SEP-Optimal Transmit Power Policy for Peak Power and Interference Outage Probability Constrained Underlay Cognitive Radios

Salil Kashyap, *Student Member, IEEE*, and Neelesh B. Mehta, *Senior Member, IEEE*

***Abstract*—In underlay cognitive radio (CR), a secondary user (SU) can transmit concurrently with a primary user (PU) provided that it does not cause excessive interference at the primary receiver (PRx). The interference constraint fundamentally changes how the SU transmits, and makes link adaptation in underlay CR systems different from that in conventional wireless systems. In this paper, we develop a novel, symbol error probability (SEP)-optimal transmit power adaptation policy for an underlay CR system that is subject to two practically motivated constraints, namely, a peak transmit power constraint and an interference outage probability constraint. For the optimal policy, we derive its SEP and a tight upper bound for MPSK and MQAM constellations when the links from the secondary transmitter (STx) to its receiver and to the PRx follow the versatile Nakagami-m fading model. We also characterize the impact of imperfectly estimating the STx-PRx link on the SEP and the interference. Extensive simulation results are presented to validate the analysis and evaluate the impact of the constraints, fading parameters, and imperfect estimates.**

***Index Terms*—Cognitive radio, underlay, fading channels, peak power, interference outage probability, imperfect channel estimates, symbol error probability.**

I. Introduction

With the increase in the number of wireless applications and services, the demand for bandwidth has increased substantially over the years. Cognitive radio (CR) technology promises to quench this demand by allowing the users to access the spectrum opportunistically [1]–[3]. A popular paradigm of CR called the *commons spectrum usage model* separates the users into two classes, namely, primary and secondary users. A secondary user (SU) can share the spectrum with a primary user (PU) but must ensure that its transmissions do not adversely interfere with the PU [4], [5].

For this usage model, various modes such as interweave and underlay have been considered in the literature [6]. In the interweave mode, the SU transmits only when it senses that the PU is off. On the other hand, in the underlay mode, which is the focus of this paper, the secondary transmitter (STx) can transmit even when the primary is on provided that it does not cause excessive interference at the primary receiver (PRx). The interference constraint fundamentally affects how the SU transmits, and sets the underlay CR mode apart from conventional wireless communication systems [7].

Several interference constraints have been investigated in the literature, and lead to different optimal transmission policies. Broadly, they can be classified into three categories: (i) average interference constraint, in which the STx needs to ensure that the interference it causes to the PRx, when averaged over the channel gains, is below a threshold [8]–[11], (ii) instantaneous interference constraint, in which the instantaneous interference must not cross a threshold with a pre-specified probability [8]–[10], [12], and (iii) signal-to-interference-plus-noise-ratio (SINR)-based outage constraint, in which the instantaneous SINR of the primary signal at the PRx must not fall below a threshold [4], [13], [14]. The third category is most well-suited to guarantee a quality of service (QoS) at the PU. However, unlike the previous two constraints, it requires channel state information (CSI) about the primary transmitter (PTx)-PRx link at the secondary receiver (SRx). This is practically infeasible when the PUs operate oblivious to the presence of the SUs.

A. Related Literature on Transmission Policies

Recent papers on CR have developed different optimal SU transmission policies for some of the above constraint categories [4], [8]–[14]. The papers also differ in whether they impose a peak or average transmit power constraint on the STx. We discuss and categorize them below.

1) Assuming CSI about the STx-SRx and STx-PRx links: Ergodic capacity of a single SU link, which is subject to a peak or average interference power constraint at the PRx, is derived in [8]. However, no transmit power constraint is imposed on the STx. In [15], the channel capacity of SUs is studied under an average receive interference power constraint. While [10] derives the capacity of an SU subject to an average transmit power constraint and an additional constraint on the peak or average interference power at the PRx, [9] develops the ergodic, outage, and delay-limited capacities for all the four possible combinations of peak or average transmit power constraints and peak or average interference power constraints. A capacity-optimal power allocation scheme for a CR that is subject to both peak and average interference

Manuscript received April 6, 2013; revised July 16, 2013; accepted October 1, 2013. The associate editor coordinating the review of this paper and approving it for publication was M. Bennis.

The authors are with the Dept. of Electrical Communication Eng., Indian Institute of Science (IISc), Bangalore, India (e-mail: salilkashyap@gmail.com, nbmehta@ece.iisc.ernet.in).

A part of this paper will be presented at the IEEE Global Communications Conference (Globecom) 2013, Atlanta, GA, USA.

This research has been partially supported by grants from the Broadcom Foundation, USA, Indo-UK Advanced Technology Consortium (IUATC), and ANRC.

Digital Object Identifier 10.1109/TWC.2013.111013.130615

1536-1276/13$31.00 © 2013 IEEE

power constraints but without a transmit power constraint is developed in [16].

The maximum power with which the STx can transmit without violating a target outage probability is characterized in [14]. In [11], the optimal transmit policy that minimizes the SEP of a peak transmit power and an average interference power constrained STx is derived, whereas [12] derives the SEP-optimal power policy when the STx is subject to peak transmit and peak interference power constraints. The impact of imperfect CSI of the STx-PRx link on the mean capacity of a peak interference power constrained SU is analyzed in [17].

