# En Masse Relay Selection Algorithms for Multi-Source, Multi-Relay, Decode-and-Forward Cooperative Systems

A. Karthik and Neelesh B. Mehta, *Senior Member, IEEE*

***Abstract*—Opportunistic relay selection in a multiple source-destination (MSD) cooperative system requires quickly allocating to each source-destination (SD) pair a suitable relay based on channel gains. Since the channel knowledge is available only locally at a relay and not globally, efficient relay selection algorithms are needed. For an MSD system, in which the SD pairs communicate in a time-orthogonal manner with the help of decode-and-forward relays, we propose three novel relay selection algorithms, namely, contention-free en masse assignment (CFEA), contention-based en masse assignment (CBEA), and a hybrid algorithm that combines the best features of CFEA and CBEA. En masse assignment exploits the fact that a relay can often aid not one but multiple SD pairs, and, therefore, can be assigned to multiple SD pairs. This drastically reduces the average time required to allocate an SD pair when compared to allocating the SD pairs one by one. We show that the algorithms are much faster than other selection schemes proposed in the literature and yield significantly higher net system throughputs. Interestingly, CFEA is as effective as CBEA over a wider range of system parameters than in single SD pair systems.**

***Index Terms*—Relay selection, Multiple source-destination networks, Splitting algorithms, Cooperative communications, Cross-layer design, Fading channels, Multiple access**

## I. Introduction

COOPERATIVE relay-aided transmission schemes have emerged as promising techniques for harnessing spatial diversity in wireless systems. In these schemes, the destination combines signals from the source and the intermediate relaying nodes that overhear the source, which exploits the broadcast nature of the wireless channel. Traditionally, cooperative relaying schemes have focused on aiding a single source-destination (SSD) pair with one or more intermediate relays [1]–[4]. Recently, cooperative schemes with multiple source-destination (MSD) pairs have also attracted considerable interest [5]–[21].

### A. Relay selection: Motivation and literature

In several SSD [1], [3], [4] and MSD [7]–[13], [19]–[21] cooperative relay systems, relay selection is employed, in which a single 'best' relay is selected to aid data transmission between a source-destination (SD) pair. It avoids the need for tight symbol-level synchronization among simultaneously transmitting relays and yet achieves full diversity order [2]. Depending on the selection criterion used, every relay maintains a certain *metric* that is a function of its local channel gains. The 'best' relay is the relay with the largest metric. For example, in reactive decode-and-forward (DF) systems, the best relay is the one that has decoded the source's message and has the highest relay-destination (RD) link SNR.

While selection is appealing, global knowledge about the metrics is not available anywhere in the system because the relays are geographically separated. Hence, a selection algorithm is required to search and discover the best relay. Selection algorithms can be broadly classified as *contention-free* and *contention-based*. In contention-free algorithms, a common coordinator (CC) seeks information from each of the relays one after the other. After listening to every relay, the CC selects the best one. On the other hand, in contention-based algorithms, not all relays need transmit. Relays with larger metrics are prioritized to transmit earlier to the CC. Due to their distributed nature, a random number of relays may transmit at any time during the selection process. This may lead to collisions, which then need to be resolved.

Though simple to implement, contention-free algorithms do not scale well with the number of relays in an SSD system. For example, with $N$ relays, polling each relay requires a total of $N$ slots. Thus, the time overhead increases linearly with $N$. On the other hand, the contention-based splitting algorithm takes only 2.47 slots, on average, to select the best relay even when $N \to \infty$ [22], [23]. Since no data transmission takes place during selection, the time overhead of selection directly affects the net throughput of the system because it reduces the time available for data transmission using the selected relay. Therefore, the overhead of relay selection must be minimized.

Research on relay selection in MSD systems has focused on characterizing the diversity, multiplexing, and outage gains achieved by selection, but has typically neglected the selection overhead [6]–[9], [12], [13], [15], [18]–[21].[1] For example, in [7]–[9], it is assumed that the relay with the highest end-to-end SNR is somehow selected. However, a selection scheme for achieving this is not discussed and its overheads are not accounted for. In [6]–[9], [12], [13], it is assumed that either

Manuscript received November 2, 2012; revised March 27, 2013. The editor coordinating the review of this paper and approving it for publication was M. Uysal.

The authors are with the Dept. of Electrical Communication Eng., Indian Institute of Science (IISc), Bangalore, India (e-mail: {karthik_ab, nbmehta}@ece.iisc.ernet.in).

A part of this paper was presented at the IEEE Int. Conf. Commun. (ICC), Jun. 2013.

This work was partially supported by a grant from the Broadcom Foundation, USA.

Digital Object Identifier 10.1109/TCOMM.2013.070213.120839

[1] The selection overhead and the throughput tradeoff have been well studied in SSD systems, see, for example, [24], [25].

the source or the destination knows the channel gains of all the SD, RD, and source-relay (SR) links in the system. In [15], every SD pair is assumed to have already been pre-allocated a relay. In [19], a relay selection criterion for a two SD pair, single relay network is proposed and its outage probability is studied. In [5], a simple contention-free relay selection scheme for MSD systems is proposed. In it, every relay sequentially communicates to the CC a binary vector that indicates the SD pairs it can aid. In [26], a contention-based distributed relay selection scheme is proposed. We explain it in detail later in Sec. IV-A.

### B. Focus and contributions

We develop novel contention-free and contention-based relay selection algorithms for an MSD system with DF relays. The sources first transmit data to the relays in a time-orthogonal manner over a common channel. The proposed selection algorithms then assign relays to the SD pairs. Thereafter, the assigned relays transmit the data to their respective destinations. In such an MSD system, a relay may be capable of assisting multiple SD pairs depending on its relay channel gains. This observation motivates a simple, yet powerful, new concept of *en masse relay assignment* that is used by the proposed schemes. In it, a selected relay is assigned to as many unallocated SD pairs as it can aid.[2]

We first introduce a simple contention-free en masse algorithm (CFEA). In it, a new relay is polled in every slot and is assigned to all the unallocated SD pairs it can aid. We analyze the time required by CFEA to allocate as many SD pairs as possible and verify the analysis with simulations.

We then propose a novel contention-based en masse algorithm (CBEA) that generalizes the splitting-based selection scheme for SSD systems proposed in [22], [23]. In it, in any slot, a relay contends on the basis of the number of unallocated SD pairs it can aid, which we refer to as its *availability*. CBEA takes the idea of en masse selection even further by incentivizing relays with higher availabilities to transmit earlier. In every slot, two thresholds determine which relays transmit in it. At the end of the slot, the thresholds are updated based on whether an idle, success, or collision occurred. In a success slot, exactly one relay transmits and all the unallocated SD pairs that it can aid are assigned en masse to it. While splitting-based selection algorithms have been studied extensively in the literature [22], [23], [28]–[30], CBEA is novel because it addresses challenges that are unique to the relay assignment problem in MSD cooperative systems. For example, the availability of a relay changes during the algorithm and is a function of the previous relay assignments.

We show that both algorithms assign relays very quickly and are much faster, per SD pair, than in an SSD system. To evaluate the efficiency of a selection algorithm in allocating SD pairs to relays over time, we introduce a new measure called *selection efficiency per slot* (SEPS). It is the average number of SD pairs that an algorithm can allocate per slot. We observe that the SEPS of CFEA is larger than that of CBEA in the initial slots when there are many unallocated SD pairs. However, the reverse is true for later slots in which fewer SD pairs remain unallocated. This motivates the development of a novel hybrid algorithm that combines the best features of both CFEA and CBEA, and outperforms them both. Finally, we benchmark the net system throughput, which accounts for the time overhead of selection, and show that it is higher than other relay selection schemes proposed in the literature.

The paper is organized as follows. The system model is presented in Sec. II. The en masse selection algorithms are developed in Sec. III. Simulation and benchmarking results are presented in Sec. IV, and are followed by our conclusions in Sec. V. Proofs are given in the Appendix.

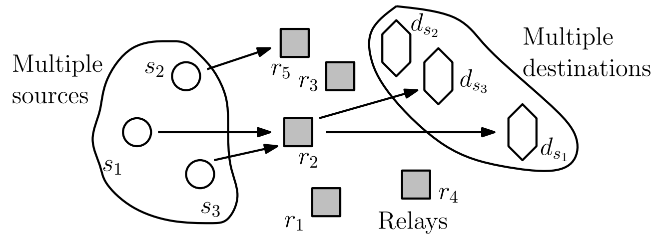


Fig. 1. Multi-source, multi-relay, and multi-destination system model ($D = 3$ and $N = 5$).

## II. System Model

As shown in Figure 1, we consider an MSD relay system with $D$ SD pairs and $N$ DF relays, which aid data transmission between the SD pairs. Let $\mathcal{S}$ denote the set of source nodes $\{s_1, \ldots, s_D\}$ and let $\mathcal{R}$ denote the set of relay nodes $\{r_1, \ldots, r_N\}$. The SD pair $s$-$d_s$ denotes the source $s$ and its destination $d_s$. This model also encompasses many existing models in the literature such as the single source, multiple destinations model of [9], [12], the single destination, multiple sources model of [6]–[8], [13], and the multiple source, multiple destination models of [5], [15], [16].

