectively [22, Ch. 6].

Proof: The derivation is relegated to Appendix A. ■

The lower bound in (3) becomes tighter as the number of terms L in (4) increases. Since it is tight, we show it as being approximately equal to the exact expression henceforth.

We now evaluate the outage probability. Outage occurs in a subchannel due to two reasons: 1) The gains of all the users for that subchannel at the time of feedback are less than λ, as a result of which the subchannel does not get assigned, or 2) due to outdated channel estimates.

Result 2: The outage probability for a subchannel for the threshold-based feedback scheme with the greedy scheduler for i.i.d. users is given by

$$P_G^{\text{out}} = \left(1 - e^{-\frac{\lambda}{\Omega}}\right)^k + \sum_{l=1}^{k} l \binom{k}{l} e^{-\frac{\lambda}{\Omega}(l-1)} \times \left(1 - e^{-\frac{\lambda}{\Omega}}\right)^{k-l} \sum_{j=w}^{M-1} \int_{\max(T_j,\lambda)}^{T_{j+1}} \int_{0}^{T_j} \times \frac{\left(1-e^{-\frac{x-\lambda}{\Omega}}\right)^{l-1} e^{-\frac{x+y}{\Omega(1-\rho)}} I_0\left(\frac{2\sqrt{\rho}\sqrt{xy}}{\Omega(1-\rho)}\right)}{\Omega^2(1-\rho)} dy dx. \tag{5}$$

Further, it can be approximated as

$$P_G^{\text{out}} \approx \left(1 - e^{-\frac{\lambda}{\Omega}}\right)^k + \sum_{l=1}^{k} \frac{l\binom{k}{l} e^{-\frac{\lambda(l-1)}{\Omega}} \left(1 - e^{-\frac{\lambda}{\Omega}}\right)^{k-l}}{\Omega^2(1-\rho)} \times \sum_{q=0}^{l-1} (-1)^q \binom{l-1}{q} e^{\frac{q\lambda}{\Omega}} \sum_{j=w}^{M-1} \varpi_q^{[\Omega]}\left(\max(T_j,\lambda), T_{j+1}; T_j\right) \tag{6}$$

where

$$\varpi_q^{[\Omega]}(a,b;c) \triangleq \Omega^2(1-\rho)^2 \sum_{p=0}^{L} \frac{\rho^p \gamma\left(p+1, \frac{c}{\Omega(1-\rho)}\right)}{(p!)^2 (q-\rho q+1)^{p+1}} \times \left[\gamma\left(p+1, \frac{b(q-\rho q+1)}{\Omega(1-\rho)}\right) - \gamma\left(p+1, \frac{a(q-\rho q+1)}{\Omega(1-\rho)}\right)\right]. \tag{7}$$

Proof: The derivation is relegated to Appendix B. ■

2) RR Scheduler: Since the RR scheduler serves the users sequentially, its throughput and outage probability are the same as that of a greedy scheduler that serves one user [26]. Therefore, substituting $k = 1$ in (3) and (6) yields

$$\eta_{\text{RR}} \approx \frac{\sum_{j=w}^{M-1} r_j \xi_0^{[\Omega]}\left(\max(T_j,\lambda), T_{j+1}; T_j\right)}{\Omega^2(1-\rho)} \tag{8}$$

$$P_{\text{RR}}^{\text{out}} \approx 1 - e^{-\frac{\lambda}{\Omega}} + \frac{\sum_{j=w}^{M-1} \varpi_0^{[\Omega]}\left(\max(T_j,\lambda), T_{j+1}; T_j\right)}{\Omega^2(1-\rho)}. \tag{9}$$

B. Best-n Feedback Scheme

Let $\beth_{ui}$ denote the number of subchannels of user u whose gains exceed the gain G_{ui} of its subchannel i. If user u reports subchannel i, then at most $n-1$ subchannels of user u can have gains that exceed G_{ui}; thus, $\beth_{ui} \leq n-1$ [10].

1) Greedy Scheduler:

Result 3: The throughput per subchannel of best-n feedback with the greedy scheduler for i.i.d. users is

$$\eta_G \approx \frac{1}{\Omega^2(1-\rho)} \sum_{l=1}^{k} l \binom{k}{l} \left(1-\frac{n}{N}\right)^{k-l} \sum_{j=2}^{M-1} r_j \times \sum_{p=0}^{L} \frac{\rho^p}{(p!)^2 \left(\Omega(1-\rho)\right)^{p-1}} \times \Gamma\left(p+1, \frac{T_j}{\Omega(1-\rho)}\right) \sum_{m=0}^{n-1} \binom{N-1}{m} \times \int_{T_j}^{T_{j+1}} x^p e^{-x\left(\frac{m}{\Omega}+\frac{1}{\Omega(1-\rho)}\right)} \left(1-e^{\frac{x}{\Omega}}\right)^{N-1-m} \Upsilon_{\Omega}^{l-1}(x)dx \quad (10)$$

where

$$\Upsilon_{\Omega}(x) \triangleq \sum_{r=0}^{n-1} \binom{N-1}{r} \times \sum_{q=0}^{N-1-r} \frac{(-1)^q \binom{N-1-r}{q} \left(1-e^{-\frac{(q+r+1)x}{\Omega}}\right)}{q+r+1}. \quad (11)$$

Proof: The derivation is given in Appendix C. ■

The single integral in (10) is evaluated numerically.

Result 4: For best-n feedback with the greedy scheduler and i.i.d. users, the outage probability is

$$P_G^{\text{out}} \approx \left(1-\frac{n}{N}\right)^k + \frac{1}{\Omega^2(1-\rho)} \sum_{l=1}^{k} l \binom{k}{l} \left(1-\frac{n}{N}\right)^{k-l} \times \sum_{j=2}^{M-1} \sum_{p=0}^{L} \frac{\rho^p \gamma\left(p+1, \frac{T_j}{\Omega(1-\rho)}\right)}{(p!)^2 \left(\Omega(1-\rho)\right)^{p-1}} \sum_{m=0}^{n-1} \binom{N-1}{m} \times \int_{T_j}^{T_{j+1}} x^p e^{-x\left(\frac{m}{\Omega}+\frac{1}{\Omega(1-\rho)}\right)} \left(1-e^{\frac{x}{\Omega}}\right)^{N-1-m} \Upsilon_{\Omega}^{l-1}(x)dx. \quad (12)$$

Proof: The derivation is relegated to Appendix D ■

2) RR Scheduler: As in Section III-A2, substituting $k=1$ in (10) and (12) and simplifying yields

$$\eta_{\text{RR}} \approx \frac{1}{\Omega^2(1-\rho)} \sum_{j=2}^{M-1} r_j \sum_{m=0}^{n-1} \binom{N-1}{m} \sum_{q=0}^{N-m-1} (-1)^q \times \binom{N-m-1}{q} \xi_{q+m}^{[\Omega]}(T_j, T_{j+1}; T_j) \quad (13)$$

$$P_{\text{RR}}^{\text{out}} \approx 1-\frac{n}{N} + \frac{1}{\Omega^2(1-\rho)} \sum_{j=2}^{M-1} \sum_{m=0}^{n-1} \binom{N-1}{m} \times \sum_{q=0}^{N-m-1} (-1)^q \binom{N-m-1}{q} \varpi_{q+m}^{[\Omega]}(T_j, T_{j+1}; T_j). \quad (14)$$

IV. Performance Analysis: Non-Independent and Identically Distributed Users

We now analyze the general scenario in which the subchannel gains of different users are not statistically identical but are still independent. The analysis gets more involved now because we need to keep track of which specific subset of users fed back. Let ϑ denote the set $\{1, 2, \ldots, k\}$ of all the k UEs in the system. Further, let ϑ_m^l denote the mth subset of size l of ϑ; there are $\binom{k}{l}$ such subsets.

A. Threshold-Based Feedback Scheme

1) Greedy Scheduler:

Result 5: For threshold-based feedback with greedy scheduler and non-i.i.d. users, the throughput per subchannel is

$$\eta_G \approx \sum_{l=1}^{k} \sum_{m=1}^{\binom{k}{l}} \left[\prod_{q\in\vartheta\setminus\vartheta_m^l} \left(1-e^{-\frac{\lambda}{\Omega_q}}\right)\right] \sum_{u\in\vartheta_m^l} \sum_{p=0}^{L} \sum_{j=w}^{M-1} r_j \times \frac{\rho^p \Gamma\left(p+1, \frac{T_j}{\Omega_u(1-\rho)}\right)}{(p!)^2 \Omega_u^{p+1}(1-\rho)^p} \int_{\max(T_j,\lambda)}^{T_{j+1}} x^p e^{-\frac{x}{\Omega_u(1-\rho)}} \times \left[\prod_{b\in\vartheta_m^l\setminus\{u\}} \left(e^{-\frac{\lambda}{\Omega_b}} - e^{-\frac{x}{\Omega_b}}\right)\right] dx. \quad (15)$$

Proof: The derivation is given in Appendix E. ■

Similarly, it can be shown that

$$P_G^{\text{out}} \approx \prod_{v=1}^{k} \left(1-e^{-\frac{\lambda}{\Omega_v}}\right) + \sum_{l=1}^{k} \sum_{m=1}^{\binom{k}{l}} \left[\prod_{q\in\vartheta\setminus\vartheta_m^l} \left(1-e^{-\frac{\lambda}{\Omega_q}}\right)\right] \times \sum_{u\in\vartheta_m^l} \sum_{p=0}^{L} \sum_{j=w}^{M-1} \frac{\rho^p \gamma\left(p+1, \frac{T_j}{\Omega_u(1-\rho)}\right)}{(p!)^2 \Omega_u^{p+1}(1-\rho)^p} \int_{\max(T_j,\lambda)}^{T_{j+1}} x^p \times e^{-\frac{x}{\Omega_u(1-\rho)}} \left[\prod_{b\in\vartheta_m^l\setminus\{u\}} \left(e^{-\frac{\lambda}{\Omega_b}} - e^{-\frac{x}{\Omega_b}}\right)\right] dx. \quad (16)$$

The single integrals in (15) and (16) are evaluated numerically.

2) PF Scheduler: For subchannel i, let the normalized channel gain of user u be denoted by $\widetilde{G}_{ui} = (G_{ui})/(\Omega_u)$. It is an exponential RV with unit mean. Thus, user u feeds back it subchannel gain G_{ui} only if $\widetilde{G}_{ui} \geq \lambda$. Similarly, let the normalized subchannel gain at the time of transmission be denoted by $\widetilde{G}_{ui}^d = (G_{ui}^d/\Omega_u)$. For user u, let w_u be the index such that $(T_{w_u}/\Omega_u) \leq \lambda < (T_{w_u+1}/\Omega_u)$.

Result 6: For threshold-based feedback with the PF scheduler and non-i.i.d. users, the throughput per subchannel is

$$\eta_{\text{PF}} \approx \frac{1}{1-\rho} \sum_{l=1}^{k} e^{-\lambda(l-1)} (1-e^{-\lambda})^{k-l} \sum_{m=1}^{\binom{k}{l}} \sum_{u\in\vartheta_m^l} \sum_{q=0}^{l-1} (-1)^q \times \binom{l-1}{q} e^{q\lambda} \sum_{j=w_u}^{M-1} r_j \xi_q^{[1]} \left(\max\left(\frac{T_j}{\Omega_u}, \lambda\right), \frac{T_{j+1}}{\Omega_u}; \frac{T_j}{\Omega_u}\right). \quad (17)$$

Proof: The derivation is relegated to Appendix F. ■

Similarly, the outage probability is given by

$$P_{\mathrm{PF}}^{\mathrm{out}} \approx (1-e^{-\lambda})^k + \frac{1}{1-\rho}\sum_{l=1}^{k} e^{-\lambda(l-1)}(1-e^{-\lambda})^{k-l} \times \sum_{m=1}^{\binom{k}{l}} \sum_{u\in\vartheta_m^l} \sum_{q=0}^{l-1} (-1)^q \binom{l-1}{q} e^{q\lambda} \sum_{j=w_u}^{M-1} \varpi_q^{[1]} \times \left(\max\left(\frac{T_j}{\Omega_u},\lambda\right), \frac{T_{j+1}}{\Omega_u}; \frac{T_j}{\Omega_u}\right). \quad (18)$$

3) RR Scheduler: For the RR scheduler, the corresponding expressions for the average throughput and the outage probability can be shown to be

$$\eta_{\mathrm{RR}} \approx \frac{1}{k}\sum_{u=1}^{k} \frac{1}{\Omega_u^2(1-\rho)} \sum_{j=w}^{M-1} r_j \xi_0^{[\Omega_u]}\left(\max(T_j,\lambda), T_{j+1}; T_j\right) \quad (19)$$

$$P_{\mathrm{RR}}^{\mathrm{out}} \approx \frac{1}{k}\sum_{u=1}^{k}\left[1-e^{-\frac{\lambda}{\Omega_u}} + \frac{1}{\Omega_u^2(1-\rho)}\sum_{j=w+1}^{M-1} \varpi_0^{[\Omega_u]} \times \left(\max(T_j,\lambda), T_{j+1}; T_j\right)\right] \quad (20)$$

where $\xi_0^{[\Omega_u]}(.,.;.)$ and $\varpi_0^{[\Omega_u]}(.,.;.)$ are defined in (4) and (7), respectively.

B. Best-n Feedback Scheme

1) Greedy Scheduler:

Result 7: The throughput per subchannel of best-n feedback with greedy scheduler and non-i.i.d. users is

$$\eta_G \approx \sum_{l=1}^{k}\left(1-\frac{n}{N}\right)^{k-l} \sum_{m=1}^{\binom{k}{l}} \sum_{u\in\vartheta_m^l} \sum_{j=2}^{M-1} r_j \sum_{s=0}^{n-1}\binom{N-1}{s} \times \sum_{p=0}^{L} \frac{\rho^p \Gamma\left(p+1, \frac{T_j}{\Omega_u(1-\rho)}\right)}{(p!)^2 \Omega_u^{p+1}(1-\rho)^p} \int_{T_j}^{T_{j+1}} x^p e^{-x\left(\frac{s}{\Omega_u}+\frac{1}{\Omega_u(1-\rho)}\right)} \times \left(1-e^{\frac{x}{\Omega_u}}\right)^{N-1-s}\left[\prod_{b\in\vartheta_m^l\setminus\{u\}} \Upsilon_{\Omega_b}(x)\right] dx. \quad (21)$$

Proof: The derivation is given in Appendix G. ■

Similarly, the outage probability of a subchannel is given by

$$P_G^{\mathrm{out}} \approx \left(1-\frac{n}{N}\right)^k + \sum_{l=1}^{k}\left(1-\frac{n}{N}\right)^{k-l} \sum_{m=1}^{\binom{k}{l}} \sum_{u\in\vartheta_m^l} \sum_{j=2}^{M-1} \sum_{s=0}^{n-1} \times \binom{N-1}{s} \sum_{p=0}^{L} \frac{\rho^p \gamma\left(p+1, \frac{T_j}{\Omega_u(1-\rho)}\right)}{(p!)^2 \Omega_u^{p+1}(1-\rho)^p} \int_{T_j}^{T_{j+1}} x^p e^{-\frac{xs}{\Omega_u}} \times e^{-\frac{x}{\Omega_u(1-\rho)}}\left(1-e^{\frac{x}{\Omega_u}}\right)^{N-1-s}\left[\prod_{b\in\vartheta_m^l\setminus\{u\}} \Upsilon_{\Omega_b}(x)\right] dx. \quad (22)$$

The single integrals above are evaluated numerically.