Let $H_{sr}$ denote the power gain of the channel fading from a source $s \in \mathcal{S}$ to relay node $r \in \mathcal{R}$. Similarly, let $H_{rd_s}$ denote the power gain of the channel fading from relay node $r$ to destination node $d_s$. The SR channel power gains are independent and identically distributed (i.i.d.) with unit mean, and so are the RD channel power gains [5], [24]. They remain constant during a coherence interval. We consider no direct link between any SD pair, as has also been assumed in [8], [15], [16], [18], [24]. This simplification helps us focus on relay selection and its implications. A relay $r$ only knows its local channel gains $H_{sr}$ and $H_{rd_s}$, for $s \in \mathcal{S}$. It does not know any other relay's channel gains [24]. Every source always has data to transmit and transmits at a rate $R$ bits per symbol (bps) [5], [13].

A CC coordinates the relay selection process. It could be, for instance, a pre-chosen source or a destination node. The CC can decode a message when exactly one relay transmits.

[2]Note that, in this model, a relay expends a fixed amount of energy for every SD pair that it aids. When it is assigned to multiple SD pairs, it spends proportionally more total energy during transmissions to the respective destinations. However, since the transmissions are time-orthogonal in our model, a relay's transmit power remains fixed. This model is different from that in [5], in which a relay proportionally reduces the energy and power it expends on an SD pair depending on the number of SD pairs it aids. Our model is justified because over time, a relay aids different number of SD pairs, and the average energy consumption per relay will be the same across relays when the SR links are statistically identical. If they are not, physical layer fairness mechanisms, e.g., [27], can be used.

However, when multiple relays transmit simultaneously, a collision occurs and the CC cannot decode any of the transmissions [22], [23], [28]. Every relay is assumed to receive all broadcast messages from the CC without error, which is justifiable given its low payload [22], [25].

In the following, the probability of an event $A$ is denoted by $\mathbf{P}[A]$ and the expectation operator is denoted by $\mathbf{E}[\cdot]$. The conditional probability of an event $A$ given the event $B$ is denoted by $\mathbf{P}[A|B]$.

### A. Data transmission model

We consider a time-slotted data transmission model. A coherence interval, during which data packet transmission occurs from the $D$ sources to their destinations, is divided into the following three phases:

1) *Source-to-relay transmissions:* In this phase, the sources broadcast their data to the relays sequentially in a time-division multiplexed (TDM) manner over a common channel [5]–[7], [10], [12], [16]. Each source, in its turn, broadcasts its data packet for $T_0$ slots at a rate of $R$ bps. Thus, the duration of this phase is $DT_0$ slots. We use the Shannon formula to ascertain whether a relay can decode a source's transmission. A relay $r$ can decode the signal from source $s$ only if
$$R \leq \log_2(1+\gamma_0 H_{sr}), \tag{1}$$
where $\gamma_0$ is the fading-averaged SNR of the SR link.
2) *Relay assignment:* In this phase, each SD pair is allocated to its best relay. The assignment criterion is detailed below in Sec. II-B. This phase lasts for a maximum duration of $T_s$ slots. If all the SD pairs are allocated before $T_s$ slots, the relay assignment phase ends as well.
3) *Relay-to-destination transmissions:* This phase occurs after the relay selection phase. In it, the assigned relays transmit data in a TDM manner to the destinations of the SD pairs they were assigned to. An assigned relay transmits at a rate $R$ bps to its destination over the common channel for a duration of $T_0$ slots. Thus, this phase also lasts for $DT_0$ slots. A destination $d_s$ can decode the signal transmitted by a relay $r$ only if
$$R \leq \log_2(1+\delta_0 H_{rd_s}), \tag{2}$$
where $\delta_0$ is the fading-averaged SNR of the RD link. If no relay is assigned to an SD pair, no data will be transmitted in the $T_0$ slots corresponding to that pair in this phase. Thus, the SD pairs that are not allocated a relay at the end of the relay selection phase cannot communicate.

### B. Relay selection criterion

For a relay $r$ to be able to aid the $s$-$d_s$ SD pair, it must not only decode the source's signal (as per (1)), but it also must transmit successfully to the destination (as per (2)). Relay $r$ is said to be *feasible* for the $s$-$d_s$ SD pair if it satisfies both (1) and (2). Clearly, any one among the DF relays that are feasible for an SD pair can be selected. Thus, the relay selection problem is to assign to every SD pair a feasible relay, if any.

### C. Terminology

We shall henceforth use the terms selection and assignment interchangeably. We now introduce some key terms that will be used throughout the paper.

*1) Feasibility bit and feasibility vector:* The feasibility bit indicates the feasibility of a relay with respect to an SD pair. For a relay $r$, its feasibility bit (FB) associated with the $s$-$d_s$ SD pair is denoted by $X_{sr}$, and is defined as
$$X_{sr} = \begin{cases} 1, & R \leq \log_2(1+\gamma_0 H_{sr}) \text{ and} \\ & R \leq \log_2(1+\delta_0 H_{rd_s}) \\ 0, & \text{otherwise} \end{cases}. \tag{3}$$
We refer to the $D$-tuple $(X_{s_1 r}, \ldots, X_{s_D r})$ of all the FBs of a relay $r$ as its *feasibility vector*. Since the channel gains are i.i.d., it follows that the FBs are also i.i.d. The probability $p_f$ that a relay $r$ is feasible for the $s$-$d_s$ SD pair is given by
$$p_f = \mathbf{P}[X_{sr}=1] = \mathbf{P}\left[H_{sr} > \frac{2^R-1}{\gamma_0}\right] \mathbf{P}\left[H_{rd_s} > \frac{2^R-1}{\delta_0}\right]. \tag{4}$$
When the channel gains are exponential RVs with unit mean, which corresponds to Rayleigh fading, (4) simplifies to
$$p_f = \exp\left(-(2^R-1)(\gamma_0^{-1}+\delta_0^{-1})\right). \tag{5}$$
Observe that $p_f$ decreases as $R$ increases.

*2) Relay availability:* At any time, the *availability* of a relay is the number of unallocated SD pairs it is feasible for.

## III. En Masse Relay Selection Algorithms

We now present the CFEA and CBEA selection algorithms. Thereafter, we develop a hybrid algorithm that combines the best features of both.

### A. Contention-free en masse assignment (CFEA)

In this algorithm, the relays transmit sequentially to the CC, one slot after the other. In its turn, each relay transmits its feasibility vector. A key idea of the algorithm is that the CC then assigns en masse to the relay all the currently unallocated SD pairs for which it is feasible. The set of allocated SD pairs is then broadcast by the CC. This proceeds until all the $D$ SD pairs are allocated, or all the $N$ relays have transmitted, or $T_s$ slots have been used up. Figure 2 illustrates how the SD pairs can get allocated in this algorithm.

Note that the algorithm requires no contention among the relays and can terminate in less than $T_s$ slots. Conservatively, every relay sends $\lceil \log_2 D \rceil$ bits if it transmits, where $\lceil\cdot\rceil$ denotes the ceil function. The average energy consumption per relay can be made the same across all the relays by randomizing the transmission order.

*1) Analysis:* We now analyze the average selection time and the net throughput of CFEA.

*Average selection time:* The following result recursively characterizes the probability mass function (pmf) of the total number of SD pairs $Q_n$ that have been allocated up to and including slot $n$, for $n \geq 1$.

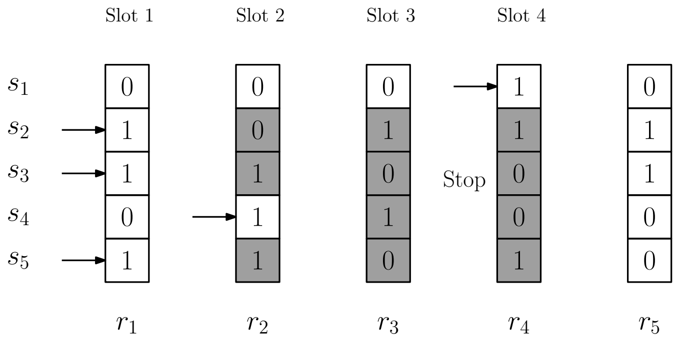


Fig. 2. Illustration of CFEA for a system with $D = 5$ SD pairs and $N = 5$ relays. The numbers inside the boxes represent feasibility bits; for example, $X_{s_2 r_1} = 1$ and $X_{s_3 r_4} = 0$. An arrow indicates an SD pair allocation. Shaded boxes indicate the SD pairs that have already been allocated. The algorithm terminates after 4 slots in this example.

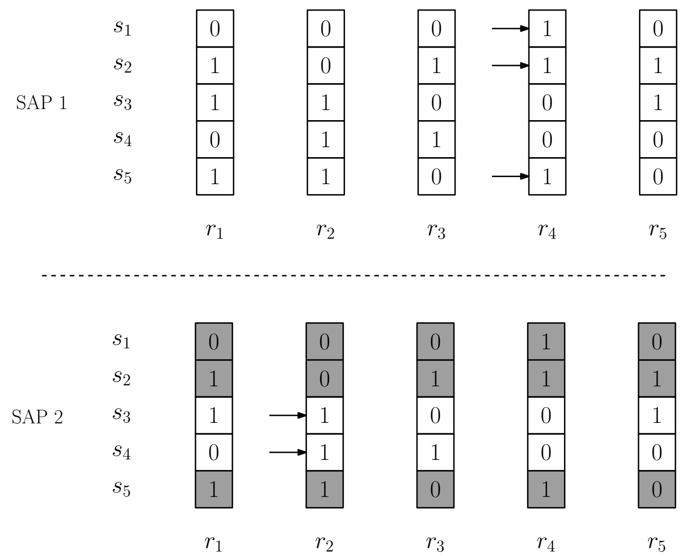


Fig. 3. Illustration of CBEA under the same settings as the illustration of Figure 2. In each SAP, the relay with the highest availability gets assigned to all the unallocated SD pairs it is feasible for. For example, in the first SAP, $r_4$ is assigned the SD pairs $s_1$-$d_{s_1}$, $s_2$-$d_{s_2}$, and $s_5$-$d_{s_5}$.