2) PF Scheduler:

Result 8: The throughput per subchannel of best-n feedback with the PF scheduler and non-i.i.d. users is

$$\eta_{\mathrm{PF}} \approx \sum_{l=1}^{k}\left(1-\frac{n}{N}\right)^{k-l} \sum_{m=1}^{\binom{k}{l}} \sum_{u\in\vartheta_m^l} \sum_{j=2}^{M-1} r_j \sum_{s=0}^{n-1}\binom{N-1}{s} \times \sum_{p=0}^{L} \frac{\rho^p \Gamma\left(p+1, \frac{T_j}{\Omega_u(1-\rho)}\right)}{(p!)^2 \Omega_u^{p+1}(1-\rho)^p} \int_{T_j}^{T_{j+1}} x^p e^{-x\left(\frac{s}{\Omega_u}+\frac{1}{\Omega_u(1-\rho)}\right)} \times \left(1-e^{\frac{x}{\Omega_u}}\right)^{N-1-s}\left[\prod_{b\in\vartheta_m^l\setminus\{u\}} \Upsilon_{\Omega_b}\left(\frac{\Omega_b x}{\Omega_u}\right)\right] dx. \quad (23)$$

Proof: The derivation is relegated to Appendix H. ■

Similarly, the outage probability is given by

$$P_{\mathrm{PF}}^{\mathrm{out}} \approx \left(1-\frac{n}{N}\right)^k + \sum_{l=1}^{k}\left(1-\frac{n}{N}\right)^{k-l} \sum_{m=1}^{\binom{k}{l}} \sum_{u\in\vartheta_m^l} \sum_{j=2}^{M-1} \sum_{s=0}^{n-1} \times \binom{N-1}{s} \sum_{p=0}^{L} \frac{\rho^p \gamma\left(p+1, \frac{T_j}{\Omega_u(1-\rho)}\right)}{(p!)^2 \Omega_u^{p+1}(1-\rho)^p} \int_{T_j}^{T_{j+1}} x^p \times e^{-x\left(\frac{s}{\Omega_u}+\frac{1}{\Omega_u(1-\rho)}\right)}\left(1-e^{\frac{x}{\Omega_u}}\right)^{N-1-s} \times \left[\prod_{b\in\vartheta_m^l\setminus\{u\}} \Upsilon_{\Omega_b}\left(\frac{\Omega_b x}{\Omega_u}\right)\right] dx. \quad (24)$$

3) RR Scheduler: The expressions for the throughput and outage probability are

$$\eta_{\mathrm{RR}} \approx \frac{1}{k(1-\rho)}\sum_{u=1}^{k}\frac{1}{\Omega_u^2}\sum_{j=2}^{M-1} r_j \sum_{m=0}^{n-1}\binom{N-1}{m}\sum_{q=0}^{N-m-1}(-1)^q \times \binom{N-m-1}{q}\xi_{q+m}^{[\Omega_u]}(T_j, T_{j+1}; T_j) \quad (25)$$

$$P_{\mathrm{RR}}^{\mathrm{out}} \approx 1-\frac{n}{N} + \frac{1}{k(1-\rho)}\sum_{u=1}^{k}\frac{1}{\Omega_u^2}\sum_{j=2}^{M-1}\sum_{m=0}^{n-1}\binom{N-1}{m} \times \sum_{q=0}^{N-m-1}(-1)^q\binom{N-m-1}{q}\varpi_{q+m}^{[\Omega_u]}(T_j, T_{j+1}; T_j). \quad (26)$$

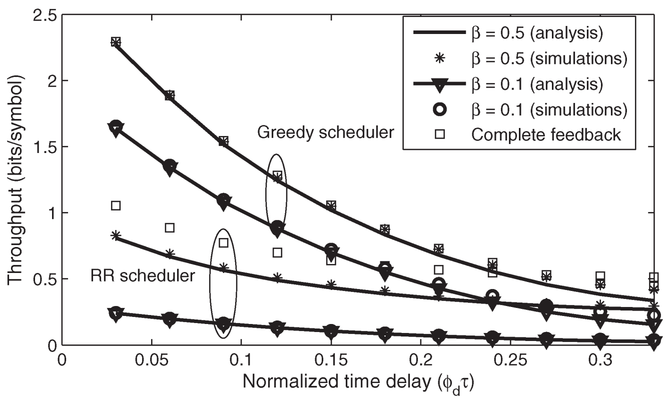

Fig. 2. Throughput as a function of $\phi_d\tau$ for threshold-based feedback with greedy and RR schedulers ($k = 10$ i.i.d. users).

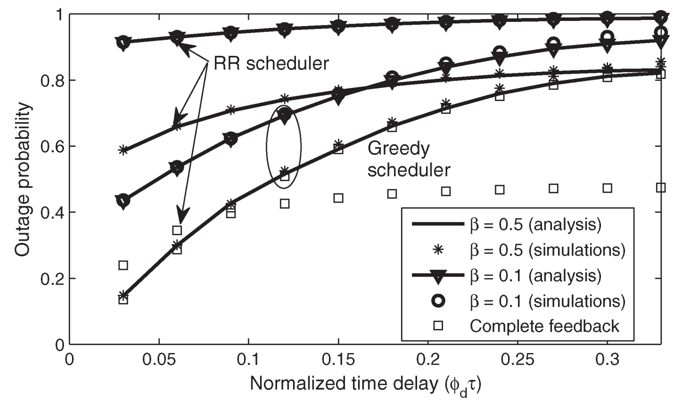

Fig. 3. Outage probability as a function of $\phi_d\tau$ for threshold-based feedback with greedy and RR schedulers ($k = 10$ i.i.d. users).

V. Simulation Results and Comparisons

We now present extensive results from Monte Carlo simulations that average over 10^5 samples to independently verify our analytical results. The independent time-varying subchannel fading gain time traces for the N subchannels of each of the k users are generated using the modified Jakes' simulator [21] with 512 oscillators. Note that the alternative simulator given in [28] can also be used. We use $\Omega = 6$ (7.78 dB) and $N = 20$ subchannels in our simulations. The rate adaptation thresholds are generated as per

$$r_i = \log_2(1 + \zeta T_i) \tag{27}$$

where ζ models the coding loss of a practical code [29]. In our simulations, we set $\zeta = 0.398$ as per [29]. Since these thresholds are not chosen based on the feedback delay, we will refer to them as zero-Doppler thresholds. The $M - 1 = 16$ rates are the ones fed back in LTE [1, Tbl. 7.2.3-1] and range from $r_1 = 0$, $r_2 = 0.15$ bits/symbol to $r_{16} = 5.55$ bits/symbol.

A. I.I.D. Users

Fig. 2 plots the throughput as a function of the normalized feedback delay $\phi_d\tau$ for the threshold-based feedback scheme for the greedy and RR schedulers for $\beta = 0.1$ and 0.5.[3] Also shown is the corresponding throughput for complete feedback ($\beta = 1$), in which each user always feeds back the gains of all its subchannels. The corresponding outage probability curves are shown in Fig. 3. We see that the analytical results match the simulation results well. The minor mismatch between the two occurs because of the limitations of the Jakes' simulator in generating multiple completely independent Rayleigh fading processes. The mismatch increases as $\phi_d\tau$ increases because to the inability of the Jakes' simulator to generate a correlation that is exactly given by Jakes' model.

As expected, the throughput degrades as the feedback delay increases. However, the degradation is quite marked. For example, when $\phi_d\tau = 0.06$, the throughput decreases by 29%. At a carrier frequency of 2 GHz and a user speed of 30 km/h, this corresponds to a feedback delay of just 1.1 ms, which is smaller than what next-generation systems such as LTE are designed for. For a feedback delay of 5 ms, the throughput decreases by 78%.

Further, the outage probabilities of both schedulers can be quite large. It is largest for the RR scheduler since it does not exploit multiuser diversity. The large outage probability arises due to two reasons. First, for smaller feedback overheads, the odds that none of the users report their gain for some of the subchannels increase. Second, as $\phi_d\tau$ increases, the odds that the subchannel gain at the time of transmission falls below the lower threshold required for ensuring successful reception for the chosen rate increase. For example, for $\phi_d\tau = 0.1$ and $k = 10$ users, the outage probability of complete feedback is 0.43, which clearly is entirely due to feedback delays. When $\beta = 0.5$ is instead used, the outage probability marginally increases by 1%. However, it increases by 69% when $\phi_d\tau$ increases from 0.05 to 0.1.

We also see that the different schedulers exhibit different sensitivities to the feedback overhead β. For the greedy scheduler, $\beta = 0.5$ entails a negligible decrease in throughput and a negligible increase in outage probability compared to complete feedback. However, this is not the case for the RR scheduler, for which the throughput degrades by 27%. When $\beta = 0.1$, the throughputs of the greedy and RR schedulers decrease by 45% and 72%, respectively, compared with complete feedback. Thus, the RR scheduler is more sensitive to the feedback overhead than the greedy scheduler. The rate of decrease of throughput with feedback delay is more for the greedy scheduler.

Figs. 4 and 5 plot the throughputs and outage probabilities, respectively, as a function of $\phi_d\tau$ for the best-n feedback scheme for the greedy and RR schedulers. Also plotted are the corresponding results for complete feedback. We again observe a good match between the analytical and simulation results and the sensitivity of the throughput to $\phi_d\tau$. Comparing these two figures with their counterparts for the threshold-based feedback scheme, we observe that, given $\phi_d\tau$ and β, the relative reduction in throughput and the relative increase in outage probability are almost the same for the two feedback schemes.

[3] The number of terms L used to accurately compute the throughput depends on $\phi_d\tau$. For $\phi_d\tau \geq 0.15$, $L = 10$ suffices, for $0.06 \leq \phi_d\tau < 0.15$, $L = 25$ suffices, and for $\phi_d\tau < 0.06$, $L = 50$ suffices. As $\phi_d\tau$ decreases, ρ approaches 1. As a result, L increases because of the presence of the $1 - \rho$ term in the denominators in (3) and (4), which requires more accurate computations.

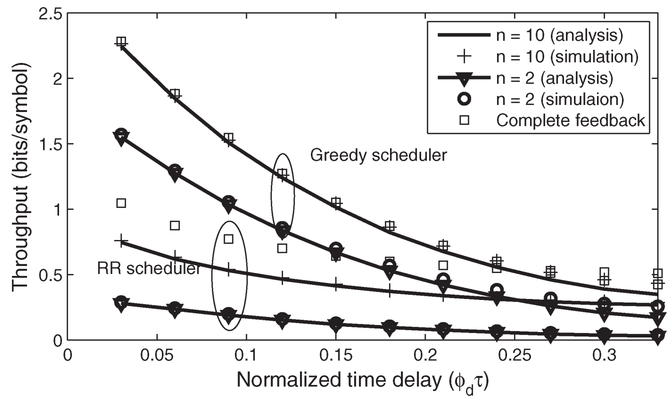

Fig. 4. Throughput as a function of $\phi_d\tau$ for best-n feedback with greedy and RR schedulers ($k = 10$ i.i.d. users).

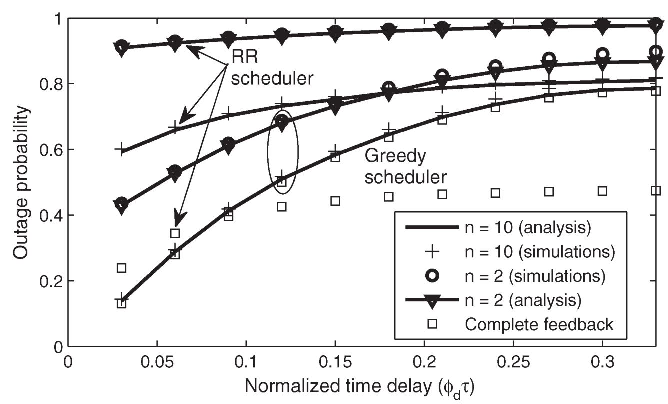

Fig. 5. Outage probability as a function of $\phi_d\tau$ for best-n feedback with greedy and RR schedulers ($k = 10$ i.i.d. users).

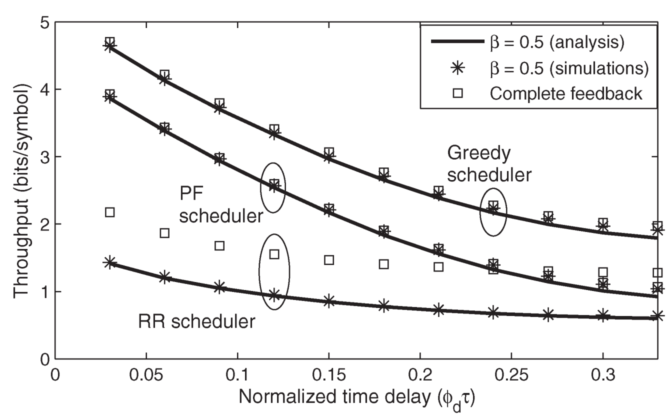

Fig. 6. Non-i.i.d. users: throughput as a function of $\phi_d\tau$ for threshold-based feedback with greedy, RR, and PF schedulers ($k = 10$ users and $\alpha = 1.4$).

B. Non-i.i.d. Users

To model non-i.i.d. users, we set the mean subchannel gain of user u to be $\Omega_u = \Omega\alpha^{u-1}$, where $\alpha > 1$ and $1 \leq u \leq k$. Increasing α makes the users more asymmetric. This parametric model ensures asymmetry while also providing a way to control it and study its effects. Fig. 6 plots the throughput as a function of $\phi_d\tau$ for the threshold-based feedback scheme for all the three schedulers. Also shown is the throughput with complete feedback. As expected, the throughput of the RR scheduler is lower than that of the PF scheduler, which is lower than

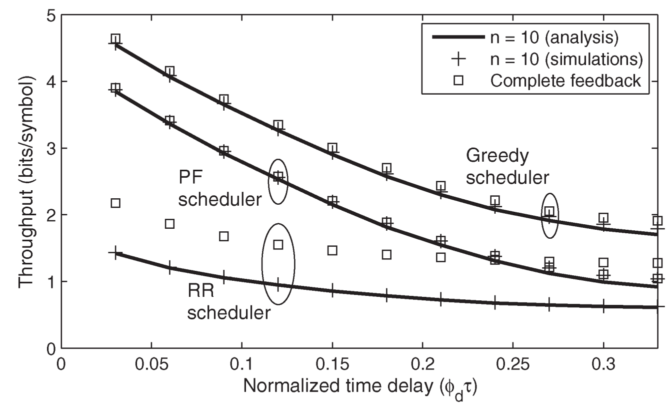

Fig. 7. Non-i.i.d. users: throughput as a function of $\phi_d\tau$ for best-n feedback with greedy, RR, and PF schedulers ($k = 10$ users and $\alpha = 1.4$).

that of the greedy scheduler. For $\alpha = 1.4$, $\phi_d\tau = 0.06$, $\beta = 0.5$, and $k = 10$ users, the throughputs of the greedy, PF, and RR schedulers degrade by 19%, 21%, and 30%, respectively, compared to the zero feedback delay case. This is smaller than the respective degradations of 29%, 29%, and 33% for the i.i.d. users ($\alpha = 1$) scenario. Fig. 7 shows the corresponding results for the best-n feedback scheme. As in the i.i.d. users scenario, given the feedback overhead and the normalized feedback delay, the relative degradation in throughput is almost the same for the two feedback schemes. In addition, as in the i.i.d. user scenario, a minor gap between the simulation and analysis curves arises for large values of $\phi_d\tau$.