*Proposition 1:* For $n \geq 1$ and $0 \leq i \leq D$,

$$\mathbf{P}[Q_n = i] = \sum_{j=0}^{i} \mathbf{P}[Q_{n-1} = j] \binom{D-j}{i-j} p_f^{i-j} (1-p_f)^{D-i}, \tag{6}$$

where $\mathbf{P}[Q_0 = 0] = 1$.

*Proof:* The proof is given in Appendix A. ■

Let $\tau_{N,T_s}$ denote the time duration in slots of CFEA. The following result obtains the average time duration of CFEA in terms of the probabilities given in Proposition 1.

*Proposition 2:* The average time duration of CFEA is given by

$$\mathbf{E}\left[\tau_{N,T_s}\right] = \sum_{k=0}^{\min\{N,T_s\}-1} \sum_{j=0}^{D-1} \mathbf{P}\left[Q_k = j\right]. \tag{7}$$

*Proof:* The proof is given in Appendix B. ■

*Net Throughput:* The net throughput $\eta$ is defined as the average of the number of bits per slot that a destination decodes per SD pair after accounting for the time spent on relay selection. It is given by

$$\eta = \frac{R T_0 \mathbf{E}[Q_{\tau_{N,T_s}}]}{(2T_0 + \mathbf{E}[\tau_{N,T_s}])D}, \tag{8}$$

where $Q_{\tau_{N,T_s}}$ is the number of allocated SD pairs when the relay assignment phase ends. Its expected value is given by $\mathbf{E}\left[Q_{\tau_{N,T_s}}\right] = \mathbf{E}\left[Q_{\min\{N,T_s\}}\right]$ since if $D$ SD pairs have been allocated before $\min\{N, T_s\}$ slots, the number of allocated SD pairs in the succeeding slots remains at $D$. The numerator and denominator of (8) are evaluated using Propositions 1 and 2, respectively.

### B. Contention-based en masse assignment (CBEA)

In this algorithm, the relays are not polled sequentially. Instead, they contend to transmit on the basis of their availabilities. Conceptually, the algorithm operates as follows. Based on whether its availability lies between two thresholds, every relay decides whether to transmit or not in a slot. At the end of every slot, the CC broadcasts an idle, success, or collision outcome indicating whether zero, one, or multiple relays, respectively, transmitted. The thresholds for the next slot are accordingly updated. After every success slot, the CC assigns en masse to the relay that successfully transmitted all the unallocated SD pairs for which it is feasible. The design of CBEA inherently ensures that the relay that transmits in a success slot has the highest availability among all the contending relays. The rest of the relays then update their availabilities, and the process is repeated.

We shall term the time duration between subsequent successes as an *SD allocation period* (SAP). Since a random number of relays contend for transmission in different slots of a SAP, the time until the success slot is also random. After every success slot, a new SAP starts. Figure 3 illustrates how the SD pairs get allocated in CBEA.

CBEA differs from splitting-based algorithms proposed in the literature [22], [23], [28]–[30] in several ways given the unique features of en masse selection. First, the availability is a discrete RV; thus, more than one relay can have the same availability with non-zero probability. This also means that all the relays with the same availability are equivalent. Secondly, the availability of a relay can change every time a success occurs because its FBs are updated. The changes in the availabilities depend on the history of SD pair assignments in previous SAPs and plays a key role in the design of CBEA.

We now introduce the state variables that CBEA employs. Then, we characterize the effect of the changing availabilities, define the algorithm, and finally explain its design.

*1) State variables:* The algorithm maintains three state variables in every slot $k$ of the $n$th SAP: left threshold $L_n(k)$, right threshold $R_n(k)$, and a collision bit $C_n(k) \in \{0, 1\}$.[3] Zero indicates that no collision has occurred in the first $k$ slots of the SAP and one indicates otherwise. The algorithm is said to be in *collision mode* if $C_n(k) = 1$, and is in *non-collision mode* otherwise.

*2) Pmf of availability:* Let $\nu_n(r)$ denote the availability of relay $r$ during the $n$th SAP. As observed earlier, it is a discrete

[3]Every relay stores and computes these variables independently based only on the outcomes broadcast by the CC. The values of the state variables turn out to be the same across all the relays in every slot.

RV. It remains constant during a SAP and can only change at the start of a new SAP. As shown below, its statistics are also different from one SAP to the other. Let $F_n(\cdot)$ denote the cumulative distribution function (CDF) of $\nu_n(r)$. For $n \geq 1$, let $a_n$ denote the number of SD pairs allocated in the $n$th SAP and let $b_n$ denote the number of unallocated SD pairs at the end of the $n$th SAP. Hence,

$$b_n = D - \sum_{i=1}^{n} a_i, \quad n \geq 1. \tag{9}$$

Furthermore, $a_0 = 0$ and $b_0 = D$. Since all the unallocated SD pairs of the selected relay in a success slot get allocated, it does not participate in the algorithm thereafter. Let $u_n$ denote the number of relays contending in the $n$th SAP. We then have

$$u_n = N - n + 1, \quad n \geq 1. \tag{10}$$

In the 1st SAP, $\nu_1(r)$ has a binomial distribution with parameters $D$ and $p_f$, i.e., for $0 \leq k \leq D$, $\mathbf{P}[\nu_1(r) = k] = \binom{D}{k} p_f^k (1-p_f)^{D-k}$. If $a_1$ SD pairs are allocated at the end of the 1st SAP, we know that

$$\nu_2(r) \leq \min\{a_1, D - a_1\}. \tag{11}$$

Thus, the pmf of $\nu_2(r)$ depends on $a_1$. In general, for $n \geq 2$, $F_n$ depends on the history of previous relay assignments. To design a splitting-based algorithm such as CBEA, knowledge of the CDFs of the relay availabilities in each SAP is essential. The following key result shows how $F_{n+1}$ is updated using $F_n$.

*Result 1:* For $0 \leq x \leq \min\{a_n, b_n\}$ and $x \in \mathbb{Z}$,

$$\begin{aligned} F_{n+1}(x) &= F_{n+1}(x-1) \\ &\quad + \frac{\sum_{t=0}^{u_n-2} \frac{\binom{u_n-2}{t}}{(t+1)} w^{t+1} y^{u_n-2-t} \sum_{j=0}^{a_n-1} d_{jx}}{\frac{1}{u_n}\left[(w+y)^{u_n} - y^{u_n}\right]} \\ &\quad + \frac{d_{a_n x} \sum_{t=1}^{u_n-1} \frac{\binom{u_n-2}{t-1}}{(t+1)} w^t y^{u_n-t-1}}{\frac{1}{u_n}\left[(w+y)^{u_n} - y^{u_n}\right]}, \end{aligned} \tag{12}$$

where $d_{jx} = \frac{(F_n(j)-F_n(j-1))\binom{a_n}{j-x}\binom{b_n}{x}}{\binom{a_n+b_n}{j}}$, for $0 \leq j \leq a_n$, $w = F_n(a_n) - F_n(a_n-1)$, $y = F_n(a_n-1)$, and $F_{n+1}(-1) = 0$. Initially, $F_1(l) = \sum_{i=0}^{l} \binom{D}{i} p_f^i (1-p_f)^{D-i}$, for $0 \leq l \leq D$.

*Proof:* The proof is given in Appendix C. ■

*3) Algorithm definition:* In the following, we specify the algorithm's operation during the $n$th SAP, for all $n \geq 1$. The slots are numbered relative to the beginning of a SAP.

*Initialization ($k = 1$):* At the start of the first slot of a SAP, a relay $r$ generates a new RV $U_r$ that is uniform in $(0, 1)$ and independent of $\nu_n(r)$. It then forms its *selection metric* using the pmf of its availability as follows:

$$\mu_n(r) = F_n(\nu_n(r) - 1) + U_r\left[F_n(\nu_n(r)) - F_n(\nu_n(r) - 1)\right]. \tag{13}$$

As shown in Appendix D, the selection metric $\mu_n(r)$ is then uniformly distributed in $(0, 1)$. Therefore, with probability one, no two relays have the same selection metric, even though they might have the same availability. Further, a relay with a lower availability will necessarily have a lower metric than a relay with a higher availability. This idea is a generalization of proportional expansion that was used in [23] for SSD systems. Now, the relay with the highest availability is also the relay with the highest metric.

Initially, $L_n(1) = 1 - \frac{1}{u_n}$, $R_n(1) = 1$, and $C_n(1) = 0$.

*Transmission rule in slot $k$:* A relay $r$ transmits in slot $k$ of the $n$th SAP only if its selection metric lies between $L_n(k)$ and $R_n(k)$:

$$L_n(k) \leq \mu_n(r) < R_n(k). \tag{14}$$

When a relay transmits, it sends its feasibility vector to the CC.