C. Optimization of Rate Adaptation Thresholds

Thus far, we have plotted the throughput and outage probability when zero-Doppler thresholds, derived as per (27), are used as rate adaptation thresholds. We now investigate how the thresholds should be optimized as a function of feedback delay. The general problem requires optimizing $M - 2$ thresholds $T_2, \ldots, T_{M-1}$, which is analytically intractable and computationally cumbersome, even for M as small as 8. We therefore consider a single parameter optimization in which the adaptation thresholds are scaled by a factor θ. Thus, the selected user is served with rate r_j if its subchannel gain lies in the interval $[\theta T_j, \theta T_{j+1})$. An outage occurs only if the subchannel gain at the time of transmission falls below T_j.

We first evaluate the efficiency of the single parameter optimization technique for a system with $M = 5$, for which a brute-force optimization is computationally feasible. The rates are set at $r_1 = 0$, $r_2 = 1$, $r_3 = 2$, and $r_4 = 3$ bits/symbol. Fig. 8 plots the throughput when rate adaptation is based on the following: 1) the zero-Doppler thresholds obtained from (27), 2) the optimal thresholds that are obtained from a brute-force numerical optimization, and 3) the thresholds that are obtained by numerically optimizing the single scaling factor θ. We see that the optimal throughput of the single parameter approach is within 2% of the throughput achieved by the brute-force approach. We, therefore, use the single parameter threshold scaling approach henceforth for case where the $M - 1 = 16$ rates are as per [1, Tab. 7.2.3-1]. The process of optimization of θ is explained using Fig. 9.

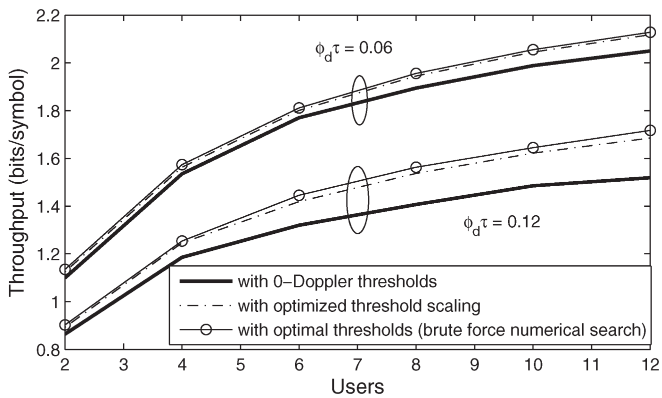

Fig. 8. Toy example: Zoomed comparison of the throughputs obtained by using the following three rate adaptation thresholds: 1) zero-Doppler thresholds; 2) single parameter-based adaptation threshold scaling; and 3) thresholds obtained from a brute-force numerical search, for the threshold-based feedback scheme with the greedy scheduler ($r_1 = 0$, $r_2 = 1$, $r_3 = 2$, $r_4 = 3$ bits/symbol, and $\beta = 0.5$).

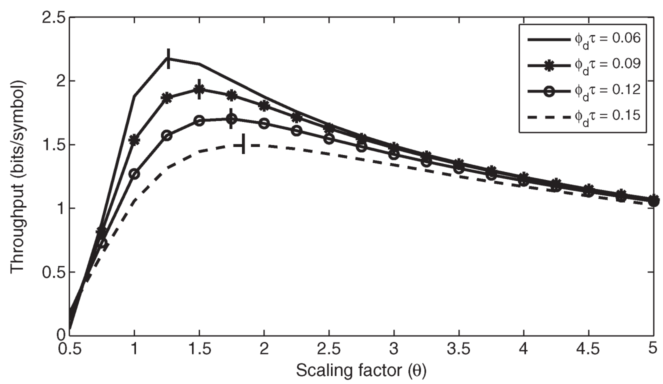

Fig. 9. Throughput of the threshold-based feedback scheme as a function of the threshold scaling factor θ (greedy scheduler, $\beta = 0.5$ and $k = 10$ i.i.d. users). Short vertical bars show where the optimal throughput occurs.

Fig. 9 plots the throughput as a function of θ for different values of $\phi_d\tau$ to determine the optimal value of θ. It shows that the optimal θ, which is the value at which the throughput curve peaks, is a function of $\phi_d\tau$ and is indeed different from unity, which corresponds to the zero-Doppler thresholds. It increases as $\phi_d\tau$ increases, which implies that lower rates are preferable at larger feedback delays. The dependence of the optimal scaling factor on k is marginal and is not shown in the figure.

Fig. 10 compares the throughput of the threshold-based feedback scheme using the zero-Doppler thresholds and the optimal scaled thresholds for different values of $\phi_d\tau$ and k. When $\phi_d\tau = 0.06$ and $k = 5$ users, the optimal thresholds increase throughput by 15% compared with using the zero-Doppler thresholds. The gains increase to 34% for $\phi_d\tau = 0.12$ and $k = 10$ users. Thus, optimizing the rate adaptation thresholds as a function of $\phi_d\tau$ improves throughput significantly.[4] The behavior is similar for the best-n scheme for a given feedback overhead and when the users are not i.i.d., and is not shown.

[4]This requires the system to know $\phi_d\tau$, which can be estimated using the techniques developed, for example, in [30] and the references therein.

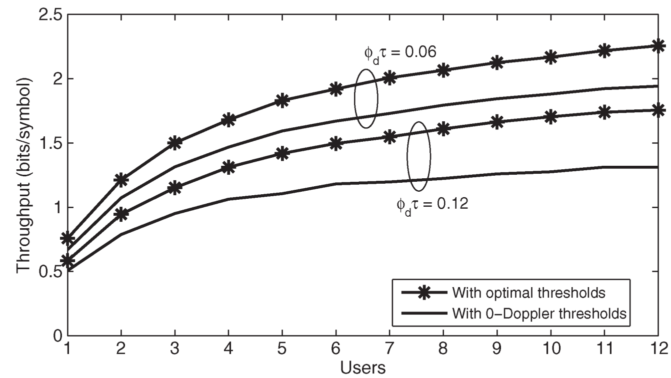

Fig. 10. Comparison of the throughputs achieved by the zero-Doppler thresholds and the optimized rate adaptation thresholds for the threshold-based feedback scheme with the greedy scheduler ($\beta = 0.5$).

VI. Conclusion

We have analyzed the impact of feedback delays on the throughput and outage probability of two common OFDMA feedback reduction schemes, namely, threshold-based feedback and best-n feedback. The comprehensive analysis accounted for their interactions with different frequency-domain schedulers and the rate adaptation rule. Compared with the RR scheduler, the greedy and PF schedulers are less sensitive to the feedback overhead. Further, the throughput and outage probability depend on the normalized feedback delay but not the feedback scheme. For all the three schedulers, the throughput degrades markedly and the outage probability increases many fold, even at low vehicular speeds and for feedback delays that are smaller than what next-generation cellular systems are designed for. Thus, it is important to optimize the rate adaptation thresholds. For the scenarios considered in this paper, the single parameter scaling approach achieves throughputs that are close to optimal. The optimal scaling factor depends on the normalized feedback delay.

Future work involves incorporating HARQ and multiple antenna techniques, such as transmit diversity and spatial multiplexing and their associated feedback overheads, in the model. Moreover, generalizing our results to a multicell environment is of great interest. However, the analysis gets considerably involved, as can be seen from [26], which only considered flat-fading channels and did not consider feedback delays and feedback schemes.

Appendix A
Derivation of Result 1

The probability that the subchannel gains of any l out of k users exceed the threshold λ is $\binom{k}{l}e^{-(\lambda l/\Omega)}(1 - e^{-(\lambda/\Omega)})^{k-l}$. A user u feeds back only if $G_{ui} \geq \lambda$. Thus, if user u fed back, G_{ui} can lie in one of the following intervals: $[\max(T_w, \lambda), T_{w+1}), \ldots, [T_{M-1}, T_M)$, based on which the rate is determined. Since $T_w \leq \lambda < T_{w+1}$, these adaptation intervals can be rewritten as $[\max(T_w, \lambda), T_{w+1})$, $[\max(T_{w+1}, \lambda), T_{w+2}), \ldots, [\max(T_{M-1}, \lambda), T_M)$. Hence, as per the rate adaptation rule in Section II-C, the throughput is given by

$$\eta_G = \sum_{l=1}^{k}\sum_{j=w}^{M-1} r_j \binom{k}{l} e^{-\frac{\lambda l}{\Omega}} \left(1-e^{-\frac{\lambda}{\Omega}}\right)^{k-l} \Pr\left(G_{S_i i}^d \geq T_j, \max(T_j,\lambda) \leq G_{S_i i} < T_{j+1} | l \text{ users fed back}\right). \quad (28)$$

Since the subchannel gains of different users are i.i.d., the (l,j)th probability term in the double summation above can be simplified as follows:

$$\begin{aligned}
&\Pr(\max(T_j,\lambda) \leq G_{S_i i} < T_{j+1}, G_{S_i i}^d \geq T_j | l \text{ users fed back}) \\
&= l \Pr\left(\max(T_j,\lambda) \leq G_{1i} < T_{j+1}, G_{1i}^d \geq T_j, \text{user 1 selected} | \text{users } 1,2,\ldots,l \text{ fed back}\right), \\
&= l \Pr\left(\max(T_j,\lambda) \leq G_{1i} < T_{j+1}, G_{1i}^d \geq T_j, G_{1i} \geq \{G_{ui}\}_{u=2}^{l} | \{G_{ui}\}_{u=1}^{l} \geq \lambda, \{G_{vi}\}_{v=l+1}^{k} < \lambda\right), \\
&= l \Pr\left(\max(T_j,\lambda) \leq G_{1i} < T_{j+1}, G_{1i}^d \geq T_j, G_{1i} \geq \{G_{ui}\}_{u=2}^{l} | \{G_{ui}\}_{u=1}^{l} \geq \lambda\right)
\end{aligned} \quad (29)$$

where the last step again follows from the independence of the gains of subchannel i of different users.

Using Bayes' rule and conditioning on the values of G_{1i} and G_{1i}^d, (29) becomes

$$\begin{aligned}
&l \Pr\left(\max(T_j,\lambda) \leq G_{1i} < T_{j+1}, G_{1i}^d \geq T_j, G_{1i} \geq \{G_{ui}\}_{u=2}^{l} | \{G_{ui}\}_{u=1}^{l} \geq \lambda\right) \\
&= \frac{l \int_{\max(T_j,\lambda)}^{T_{j+1}} \int_{T_j}^{\infty} \Pr\left(\lambda \leq \{G_{ui}\}_{u=2}^{l} < x\right) f_{G_{1i},G_{1i}^d}(x,y)\,dy\,dx}{e^{-\frac{\lambda l}{\Omega}}} \\
&= l e^{\frac{\lambda}{\Omega}} \int_{\max(T_j,\lambda)}^{T_{j+1}} \int_{T_j}^{\infty} \left(1-e^{-\frac{x-\lambda}{\Omega}}\right)^{l-1} f_{G_{1i},G_{1i}^d}(x,y)\,dy\,dx.
\end{aligned} \quad (30)$$

To get (30), we have used the fact that the gains of subchannel i of different users are i.i.d. Substituting (30) in (28) yields (2).

The expression in (2) consists of a sum of double integrals. From (1), each double integral is of the form

$$\Delta(a,b,c) = \int_a^b \int_c^\infty \frac{\left(1-e^{-\frac{x-\lambda}{\Omega}}\right)^{l-1} e^{-\frac{x+y}{\Omega(1-\rho)}} I_0\left(\frac{2\sqrt{\rho x y}}{\Omega(1-\rho)}\right)}{\Omega^2(1-\rho)}\,dy\,dx. \quad (31)$$

We simplify it below. The modified Bessel function term can be lower bounded as [22]

$$I_0\left(\frac{2\sqrt{\rho x y}}{\Omega(1-\rho)}\right) \geq \sum_{p=0}^{L} \frac{\rho^p}{(p!)^2 \Omega^{2p} (1-\rho)^{2p}} x^p y^p. \quad (32)$$

We replace $I_0(\cdot)$ in $\Delta(a,b,c)$ with its truncated expansion that consists of $L+1$ terms, and we replace $(1-e^{-(x-\lambda/\Omega)})^{l-1}$ with its binomial expansion. Then, $\Delta(a,b,c)$ is lower bounded by the following summation of products of two single integrals:

$$\Delta(a,b,c) \geq \sum_{q=0}^{l-1} \binom{l-1}{q} \frac{(-1)^q e^{\frac{q\lambda}{\Omega}}}{\Omega^2(1-\rho)} \sum_{p=0}^{L} \frac{1}{(p!)^2} \times \left(\frac{\rho}{\Omega^2(1-\rho)^2}\right)^p \int_c^\infty y^p e^{-\frac{y}{\Omega(1-\rho)}}\,dy \int_a^b x^p e^{-x\left(\frac{q}{\Omega}+\frac{1}{\Omega(1-\rho)}\right)}\,dx.$$

Writing the single integrals above in terms of incomplete Gamma functions and simplifying further, we get

$$\Delta(a,b,c) \geq \sum_{q=0}^{l-1} \binom{l-1}{q} \frac{(-1)^q e^{\frac{q\lambda}{\Omega}}}{\Omega^2(1-\rho)} \xi_q^{[\Omega]}(a,b;c) \quad (33)$$

where $\xi_q^{[\Omega]}(\cdot,\cdot;\cdot)$ is defined in (4). Substituting (33) in (2) yields (3).

Appendix B
Brief Derivation of Result 2

An outage occurs in a slot if no user feeds back its gain for subchannel i in that slot or due to feedback delays. Hence

$$P_G^{\text{out}} = \Pr(\text{No users fed back}) + \sum_{l=1}^{k}\sum_{j=w}^{M-1} \times \Pr\left(\max(T_j,\lambda) \leq G_{S_i i} < T_{j+1}, G_{S_i i}^d < T_j | l \text{ users fed back}\right) \Pr(l \text{ users fed back}). \quad (34)$$

The probability that no user fed back its gain for subchannel i is $(1-e^{-(\lambda/\Omega)})^k$. The (l,j)th probability term within the double summation above is then evaluated along lines similar to Appendix A, except for the following two differences: 1) The limits of integration for $G_{S_i i}^d$ change from 0 to T_j, and 2) there is no rate term r_j in the integrand.