*Outcomes:* At the end of each slot, the CC broadcasts one of the following three outcomes: (i) *idle*, if no relay transmitted, or (ii) *collision*, if at least two relays transmitted and collided, or (iii) *success*, when one relay transmitted and was, thus, decoded by the CC. The unallocated SD pairs for which this relay is feasible are all assigned to it and its feasibility vector is broadcast by the CC. Conservatively, the CC broadcast overhead is $\lceil \log_2 D \rceil$ bits. Every relay then updates its feasibility vector by setting to zero all the FBs whose SD pairs got allocated.

*Variable update rules in the $n$th SAP:* Let $\Delta_n(k) = R_n(k) - L_n(k)$ denote the interval length in slot $k$. The state variables are updated as follows:

- *If slot $k$ is an idle:* $\Delta_n(k+1)$ is updated as follows[4]:

$$\Delta_n(k+1) = \begin{cases} \frac{L_n(k)}{u_n}, & C_n(k) = 0 \text{ (Non-collision mode)} \\ \frac{\Delta_n(k)}{2}, & C_n(k) = 1 \text{ (Collision mode)} \end{cases}. \tag{15}$$

  Then, $R_n(k+1) = L_n(k)$, $L_n(k+1) = R_n(k+1) - \Delta_n(k+1)$, and $C_n(k+1) = C_n(k)$.
- *If slot $k$ is a collision:* In this case, $R_n(k+1) = R_n(k)$, $\Delta_n(k+1) = \frac{\Delta_n(k)}{2}$, $L_n(k+1) = R_n(k+1) - \Delta_n(k+1)$, and $C_n(k+1) = 1$.
- *If slot $k$ is a success:* Then the $(n+1)$th SAP begins unless the algorithm terminates.

*Termination:* The algorithm terminates when all the $D$ SD pairs have been allocated, or $R_n(k) \leq F_n(0)$, or $T_s$ slots have been used up for selection.

*4) Explanation of the algorithm:* In every SAP, CBEA essentially scans the interval $(0, 1)$ from the right, searching for the relay with the highest metric. The thresholds are successively lowered in every slot until a non-idle slot occurs. At this point the algorithm enters the collision mode, in the case of a collision, or it starts the next SAP in the case of a success. CBEA behaves differently during the collision and the non-collision modes. It adheres to the following design rules: (i) *Non-collision mode:* Here, one node, on average, transmits in a slot. This maximizes the probability of success in the slot [22]. The interval length update $\Delta_n(k+1) = \frac{L_n(k)}{u_n}$ in (15) ensures this since, after $k$ consecutive idle slots, the selection metrics of the $u_n$ unassigned relays are uniformly distributed in $(0, L_k)$, (ii) *Collision mode:* The average number of relays that transmit in a slot is half the number that had collided previously. The interval length update $\Delta_n(k+1) = \frac{\Delta_n(k)}{2}$ in (15) ensures this. This continues until a success occurs.

At the end of $(n-1)$th SAP, the availabilities of *all* the relays might become zero in case there are no feasible relays

[4] $C_n(k)$ is one when a collision has occured in any of the slots $1, \ldots, k-1$ of the $n$th SAP. Hence, slot $k$ can be an idle even in the collision mode.

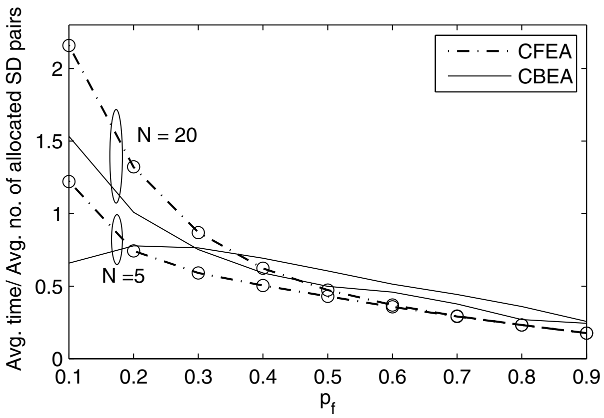


Fig. 4. Comparison of the ratio of average total time taken to the average total number of allocated SD pairs for CFEA and CBEA as a function of $p_f$ ($D = 10$ and $T_s \to \infty$). Analytical values of the ratio $\frac{\mathbf{E}[Q_N]}{\mathbf{E}[\tau_{N,\infty}]}$ for CFEA (cf. Sec. III-A1) are shown using the marker '○'.

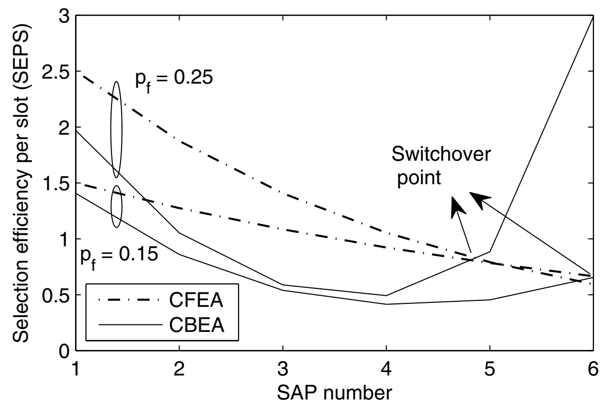


Fig. 5. Comparison of the ratio of average number of allocated SD pairs to the average SAP duration ($D = N = 10$).

for the unallocated SD pairs. However, the CC can only learn about this by proceeding $k$ slots into the next SAP until $R_n(k) \leq F_n(0)$, at which time it becomes obvious that no more relay assignments are possible since this means that the selection metrics of all the relays are lower than $F_n(0)$. Therefore, the algorithm then terminates.

### C. Selection speed comparisons

We now compare the selection speeds of CBEA and CFEA and develop insights into their working, which shall lead to an even better hybrid algorithm in Sec. III-D. To do so, we initially consider the case $T_s \to \infty$.

First, we measure the speed of relay assignment of the CFEA and CBEA algorithms. Figure 4 plots the ratio of the average total time in slots to the average total number of allocated SD pairs as a function of $p_f$.[5] This is done for a system with $D = 10$ SD pairs for $N = 5$ and 20 relays. The SR and RD channels are exponentially distributed with unit mean. This ratio measures how fast a selection algorithm is in allocating SD pairs. We observe that both algorithms are extremely fast over a wide range of values of $p_f$. For example, for $N = 20$, CFEA and CBEA take $0.87$ and $0.75$ slots, respectively, on average, to assign a feasible relay to an SD pair when $p_f = 0.3$. These numbers decrease to $0.37$ and $0.46$ slots, respectively, when $p_f$ increases to $0.6$. The assignment times decrease as $p_f$ increases since the probability that a relay is feasible for an SD pair increases. For CFEA, the ratio decreases to $\frac{1}{D}$ as $p_f \to 1$ since then only one slot would then be required to allocate all the $D$ SD pairs. For CBEA, the ratio decreases to $\frac{2.47}{D}$ for large $N$ as $p_f \to 1$ since one SAP is required to allocate all the $D$ SD pairs.

CBEA outperforms CFEA for $p_f \leq 0.2$ for $N = 5$ and $p_f \leq 0.45$ for $N = 20$. However, as $p_f$ increases, CFEA becomes faster. This is because a relay is feasible for a larger number of SD pairs, which enables CFEA to allocate most of them early on without wasting any time on collisions. Further, for larger $N$, CBEA outperforms CFEA over a larger $p_f$ regime. For CFEA, notice that the analysis and simulation results match each other very well.

*1) Selection efficiency per slot:* We now delve deeper into the two algorithms by observing their behaviour over different time slots. To this end, we introduce the notion of *selection efficiency per slot* (SEPS), which is defined as the ratio of the average number of relays assigned in a SAP to the average duration of a SAP (in slots). It measures how many SD pairs, on average, get assigned in each slot. For CFEA, each slot is effectively a SAP. We denote by $U_{CFEA}^{(n)}$ and $U_{CBEA}^{(n)}$ the SEPS of CFEA and CBEA, respectively, of the $n$th SAP.

Figure 5 plots the SEPS of the first five SAPs for CFEA and CBEA for $N = D = 10$ and for two different $p_f$ values. In both cases, the SEPS of CFEA is higher than that of CBEA in the first few SAPs. This is because CFEA allocates multiple SD pairs, en masse, in the initial slots without wasting anytime in contention. However, as time progresses, the fewer remaining unallocated SD pairs become increasingly harder to assign in CFEA. On the other hand, for CBEA, the fact that there are fewer remaining SD pairs does not affect it because it always seeks the relay with the highest availability and because it updates the CDF $F_n$. Hence, in the later SAPs, the SEPS of CBEA is higher.

### D. Hybrid algorithm

Based on the above observations, we propose the following hybrid algorithm. It uses CFEA in the initial slots in which SEPS of CFEA exceeds that of CBEA. Thereafter, it switches over to CBEA and persists with it. Note that, depending on $p_f$, the hybrid algorithm might end up using CFEA or CBEA exclusively. The switchover point can be pre-computed as a function of the system parameters as follows.