Appendix C
Derivation of Result 3

The probability that any l users report subchannel (subch.) i is $\binom{k}{l}(n/N)^l(1-(n/N))^{k-l}$. Therefore

$$\eta_G = \sum_{l=1}^{k}\sum_{j=2}^{M-1} r_j \binom{k}{l} \left(\frac{n}{N}\right)^l \left(1-\frac{n}{N}\right)^{k-l} \times \Pr\left(T_j \leq G_{S_i i} < T_{j+1}, G_{S_i i}^d \geq T_j | l \text{ users report subch. } i\right). \quad (35)$$

Since the subchannel gains of different users are i.i.d., the (l,j)th probability summation term in (35) can be written as

$$\begin{aligned}
&\Pr\left(T_j \leq G_{S_i i} < T_{j+1}, G_{S_i i}^d \geq T_j | l \text{ users report } i^{\text{th}} \text{ subch.}\right) \\
&= l \Pr\left(T_j \leq G_{1i} < T_{j+1}, G_{1i}^d \geq T_j, \text{user 1 selected} | \text{users } 1,\ldots,l \text{ report subch. } i\right) \\
&= l \Pr\left(T_j \leq G_{1i} < T_{j+1}, G_{1i}^d \geq T_j, G_{1i} \geq \{G_{ui}\}_{u=2}^{l} | \{\beth_{ui}\}_{u=1}^{l} \leq n-1, \{\beth_{vi}\}_{v=l+1}^{k} > n-1\right)
\end{aligned}$$

$$= l\left(\frac{N}{n}\right)^l \int_{T_j}^{T_{j+1}}\int_{T_j}^{\infty} \Pr\left(\beth_{1i} \le n-1 | G_{1i}=x, G_{1i}^d = y\right) \times \left[\Pr\left(G_{ui} \le x, \beth_{ui} \le n-1\right)\right]^{l-1} f_{G_{1i}G_{1i}^d}(x,y)dydx \quad (36)$$

where (36) follows because the gains of subchannel i of different users are independent, by conditioning on G_{1i} and G_{1i}^d, and using the relation $\Pr(\{\beth_{ui}\}_{u=1}^l \le n-1) = (n/N)^l$.

We now evaluate the terms $\Pr(\beth_{1i} \le n-1 | G_{1i}=x, G_{1i}^d = y)$ and $\Pr(G_{ui} \le x, \beth_{ui} \le n-1)$ in the above integrand. Since the gains of the different subchannels of user 1 are independent, it can be shown that

$$\Pr\left(\beth_{1i} \le n-1 | G_{1i}=x, G_{1i}^d=y\right) = \Pr(\beth_{1i} \le n-1 | G_{1i}=x). \quad (37)$$

Given that $G_{1i}=x$, the probability that the gains of m subchannels (other than subchannel i) of user 1 exceed x is

$$\Pr(\beth_{1i} = m | G_{1i}=x) = \binom{N-1}{m}\left(1-e^{-\frac{x}{\Omega}}\right)^{N-1-m} e^{-\frac{xm}{\Omega}}.$$

Hence

$$\Pr(\beth_{1i} \le n-1 | G_{1i}=x) = \sum_{m=0}^{n-1}\binom{N-1}{m}\left(1-e^{-\frac{x}{\Omega}}\right)^{N-1-m} e^{-\frac{xm}{\Omega}}. \quad (38)$$

Since the users are statistically identical, it also follows from (38) that for any user u

$$\begin{aligned}
\Pr(G_{ui} \le x, \beth_{ui} \le n-1) &= \Pr(G_{1i} \le x, \beth_{1i} \le n-1) \\
&= \int_0^x \sum_{m=0}^{n-1}\binom{N-1}{m}\left(1-e^{-\frac{z}{\Omega}}\right)^{N-1-m}\left(e^{-\frac{z}{\Omega}}\right)^m \frac{e^{-\frac{z}{\Omega}}}{\Omega}dz \\
&= \sum_{m=0}^{n-1}\binom{N-1}{m}\sum_{q=0}^{N-1-m}\binom{N-m-1}{q} \times \frac{(-1)^q\left(1-e^{-\frac{(q+m+1)x}{\Omega}}\right)}{q+m+1} \\
&= \Upsilon_\Omega(x) \qquad (39)
\end{aligned}$$

where $\Upsilon_\Omega(x)$ is defined in Result 3.

Using (38) and (39), (36) reduces to

$$l\left(\frac{N}{n}\right)^l \int_{T_j}^{T_{j+1}}\int_{T_j}^{\infty}\sum_{m=0}^{n-1}\binom{N-1}{m}e^{-\frac{xm}{\Omega}}\left(1-e^{\frac{x}{\Omega}}\right)^{N-1-m} \times \frac{e^{-\frac{x+y}{\Omega(1-\rho)}}}{\Omega^2(1-\rho)} I_0\left(\frac{2\sqrt{\rho}\sqrt{xy}}{\Omega(1-\rho)}\right)\left(\Upsilon_\Omega(x)\right)^{l-1}dydx. \quad (40)$$

As in Appendix A, replacing $I_0(\cdot)$ in (40) with its truncated series and simplifying yields (10).

Appendix D
Derivation of Result 4

Outage occurs under one of the following three scenarios: 1) No user reports subchannel i; 2) the subchannel gain of the selected user S_i is below T_2, i.e., $G_{S_i i} < T_2$; or 3) it is due to feedback delay. The probability that $G_{S_i i} < T_2$ is negligible because T_2 is the lowest threshold for transmitting at a nonzero rate and the subchannel i is one of the n best subchannels of the selected user. Hence

$$\begin{aligned}
P_G^{\text{out}} = &\Pr(\text{No users report subch. } i) \\
&+ \sum_{l=1}^{k}\sum_{j=2}^{M-1}\Pr(l \text{ users report subch. } i) \\
&\times \Pr\left(T_j \le G_{S_i i} < T_{j+1}, G_{S_i i}^d < T_j | l \text{ users report subch. } i\right). \qquad (41)
\end{aligned}$$

The first term is equal to $(1-(n/N))^k$. The (l,j)th probability term within the double summation above can be evaluated along the lines of Appendix C and is not repeated here.

Appendix E
Derivation of Result 5

We denote the probability that the users in the subset ϑ_m^l report subchannel i by $P_{l,m}$. It is equal to $[\prod_{u\in\vartheta_m^l} e^{-(\lambda/\Omega_u)}]\prod_{v\in\vartheta\setminus\vartheta_m^l}(1-e^{-(\lambda/\Omega_v)})$. By conditioning on which subset of users reported subchannel i and which user got selected from the subset, we can show that the throughput of the greedy scheduler is given by

$$\begin{aligned}
\eta_G &= \sum_{l=1}^{k}\sum_{m=1}^{\binom{k}{l}}\sum_{j=w}^{M-1} r_j P_{l,m} \\
&\quad\times \Pr\left(\max(T_j,\lambda) \le G_{S_i i} < T_{j+1}, G_{S_i i}^d \ge T_j | \text{ users in } \vartheta_m^l \text{ report subch. } i\right) \\
&= \sum_{l=1}^{k}\sum_{m=1}^{\binom{k}{l}}\sum_{j=w}^{M-1} r_j P_{l,m} \\
&\quad\times \sum_{u\in\vartheta_m^l}\Pr\left(\max(T_j,\lambda) \le G_{ui} < T_{j+1}, G_{ui}^d \ge T_j \text{ user } u \text{ selected} | \text{users in} \vartheta_m^l \text{ report subch. } i\right).
\end{aligned}$$

Along lines similar to Appendix A, we can show that

$$\begin{aligned}
&\Pr\left(\max(T_j,\lambda) \le G_{ui} < T_{j+1}, G_{ui}^d \ge T_j, \text{user } u \text{ selected} | \text{users in } \vartheta_m^l \text{ report subch. } i\right) \\
&= e^{\frac{\lambda}{\Omega_u}}\int_{\max(T_j,\lambda)}^{T_{j+1}}\int_{T_j}^{\infty}\left[\prod_{b\in\vartheta_m^l\setminus\{u\}}\left(1-e^{-\frac{x-\lambda}{\Omega_b}}\right)\right] \times f_{G_{ui},G_{ui}^d}(x,y)dydx. \qquad (42)
\end{aligned}$$

The equation is then simplified along lines similar to Appendix A. The steps are not repeated here to conserve space.