*Switchover point:* Suppose that the hybrid algorithm has been using CFEA until slot $n$ ($\geq 0$). Then, $b_n$ SD pairs remain unallocated at the start of the next slot. The use of CFEA again would result in $b_n p_f$ SD pairs, on average, getting allocated in the next slot. If CBEA is used instead, then the average number of SD pairs allocated in the next slot is calculated as follows.

[5]The average number of allocated SD pairs can be less than $D$ because some SD pairs may not have any feasible relays.

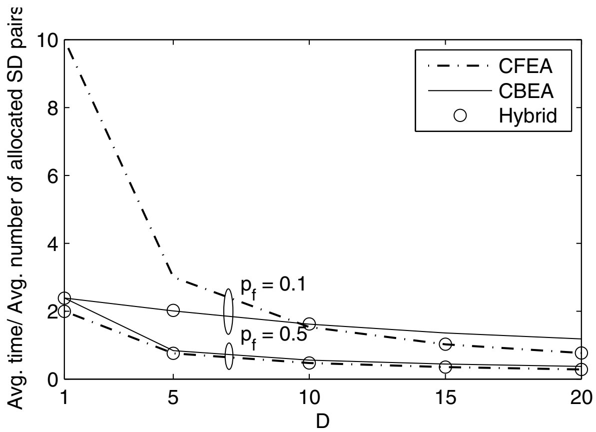


Fig. 6. Comparison of the ratio of average total time taken to the average total number of allocated SD pairs for CFEA, CBEA, and hybrid algorithm as a function of $D$ ($N = 10$).

From Result 1, the probability that a relay's availability is $j$ is given by $\binom{b_n}{j} p_f^j (1-p_f)^{b_n - j}$. Since CBEA selects the relay with the highest availability from among $u_{n+1}$ remaining relays, the probability that $a_{n+1}$ is at least $k+1$ is given by $1 - \left[\sum_{j=0}^{k} \binom{b_n}{j} p_f^j (1-p_f)^{(b_n-j)}\right]^{u_{n+1}}$. Summing over $k$, the expected value of the availability of the assigned relay is given by

$$\mathbf{E}[a_{n+1}] = b_n - \sum_{k=0}^{b_n - 1} \left[\sum_{j=0}^{k} \binom{b_n}{j} p_f^j (1-p_f)^{(b_n-j)}\right]^{u_{n+1}}. \tag{16}$$

And, from [22], [23], this assignment takes at most 2.47 slots, on average.

Thus, the hybrid algorithm switches to CBEA in slot $n+1$ if

$$U_{CFEA}^{(n+1)} = b_n p_f < \frac{\mathbf{E}[a_{n+1}]}{2.47} = U_{CBEA}^{(n+1)}, \quad n \geq 0. \tag{17}$$

*1) Performance of hybrid algorithm:* Figure 6 plots the ratio of the average total time (in slots) to the average total number of allocated SD pairs as a function of $D$ for CFEA, CBEA, and the hybrid algorithm. The number of relays is $N = 10$ relays and two different values of $p_f$ are considered. We observe that the hybrid algorithm is always faster than CFEA and CBEA. For example, for $p_f = 0.1$, the hybrid algorithm replicates the behaviour of CBEA when $D = 5$ and is $33\%$ faster than CFEA. On the other hand, when $D = 15$, the hybrid algorithm replicates the behaviour of CFEA and is $18.5\%$ faster than CBEA. For larger values of $p_f$, e.g., $p_f = 0.5$, all the three algorithms are equally fast.

The comparison with the $D = 1$ case in the figure brings out the advantage of en masse assignment in an MSD system. Here, $D = 1$ corresponds to an SSD system. It also corresponds to the case when relay selection is performed one SD pair after the other. We see that at least two slots, on average, are required to assign a relay to an SD pair for the two $p_f$ values considered in the figure. In fact, at least one slot per SD pair, on average, is required for *any* value of $p_f$. Thus, selection can be made extremely fast in an MSD system.

### *E. Comments*

All three algorithms ensure that any SD pair that has at least one feasible relay will get allocated a feasible relay. Since an assigned relay expends the same fixed amount of energy in transmitting to its corresponding destination in the RD transmission phase, the total energy consumed by all the relays is, thus, the same for all the algorithms.

The order in which different SD pairs and relays get paired depends on the realizations of the metrics. Therefore, these algorithms can be viewed as performing joint source-relay selection. Further, depending on the metric realizations, different relays may be assigned to different numbers of SD pairs, and can, thus, consume different amounts of energies. However, the probability that a relay is assigned to an SD pair is the same for all the relays in our model. Therefore, all the relays consume the same amount of energy on average. When different relays see statistically non-identical channel gains, then the definition of the metric can be changed, if desired, to implement different notions of fairness [31].

## IV. System-Level Tradeoffs and Performance Benchmarking

In this section, we study system-level tradeoffs and benchmark CFEA, CBEA, and the hybrid algorithm against other relay selection schemes that have been proposed in the literature. We use Monte Carlo simulations that use $10^6$ different channel realizations. In the simulations, the SR and RD channel gains are exponentially distributed with unit mean.

### *A. Benchmark schemes*

We consider the following two schemes:

1) *Contention-free Poll-all scheme [5]:* In it, every relay sends its feasibility vector to the CC one by one. This requires $N$ slots.
2) *Contention-based distributed pairing scheme (DPS) [26]:* In it, the selection duration $T_s$ is divided into $G$ pairing sections, each of which consists of $H$ slots. In every pairing section, relays feasible for only one SD pair pick a slot randomly and transmit in it. Each successful transmission results in an allocation of an SD pair, while relays that collided participate in the next pairing section. In the last pairing section, a relay randomly picks an SD pair that it is feasible for and contends for it. Collisions are not resolved in the last pairing section. In the simulations, we use $G = 3$, as per [26].

### *B. Net throughput comparisons*

Figure 7 plots the net throughput as a function of the source rate for CFEA, CBEA, Poll-all, DPS, and the hybrid algorithm with $D = N = 10$. We see that for all values of $R$, either CFEA or CBEA outperforms the benchmark schemes. For example, at $R = 0.5$ bps, they respectively have $29.4\%$ and $17.9\%$ larger net throughputs than Poll-all. At $R = 2.5$ bps, CBEA outperforms CFEA, Poll-all, and DPS by $42.8\%$. The hybrid algorithm always outperforms all the other schemes. Note that DPS performs markedly worse for all values of $R$.

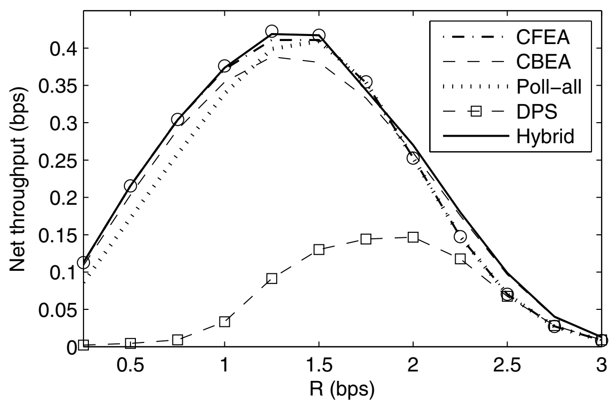


Fig. 7. Net throughput as a function of $R$ for CFEA, CBEA, Poll-all [5], DPS [26], and hybrid algorithm ($D = 10, N = 10, T_0 = 10, T_s = 9$, and $\gamma_0 = \delta_0 = 3$ dB). The net throughput of CFEA evaluated using (8) is shown using the marker '$\circ$'.

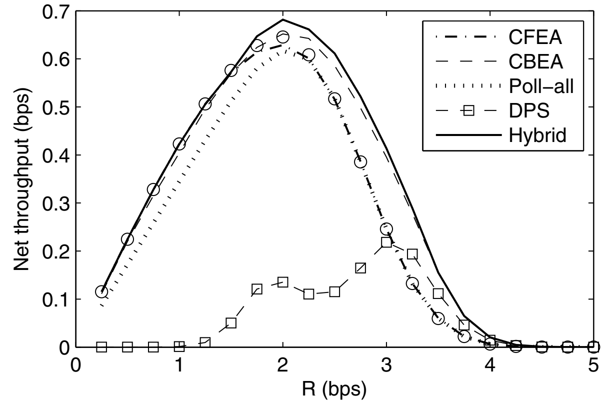


Fig. 8. Net throughput as a function of $R$ for CFEA, CBEA, Poll-all [5], DPS [26], and hybrid algorithms ($D = 10, N = 20, T_0 = 10, T_s = 9$, and $\gamma_0 = \delta_0 = 6$ dB). The net throughput of CFEA evaluated using (8) is shown using the marker '$\circ$'.

For small $R$, the net throughput of the proposed schemes increases with $R$ since the data rate has increased. As $R$ increases further, fewer of the relays are feasible. Hence, the net throughput eventually decreases.

Note that for small $R$, CFEA outperforms CBEA. This is because $p_f$ is larger (cf. (4)), which makes CFEA faster than CBEA. The throughput trend is reversed as $R$ increases. Furthermore, CFEA outperforms Poll-all because it is able to terminate as soon as all the $D$ SD pairs are allocated. However, as $R$ increases, $p_f$ decreases, and CFEA also ends up polling all the relays. Consequently the net throughputs of CFEA and Poll-all are the same for large $R$. Another important point that comes out of the figure is that the relay selection overhead is not negligible. For example, in Figure 7, the maximum net throughput of CFEA occurs at $R = 1.25$ bps. At this point, the average time spent on selection turns out to be $8.2\%$ of the SR data transmission phase duration and $7.4\%$ of the SD pairs remained unallocated. The selection overheads are even larger for the benchmark schemes.

Figure 8 plots the net throughput as a function of $R$ for all the algorithms for $D = 10$ and $N = 20$. With more relays in the system, CBEA can select relays with larger availabilities, on average, and, hence, outperforms CBEA, DPS, and Poll-all for a larger range of $R$. The net throughput of the DPS scheme is higher than that of CFEA and Poll-all when $3 \le R \le 4$. However, it underperforms CBEA and the Hybrid algorithm in this regime. As before, the hybrid algorithm is the best for all $R$.

## V. Conclusions

We proposed and studied novel contention-free and contention-based relay selection algorithms for an MSD cooperative system that uses DF relays. In both CFEA and CBEA, a relay when selected is assigned en masse to all the unallocated SD pairs that it can aid. We compared their efficiency in assigning relays to SD pairs on a per slot basis using a new performance measure called SEPS. We saw that CFEA typically assigns more SD pairs per slot than CBEA initially, but this trend is reversed in later slots. These insights led to the development of a novel hybrid algorithm, which combines the best features of CFEA and CBEA.

We showed that these three algorithms achieve a higher net throughput compared to the schemes proposed in the literature. The average time required to assign a relay to an SD pair by the proposed algorithms is much less than one slot, and is markedly lower than that required by selection algorithms for SSD systems. Interestingly, CFEA is as effective as CBEA over a wider range of system parameters than in SSD systems. Further, the time spent on selection is not negligible, and must be accounted for in the MSD system design.

## Appendix

### A. Proof of Proposition 1

Initially, $Q_0 = 0$ since no SD pair has been allocated yet. Let $X_n$ denote the number of SD pairs that get allocated during the $n$th slot, $n \ge 1$. Then, we have

$$Q_n = Q_{n-1} + X_n. \tag{18}$$

If $Q_{n-1} = q$ SD pairs have been allocated by the end of the $(n-1)$th slot, then in the $n$th slot, the allocated SD pairs, if any, must be from among the $D - q$ unallocated SD pairs. Further, since the channel gains are i.i.d., the availability $X_n$ of the polled relay is purely a function of its feasibility for these $D - q$ remaining SD pairs. Thus, the availability conditioned on $Q_{n-1} = q$, for $0 \le k \le D - q$, is given by

$$\mathbf{P}[X_n = k | Q_{n-1} = q] = \binom{D-q}{k} p_f^k (1 - p_f)^{D-q-k}. \tag{19}$$

For $n \ge 1$ and $0 \le i \le D$, from the law of total probability and (18), $\mathbf{P}\left[Q_n = i\right]$ is given by

$$\begin{aligned} \mathbf{P}[Q_n = i] &= \sum_{j=0}^{i} \mathbf{P}[Q_{n-1} = j]\mathbf{P}[Q_n = i | Q_{n-1} = j], \\ &= \sum_{j=0}^{i} \mathbf{P}[Q_{n-1} = j]\mathbf{P}[X_n = i - j | Q_{n-1} = j]. \end{aligned} \tag{20}$$

Substituting (19) in (20) yields the desired result.

### B. Proof of Proposition 2

Let $\tau$ denote the time required to allocate all the $D$ SD pairs in a system with a large number of relays and no cap on the selection duration. Since $Q_n$ cannot decrease with $n$, we have

$$\mathbf{P}[\tau > n] = \mathbf{P}[Q_1 < D, \ldots, Q_n < D] = \mathbf{P}[Q_n < D]. \quad (21)$$

Since $\tau_{N,T_s} = \min\{\tau, \min\{N, T_s\}\}$, its complementary CDF $\mathbf{P}\left[\tau_{N,T_s} > k\right]$ is given by

$$\mathbf{P}[\tau_{N,T_s} > k] = \begin{cases} \mathbf{P}[\tau > k], & 0 \le k \le \min\{N, T_s\} - 1 \\ 0, & k \ge \min\{N, T_s\} \end{cases}. \quad (22)$$

Using (21) and (22), the average duration of CFEA is then given by

$$\begin{aligned} \mathbf{E}\left[\tau_{N,T_s}\right] &= \sum_{k=0}^{\min\{N,T_s\}-1} \mathbf{P}\left[\tau_{N,T_s} > k\right], \\ &= \sum_{k=0}^{\min\{N,T_s\}-1} \mathbf{P}\left[Q_k < D\right], \\ &= \sum_{k=0}^{\min\{N,T_s\}-1} \sum_{j=0}^{D-1} \mathbf{P}\left[Q_k = j\right]. \quad (23) \end{aligned}$$

### C. Proof of Result 1

Let $\bar{r}_n$ denote the relay assigned in the $n$th SAP and let $\mathcal{H}_n$ denote the set of availabilities and identities of the relays that have been assigned in the first $n$ SAPs, i.e.,

$$\mathcal{H}_n = \{\nu_n(\bar{r}_n) = a_n, \bar{r}_n \text{ sel.}, \ldots, \nu_1(\bar{r}_1) = a_1, \bar{r}_1 \text{ sel.}\},$$

where $\bar{r}_n$ sel. indicates that relay $\bar{r}_n$ is selected in the $n$th SAP. Observe that the pmf of a relay's availability in the $(n+1)$th SAP is determined by $\mathcal{H}_n$. For an unassigned relay $r$ in the $(n+1)$th SAP, we must find the pmf of $\nu_{n+1}(r)$ given $\mathcal{H}_n$.

First, we determine the range of values that $\nu_{n+1}(r)$ can take. By definition, since at least one SD pair is assigned in every SAP, $\nu_n(r)$ cannot increase with $n$. Hence, $\nu_{n+1}(r) \le \nu_n(r)$. Since the relay with the largest availability is selected during the $n$th SAP and relay $r$ has remained unassigned, it follows that $\nu_n(r) \le a_n$. Thus, $\nu_{n+1}(r) \le \nu_n(r) \le a_n$. Furthermore, a relay's availability cannot exceed the total number of unallocated SD pairs at the start of a SAP. Hence, $\nu_{n+1}(r) \le b_n$. Therefore, $\nu_{n+1}(r) \le \min\{a_n, b_n\}$. From Bayes' rule, for $0 \le x \le \min\{a_n, b_n\}$, we have

$$\begin{aligned} &\mathbf{P}\left[\nu_{n+1}(r) = x | \mathcal{H}_n\right] \\ &\quad = \frac{\mathbf{P}\left[\nu_{n+1}(r) = x, \nu_n(\bar{r}_n) = a_n, \bar{r}_n \text{ sel.} | \mathcal{H}_{n-1}\right]}{\mathbf{P}\left[\nu_n(\bar{r}_n) = a_n, \bar{r}_n \text{ sel.} | \mathcal{H}_{n-1}\right]}. \quad (24) \end{aligned}$$

Clearly, $F_n(-1) = 0$, for all $n \ge 1$.

*1) Evaluating denominator of (24):* Since the availabilities of the $u_n$ relays that contend in the $n$th SAP are statistically identical, the probability that relay $\bar{r}_n$ is selected from among them is $\frac{1}{u_n}$. The selected relay's availability is $a_n$ if and only if the availability of all the $u_n$ relays is less than or equal to $a_n$ and at least one relay has availability equal to $a_n$. From the definition of $F_n$, the probability of this event is $(F_n(a_n))^{u_n} - (F_n(a_n - 1))^{u_n}$. Therefore,

$$\begin{aligned} &\mathbf{P}\left[\nu_n(\bar{r}_n) = a_n, \bar{r}_n \text{ sel.} | \mathcal{H}_{n-1}\right] \\ &\quad = \frac{1}{u_n}\left[(F_n(a_n))^{u_n} - (F_n(a_n - 1))^{u_n}\right]. \quad (25) \end{aligned}$$

*2) Evaluating numerator of (24):* Let $t$ relays, excluding relay $\bar{r}_n$, have availability equal to $a_n$. Then, the probability that $\bar{r}_n$ is selected is $\frac{1}{t+1}$. Let $\Omega_n$ denote the set of relays that contend in the $n$th SAP *excluding* relays $\bar{r}_n$ and $r$. It is of size $u_n - 2$. The following two mutually exclusive cases arise.

(i) *Relay $r$ was not one among the $t$ relays (i.e., $\nu_n(r) = j$, where $0 \le j \le a_n - 1$):* In this case, $t$ relays from $\Omega_n$ have availability equal to $a_n$. This can be written as $\sum_{y \in \Omega_n} 1_{\{\nu_n(y) = a_n\}} = t$, where $1_{\{W\}}$ denotes the indicator function – it is 1 if $W$ is true and is 0 otherwise. Furthermore, $u_n - 2 - t$ relays in $\Omega_n$ have availabilities strictly less than $a_n$. Clearly, $0 \le t \le u_n - 2$.

(ii) *Relay $r$ was one among the $t$ relays (i.e., $\nu_n(r) = a_n$):* In this case, $t-1$ relays in $\Omega_n$ have availability equal to $a_n$, i.e., $\sum_{y \in \Omega_n} 1_{\{\nu_n(y) = a_n\}} = t-1$, and the remaining $u_n - 1 - t$ relays in $\Omega_n$ have availabilities strictly less than $a_n$. Clearly, $1 \le t \le u_n - 1$.