Appendix F
Derivation of Result 6

Recall that for subchannel i, the PF scheduler selects the user with the highest normalized subchannel gain from among the users that reported this subchannel. The normalized channel gains of all the users are i.i.d. unit mean exponential RVs. Therefore, the probability that the users from the set ϑ_m^l (of size l) report subchannel i is given by $e^{-l\lambda}(1-e^{-\lambda})^{k-l}$. Further, in threshold-based feedback, if a user u reports subchannel i, then $(G_{ui})/(\Omega_u) = \widetilde{G}_{ui} \geq \lambda$. Since $\lambda \in [(T_{w_u}/\Omega_u), (T_{w_u+1}/\Omega_u))$, it follows that $\lambda\Omega_u \geq T_{w_u}$.

Summing over all possible subsets of users that report subchannel i and the user that gets selected, we get

$$\eta_{\mathrm{PF}} = \sum_{l=1}^{k}\sum_{m=1}^{\binom{k}{l}} e^{-l\lambda}(1-e^{-\lambda})^{k-l}\sum_{u\in\vartheta_m^l}\sum_{j=w_u}^{M-1} r_j \times \Pr\Big(\max(T_j,\lambda\Omega_u) \leq G_{ui} < T_{j+1}, G_{ui}^d \geq T_j, \text{user } u \text{ selected}|\text{users in } \vartheta_m^l \text{report subch. } i\Big). \quad (43)$$

In the following, we use the notation $Y \geq \{X_{si}\}_{s\in\vartheta_m^l}$ to denote the set of conditions $\{Y \geq X_{si} : s \in \vartheta_m^l\}$:

$$\begin{aligned}&\Pr\Big(\max(T_j,\lambda\Omega_u) \leq G_{ui} < T_{j+1}, G_{ui}^d \geq T_j, \text{user } u \text{ selected} \\ &\qquad |\text{users in } \vartheta_m^l \text{report subch. } i\Big) \\ &= \Pr\Big(\max(T_j,\lambda\Omega_u) \leq G_{ui} < T_{j+1}, G_{ui}^d \geq T_j, \widetilde{G}_{ui} \\ &\qquad \geq \{\widetilde{G}_{si}\}_{s\in\vartheta_m^l\setminus\{u\}}|\{\widetilde{G}_{si}\}_{s\in\vartheta_m^l} \geq \lambda, \{\widetilde{G}_{vi}\}_{v\in\vartheta\setminus\vartheta_m^l} < \lambda\Big) \\ &= \Pr\left(\max\left(\frac{T_j}{\Omega_u},\lambda\right) \leq \widetilde{G}_{ui} < \frac{T_{j+1}}{\Omega_u}, \widetilde{G}_{ui}^d \right. \\ &\qquad \left. \geq \frac{T_j}{\Omega_u}, \widetilde{G}_{ui} \geq \{\widetilde{G}_{si}\}_{s\in\vartheta_m^l, s\neq u}|\{\widetilde{G}_{si}\}_{s\in\vartheta_m^l} \geq \lambda\right).\end{aligned} \quad (44)$$

As in Appendix A, (44) above can be written as $e^{\lambda}\int_{\max((T_j/\Omega_u),\lambda)}^{(T_{j+1}/\Omega_u)}\int_{(T_j/\Omega_u)}^{\infty}(1-e^{-(x-\lambda)})^{l-1} f_{\widetilde{G}_{ui},\widetilde{G}_{ui}^d}(x,y)dydx$, where $f_{\widetilde{G}_{ui},\widetilde{G}_{ui}^d}$ is the joint pdf of $\widetilde{G}_{ui}$ and $\widetilde{G}_{ui}^d$. It is equal to

$$f_{\widetilde{G}_{ui},\widetilde{G}_{ui}^d}(x,y) = \frac{e^{-\frac{x+y}{1-\rho}}}{1-\rho} I_0\left(\frac{2\sqrt{\rho}\sqrt{xy}}{1-\rho}\right), \quad x,y \geq 0. \quad (45)$$

Further simplification yields (17).

Appendix G
Brief Derivation of Result 7

The probability that the users in subset ϑ_m^l report subchannel i is $P_{l,m} = (n/N)^l(1-(n/N))^{k-l}$. Along the lines of Appendix E, the throughput of the greedy scheduler with the best-n feedback scheme can be written as

$$\eta_G = \sum_{l=1}^{k}\sum_{m=1}^{\binom{k}{l}}\sum_{j=2}^{M-1}\sum_{u\in\vartheta_m^l} P_{l,m} r_j \Pr\Big(T_j \leq G_{ui} < T_{j+1}, G_{ui}^d \geq T_j, \text{user } u \text{ selected}|\text{ users in } \vartheta_m^l \text{ report subch. } i\Big). \quad (46)$$

Further, the probability term in (46) simplifies to

$$\begin{aligned}&\Pr\Big(T_j \leq G_{ui} < T_{j+1}, G_{ui}^d \geq T_j, \text{user } u \text{ selected} \\ &\qquad |\text{ users in } \vartheta_m^l \text{report subch. } i\Big) \\ &= \left(\frac{N}{n}\right)^l \int_{T_j}^{T_{j+1}}\int_{T_j}^{\infty} \Pr\left(\beth_{ui} \leq n-1|G_{ui}=x, G_{ui}^d = y\right) \\ &\quad \times \left[\prod_{b\in\vartheta_m^l\setminus\{u\}} \Pr(G_{bi} \leq x, \beth_{bi} \leq n-1)\right] \\ &\quad \times f_{G_{ui}G_{ui}^d}(x,y)\,dy\,dx.\end{aligned} \quad (47)$$

Simplifying further along lines similar to Appendix C yields (21).

Appendix H
Derivation of Result 8

The throughput of the PF scheduler for the non-i.i.d. users scenario can be written as

$$\eta_{\mathrm{PF}} = \sum_{l=1}^{k}\sum_{m=1}^{\binom{k}{l}}\left(\frac{n}{N}\right)^l\left(1-\frac{n}{N}\right)^{k-l}\sum_{u\in\vartheta_m^l}\sum_{j=2}^{M-1} r_j \times \Pr\Big(T_j \leq G_{ui} < T_{j+1}, G_{ui}^d \geq T_j, \text{user } u \text{ selected} |\text{users in } \vartheta_m^l \text{ report subch. } i\Big). \quad (48)$$

Users in the subset ϑ_m^l report subchannel i if and only if $\{\beth_{si}\}_{s\in\vartheta_m^l} \leq n-1$ and $\{\beth_{vi}\}_{v\in\vartheta\setminus\vartheta_m^l} > n-1$. Hence

$$\begin{aligned}&\Pr\Big(T_j \leq G_{ui} < T_{j+1}, G_{ui}^d \geq T_j, \text{user } u \text{ selected} \\ &\qquad |\text{users in } \vartheta_m^l \text{ report subch. } i\Big) \\ &= \Pr\Big(T_j \leq G_{ui} < T_{j+1}, G_{ui}^d \geq T_j, \widetilde{G}_{ui} \geq \{\widetilde{G}_{si}\}_{s\in\vartheta_m^l\setminus\{u\}} \\ &\qquad |\{\beth_{si}\}_{s\in\vartheta_m^l} \leq n-1, \{\beth_{vi}\}_{v\in\vartheta\setminus\vartheta_m^l} > n-1\Big) \\ &= \Pr\left(\frac{T_j}{\Omega_u} \leq \widetilde{G}_{ui} < \frac{T_{j+1}}{\Omega_u}, \widetilde{G}_{ui}^d \geq \frac{T_j}{\Omega_u},\right. \\ &\qquad \left.\widetilde{G}_{ui} \geq \{\widetilde{G}_{si}\}_{s\in\vartheta_m^l\setminus\{u\}}|\{\beth_{si}\}_{s\in\vartheta_m^l} \leq n-1\right)\end{aligned} \quad (49)$$

where the last step follows because the gains of subchannel i of different users are independent. Along lines similar to Appendices C and F, (49) can be simplified further to get (23).

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