Combining the above cases, we can write the numerator of (24) as

$$\begin{aligned} &\mathbf{P}\left[\nu_{n+1}(r) = x, \nu_n(\bar{r}_n) = a_n, \bar{r}_n \text{ sel.} | \mathcal{H}_{n-1}\right] \\ &\quad = \sum_{t=0}^{u_n-2} \frac{1}{t+1} \sum_{j=0}^{a_n-1} \psi_j + \sum_{t=1}^{u_n-1} \frac{1}{t+1} \psi_{a_n}, \quad (26) \end{aligned}$$

where, for $0 \le j \le a_n - 1$,

$$\begin{aligned} \psi_j = \mathbf{P}\Bigg[&\nu_n(\bar{r}_n) = a_n, \nu_{n+1}(r) = x, \nu_n(r) = j, \\ &\sum_{y \in \Omega_n} 1_{\{\nu_n(y) = a_n\}} = t, \\ &\sum_{z \in \Omega_n} 1_{\{\nu_n(z) < a_n\}} = u_n - 2 - t \Big| \mathcal{H}_{n-1}, \bar{r}_n \text{ sel.}\Bigg], \end{aligned}$$

and

$$\begin{aligned} \psi_{a_n} = \mathbf{P}\Bigg[&\nu_n(\bar{r}_n) = a_n, \nu_{n+1}(r) = x, \nu_n(r) = a_n, \\ &\sum_{y \in \Omega_n} 1_{\{\nu_n(y) = a_n\}} = t - 1, \\ &\sum_{z \in \Omega_n} 1_{\{\nu_n(z) < a_n\}} = u_n - 1 - t \Big| \mathcal{H}_{n-1}, \bar{r}_n \text{ sel.}\Bigg]. \end{aligned}$$

Clearly, the availabilities of the unassigned relays are statistically identical. They are independent at the start of the first SAP. To facilitate analysis, we make the decoupling approximation that they remain conditionally independent in the subsequent SAPs. Then, for $0 \le j \le a_n - 1$,

$$\begin{aligned}\psi_j &= \mathbf{P}\left[\nu_n(\bar{r}_n) = a_n | \mathcal{H}_{n-1}, \bar{r}_n \text{ sel.}\right] \\ &\quad \times \mathbf{P}\left[\nu_{n+1}(r) = x, \nu_n(r) = j | \mathcal{H}_{n-1}, \bar{r}_n \text{ sel.}\right] \\ &\quad \times \mathbf{P}\Bigg[\sum_{y \in \Omega_n} 1_{\{\nu_n(y) = a_n\}} = t, \\ &\qquad\qquad \sum_{z \in \Omega_n} 1_{\{\nu_n(z) < a_n\}} = u_n - 2 - t \Big| \mathcal{H}_{n-1}, \bar{r}_n \text{ sel.}\Bigg], \\ &= \binom{u_n - 2}{t} \left(F_n(a_n) - F_n(a_n - 1)\right)^{t+1} \left(F_n(a_n - 1)\right)^{u_n - 2 - t} \\ &\quad \times \mathbf{P}\left[\nu_{n+1}(r) = x, \nu_n(r) = j | \mathcal{H}_{n-1}, \bar{r}_n \text{ sel.}\right]. \quad (27)\end{aligned}$$

Here, (27) follows because the probability that a relay's availability is at most $a_n - 1$ in the $n$th SAP is equal to $F_n(a_n - 1)$, the probability that it is $a_n$ is equal to $F_n(a_n) - F_n(a_n - 1)$, and the fact that $t$ relays can be chosen out of $u_n - 2$ relays in $\binom{u_n - 2}{t}$ ways.

*Evaluating* $\mathbf{P}\left[\nu_{n+1}(r) = x, \nu_n(r) = j | \mathcal{H}_{n-1}, \bar{r}_n \textit{ sel.}\right]$*:* The event $\{\nu_{n+1}(r) = x, \nu_n(r) = j\}$ implies that relay $r$ is feasible for $j$ SD pairs at the start of the $n$th SAP, and $(j - x)$ of them are common to relays $\bar{r}_n$ and $r$. Consider the SD pairs for which relay $r$ is feasible. The total number of ways of finding $x$ feasible SD pairs among the $b_n$ SD pairs that remain after slot $n$ and finding $(j - x)$ feasible SD pairs among the $a_n$ SD pairs that are common with $\bar{r}_n$ is $\binom{b_n}{x}\binom{a_n}{j-x}$. Since any $j$ out of the total number of $(a_n + b_n)$ SD pairs are equally likely to be feasible, the probability that $\nu_{n+1}(r)$ is $x$ given $j$ SD pairs were feasible in slot $n$ is $\frac{\binom{b_n}{x}\binom{a_n}{j-x}}{\binom{a_n + b_n}{j}}$. Further, by definition, we have $\mathbf{P}\left[\nu_n(r) = j | \mathcal{H}_{n-1}, \bar{r}_n \text{ sel.}\right] = F_n(j) - F_n(j - 1)$. Thus,

$$\begin{aligned}\mathbf{P}&\left[\nu_n(r) = j, \nu_{n+1}(r) = x | \mathcal{H}_{n-1}, \bar{r}_n \text{ sel.}\right] \\ &= \mathbf{P}\left[\nu_n(r) = j | \mathcal{H}_{n-1}, \bar{r}_n \text{ sel.}\right] \\ &\quad \times \mathbf{P}\left[\nu_{n+1}(r) = x | \nu_n(r) = j, \mathcal{H}_{n-1}, \bar{r}_n \text{ sel.}\right], \\ &= \left(F_n(j) - F_n(j - 1)\right) \frac{\binom{b_n}{x}\binom{a_n}{j-x}}{\binom{a_n + b_n}{j}}. \quad (28)\end{aligned}$$

Substituting (28) in (27), we get, for $0 \le j \le a_n - 1$,

$$\begin{aligned}\psi_j &= \binom{u_n - 2}{t} \left(F_n(a_n) - F_n(a_n - 1)\right)^{t+1} \\ &\quad \times \left(F_n(a_n - 1)\right)^{u_n - 2 - t} \left(F_n(j) - F_n(j - 1)\right) \frac{\binom{b_n}{x}\binom{a_n}{j-x}}{\binom{a_n + b_n}{j}}. \quad (29)\end{aligned}$$

Similarly, we can show that

$$\begin{aligned}\psi_{a_n} &= \binom{u_n - 2}{t - 1} \left(F_n(a_n) - F_n(a_n - 1)\right)^{t+1} \\ &\quad \times \left(F_n(a_n - 1)\right)^{u_n - 1 - t} \frac{\binom{b_n}{x}\binom{a_n}{a_n - x}}{\binom{a_n + b_n}{a_n}}. \quad (30)\end{aligned}$$

Substituting (29) and (30) into (26) followed by substituting (25) and (26) in (24) yields (12).

### D. Proof that the selection metric $\mu_n(r)$ is uniform in $(0, 1)$

Let the availability $\nu_n(r)$ of relay $r$ in the $n$th SAP take values from the set $\{0, 1, \ldots, m\}$. From (13), the moment generating function (MGF) $\mathbf{E}\left[e^{t\mu_n(r)}\right]$ of $\mu_n(r)$ is given by

$$\mathbf{E}\left[e^{t\mu_n(r)}\right] = \mathbf{E}\left[e^{t\left(F_n(\nu_n(r) - 1) + U_r\left[F_n(\nu_n(r)) - F_n(\nu_n(r) - 1)\right]\right)}\right].$$

Conditioning on $\nu_n(r)$, we get

$$\begin{aligned}\mathbf{E}\left[e^{t\mu_n(r)}\right] &= \sum_{i=0}^{m} \mathbf{P}\left[\nu_n(r) = i\right] e^{tF_n(i-1)} \\ &\quad \times \mathbf{E}\left[e^{tU_r\left(F_n(i) - F_n(i-1)\right)} | \nu_n(r) = i\right]. \quad (31)\end{aligned}$$

From the independence of $U_r$ and $\nu_n(r)$ and the fact that $\mathbf{E}\left[e^{tU_r}\right] = \frac{e^t - 1}{t}$ (since $U_r$ is uniform over $(0, 1)$), we get

$$\mathbf{E}\left[e^{tU_r\left(F_n(i) - F_n(i-1)\right)} \big| \nu_n(r) = i\right] = \frac{e^{t(F_n(i) - F_n(i-1))} - 1}{t\left(F_n(i) - F_n(i - 1)\right)}. \quad (32)$$

Substituting (32) in (31) and using $\mathbf{P}\left[\nu_n(r) = i\right] = F_n(i) - F_n(i - 1)$, we obtain

$$\begin{aligned}\mathbf{E}\left[e^{t\mu_n(r)}\right] &= \sum_{i=0}^{m} e^{tF_n(i-1)} \frac{e^{t\left(F_n(i) - F_n(i-1)\right)} - 1}{t}, \\ &= \frac{e^{tF_n(m)} - e^{tF_n(-1)}}{t}.\end{aligned}$$

Since $F_n(m) = 1$ and $F_n(-1) = 0$, we get $\mathbf{E}\left[e^{t\mu_n(r)}\right] = \frac{e^t - 1}{t}$. Hence, the desired result follows.

## References

[1] A. Bletsas, A. Khisti, D. Reed, and A. Lippman, "A simple cooperative diversity method based on network path selection," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 3, pp. 659–672, Mar. 2006.

[2] Y. Jing and H. Jafarkhani, "Single and multiple relay selection schemes and their achievable diversity orders," *IEEE Trans. Wireless Commun.*, vol. 8, no. 3, pp. 1414–1423, Mar. 2009.

[3] Y. Zhao, R. Adve, and T. Lim, "Improving amplify-and-forward relay networks: optimal power allocation versus selection," *IEEE Trans. Wireless Commun.*, vol. 6, no. 8, pp. 3114–3123, Aug. 2007.

[4] A. Ibrahim, A. Sadek, W. Su, and K. Liu, "Cooperative communications with relay-selection: when to cooperate and whom to cooperate with?" *IEEE Trans. Wireless Commun.*, vol. 7, no. 7, pp. 2814–2827, Jul. 2008.

[5] E. Beres and R. Adve, "Selection cooperation in multi-source cooperative networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 1, pp. 118–127, Jan. 2008.

[6] S. Chen, W. Wang, and X. Zhang, "Performance analysis of multiuser diversity in cooperative multi-relay networks under Rayleigh-fading channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 7, pp. 3415–3419, Jul. 2009.

[7] L. Sun, T. Zhang, L. Lu, and H. Niu, "On the combination of cooperative diversity and multiuser diversity in multi-source multi-relay wireless networks," *IEEE Signal Process. Lett.*, vol. 17, no. 6, pp. 535–538, Jun. 2010.

[8] H. Ding, J. Ge, D. B. da Costa, and Z. Jiang, "A new efficient low-complexity scheme for multi-source multi-relay cooperative networks," *IEEE Trans. Veh. Technol.*, vol. 60, no. 2, pp. 716–722, Feb. 2011.

[9] J. Kim, D. S. Michalopoulos, and R. Schober, "Diversity analysis of multi-user multi-relay networks," *IEEE Trans. Wireless Commun.*, vol. 10, no. 7, pp. 2380–2389, Jul. 2011.

[10] Z. Zhang, T. Lv, and X. Su, "Combining cooperative diversity and multiuser diversity: a fair scheduling scheme for multi-source multi-relay networks," *IEEE Commun. Lett.*, vol. 15, no. 12, pp. 1353–1355, Dec. 2011.

[11] X. Zhang, M. Hasna, and A. Ghrayeb, "Performance analysis of relay assignment schemes for cooperative networks with multiple source-destination pairs," *IEEE Trans. Wireless Commun.*, vol. 11, no. 1, pp. 166–177, Jan. 2012.

[12] M. Beserra de Melo and D. da Costa, "An efficient relay-destination selection scheme for multiuser multi-relay downlink cooperative networks," *IEEE Trans. Veh. Technol.*, vol. 61, no. 5, pp. 2354–2360, Jun. 2012.

[13] Z. Ding, K. K. Leung, D. L. Goeckel, and D. Towsley, "A relay assisted cooperative transmission protocol for wireless multiple access systems," *IEEE Trans. Commun.*, vol. 58, no. 8, pp. 2425–2435, Aug. 2010.

[14] C. Wang, M. Xiao, and M. Skoglund, "Diversity-multiplexing tradeoff analysis of coded multi-user relay networks," *IEEE Trans. Commun.*, vol. 59, no. 7, pp. 1995–2005, Jul. 2011.

[15] K. Phan, T. Le-Ngoc, S. Vorobyov, and C. Tellambura, "Power allocation in wireless multi-user relay networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 5, pp. 2535–2545, May 2009.

[16] D. Wang, X. Wang, and X. Cai, "Optimal power control for multi-user relay networks over fading channels," *IEEE Trans. Wireless Commun.*, vol. 10, no. 1, pp. 199–207, Jan. 2011.

[17] J. Li, J. Yuan, R. Malancy, M. Xiao, and W. Chen, "Full-diversity binary frame-wise network coding for multiple-source multiple-relay networks over slow-fading channels," *IEEE Trans. Veh. Technol.*, vol. 61, no. 3, pp. 1346–1360, Mar. 2012.

[18] J. Wu, Y. D. Zhang, M. G. Amin, and M. Uysal, "Multiple-relay selection in amplify-and-forward cooperative wireless networks with multiple-source nodes," *EURASIP J. Wireless Commun. and Netw.*, vol. 2012, no. 1, p. 256, Aug. 2012.

[19] Z. Chen, P. Fan, and K. Letaief, "Cooperative multi-source-multi-destination transmission system with relay selection," in *Proc. 2011 Wireless Commun. and Mobile Comput. Conf.*, pp. 1524–1528.

[20] J. Xu, S. Zhou, and Z. Niu, "Interference-aware relay selection for multiple source-destination cooperative networks," in *Proc. 2009 Asia-Pacific Conf. Commun.*, pp. 338–341.

[21] D. Li, Y. Xu, and J. Liu, "Distributed relay selection over multi-source and multi-relay wireless cooperative networks with selfish nodes," *Comput. Commun.*, vol. 33, no. 17, pp. 2145–2153, Nov. 2010.

[22] X. Qin and R. Berry, "Opportunistic splitting algorithms for wireless networks," in *Proc. 2004 INFOCOM*, pp. 1662–1672.

[23] V. Shah, N. B. Mehta, and R. Yim, "Splitting algorithms for fast relay selection: generalizations, analysis, and a unified view," *IEEE Trans. Wireless Commun.*, vol. 9, no. 4, pp. 1525–1535, Apr. 2010.

[24] ——, "The relay selection and transmission trade-off in cooperative communication systems," *IEEE Trans. Wireless Commun.*, vol. 9, no. 8, pp. 2505–2515, Aug. 2010.

[25] N. Marchenko and C. Bettstetter, "Impact of relay selection overhead in cooperative diversity protocols," in *Proc. 2011 VTC – Fall*, pp. 1–4.

[26] Y. Yang, H. Hu, J. Xu, and G. Mao, "Relay technologies for WiMAX and LTE-Advanced mobile systems," *IEEE Commun. Mag.*, vol. 47, no. 10, pp. 100–105, Oct. 2009.

[27] D. Michalopoulos and G. Karagiannidis, "PHY-layer fairness in amplify and forward cooperative diversity systems," *IEEE Trans. Wireless Commun.*, vol. 7, no. 3, pp. 1073–1082, Mar. 2008.

[28] D. Bertsekas and R. Gallager, *Data networks*, 2nd ed. Prentice-Hall, Inc., 1992.

[29] B. Van Houdt and C. Blondia, "Analysis of an identifier splitting algorithm combined with polling (ISAP) for contention resolution in a wireless access network," *IEEE J. Sel. Areas Commun.*, vol. 18, no. 11, pp. 2345–2355, Nov. 2000.

[30] A. Karthik and N. B. Mehta, "An opportunistic, fast, and distributed subchannel and user-pairing algorithm for OFDMA," *IEEE Trans. Commun.*, vol. 60, no. 3, pp. 767–778, Mar. 2012.

[31] A. Attar, N. Devroye, H. Li, and V. C. M. Leung, "A unified scheduling framework based on virtual timers for selfish-policy shared spectrum," in *Proc. 2010 ICC*, pp. 1–5.


**A. Karthik** received his Bachelor of Engineering degree in Telecommunications Engineering from PES Institute of Technology, Bangalore in 2006. He is currently pursuing his Ph.D. in the Electrical Communication Engineering department at the Indian Institute of Science (IISc), Bangalore. During 2006–2007, he worked at CoreEL Technologies Pvt. Ltd, where he was involved in developing network interface cards on FPGAs. His research interests broadly include resource allocation in wireless networks and multiple access protocols.

**Neelesh B. Mehta** (S'98-M'01-SM'06) received his Bachelor of Technology degree in Electronics and Communications Engineering from the Indian Institute of Technology (IIT), Madras in 1996, and his M.S. and Ph.D. degrees in Electrical Engineering from the California Institute of Technology, Pasadena, CA, USA in 1997 and 2001, respectively. He is now an Associate Professor in the Dept. of Electrical Communication Eng., Indian Institute of Science (IISc), Bangalore, India. Prior to joining IISc in 2007, he was a research scientist in AT&T Laboratories, NJ, USA (2001-02), Broadcom Corp., NJ, USA (2002-03), and Mitsubishi Electric Research Laboratories (MERL), MA, USA (2003-07).

His research includes work on link adaptation, multiple access protocols, WCDMA downlinks, cellular systems, MIMO and antenna selection, energy harvesting networks, cognitive radio, and cooperative communications. He was also actively involved in the Radio Access Network (RAN1) standardization activities in 3GPP from 2003 to 2007. He was a TPC co-chair for tracks/symposia in Globecom 2013, ICC 2013, WISARD 2010 and 2011, NCC 2011, VTC-Fall 2009, and Chinacom 2008. He has co-authored 40+ IEEE transactions papers, 65+ conference papers, and 3 book chapters, and is a co-inventor in 20 issued US patents. He was an Editor of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS from 2008 to 2011. He is now an editor of IEEE WIRELESS COMMUNICATIONS LETTERS and the *Journal of Communications and Networks*. He serves as the Director of Conference Publications on the Board of Governors of the IEEE Communications Society.