2) Assuming CSI about the STx-SRx, STx-PRx, and PTx-PRx links: In [4], an adaptive transmit power policy that maximizes the data rate of a peak transmit power constrained SU is developed. In [13], transmit power policies that maximize the ergodic and outage rates of a peak or average transmit power constrained SU are investigated. In both [4] and [13], the STx is subject to an additional primary SINR-based outage probability constraint, which requires perfect knowledge of the PTx-PRx link at the STx. In [18], the optimal power control policy that maximizes the SU ergodic rate is developed for an average transmit power constrained SU. An upper bound on the rate of the cognitive users is derived in [19] under a constraint on the average or peak secondary-to-primary interference-to-signal ratio. In [20], the ergodic capacity of an SU subject to average or peak transmit power constraints and with respect to an interference outage constraint and a signal-to-interference outage constraint is derived. However, with perfect CSI, the instantaneous interference at the PRx is never allowed to exceed a threshold.

B. Underlay Constraints and CSI Model

In this paper, we consider an underlay CR system that is subject to the following two practically motivated constraints:

- *Interference outage probability constraint:* It mandates that the instantaneous interference should not exceed a threshold τ more than P_{out} fraction of the time. The constraint is well-suited for scenarios in which the PU transmissions do not last long enough to average over a sufficient number of coherence intervals of the STx-PRx channel [9]. It has been used to design the primary exclusive zone (PEZ) to protect the PUs in CR networks [1], [21]. Furthermore, it includes as a special case the rigid constraint imposed in [8]–[10], [12], [17], in which the instantaneous interference at the PRx is required to always lie below a threshold.
- *Peak transmit power constraint:* It mandates that the SU transmit power must not exceed a peak value of $P_{\max}$. It is practically well motivated by the inherent limitations of non-linear transmit power amplifiers. Clearly, it is more restrictive than an average transmit power constraint.

We assume that the STx knows the channel power gain of its local links, i.e., its links to the SRx and to the PRx, which can be obtained in practice by exploiting channel reciprocity and by measuring the signal energy received from transmissions by the PRx. The STx does not know the PTx-PRx channel gain, given that this is practically infeasible. Furthermore, no knowledge of the phases of any channel power gain is required at the STx.

C. Contributions

We make the following contributions:

Underlay CR Model and Optimal Policy: We derive a novel SEP-optimal transmit power adaptation policy for an STx that is subject to the aforementioned constraints, which have been considered in isolation in the literature. This is achieved through a sequence of insightful results that bring out the unique attributes of these constraints and show how they mould the structure of the optimal policy. The optimal policy includes as its special case the policy derived in [17], in which the STx is subject to the peak interference constraint.

SEP Analysis: We then derive the SEP of the optimal transmit power policy for both MPSK and MQAM constellations, when the STx-SRx and STx-PRx links follow the versatile Nakagami-m fading model. It covers non-line-of-sight Rayleigh fading and the net channel seen by wireless systems that exploit frequency, time, multi-antenna, or space diversity, and line-of-sight Rician fading [7, Chp. 3] and [22, Chp. 2].

Impact of Imperfect CSI: Finally, we characterize the impact of imperfect estimation of the STx-PRx channel power gain on the SEP and the interference of the optimal policy. This is an essential first step in understanding its robustness in practical scenarios, in which the CSI is likely to be imperfect. An extensive set of results – with perfect and imperfect CSI – then quantitatively characterize the effect of various system parameters.

We develop the system model and formulate the problem in Sec. II. The optimal transmit power policy is developed in Sec. III. Its SEP is derived in Sec. IV. The impact of imperfect channel estimation is analyzed in Sec. V. Numerical results and conclusions follow in Sec. VI and Sec. VII, respectively.

II. System Model

We use the following notation henceforth. The notation $X \sim$ Nakagami(m, Ω) means that X is a Nakagami-m distributed random variable (RV) with fading parameter m and mean square value Ω. Similarly, $X \sim \mathcal{CN}(0, \delta)$ means that X is a circular symmetric complex Gaussian RV with zero mean and variance δ. The expectation with respect to an RV X is denoted by $\mathbf{E}_{\text{X}}[.]$. The probability of an event A is denoted by $\Pr(A)$.

As shown in Figure 1, we consider an underlay CR network in which the SU shares the same spectrum with the PU opportunistically. The STx sends data to the SRx, and, in the process, causes interference at the PRx. The STx, SRx, and PRx are equipped with one antenna each. Let h denote the instantaneous channel power gain of the link between the STx and the SRx, which we shall refer to as the *data link*, and let g denote the instantaneous channel power gain of the link between the STx and the PRx, which we shall refer to as the *interference link*. The two links are assumed to be independent of each other, which is justified because the primary and the secondary nodes are spatially separated. However, they need not be identically distributed. Both the links undergo Nakagami-m fading. Therefore, $\sqrt{h} \sim$ Nakagami(m_s, Ω_s) and $\sqrt{g} \sim$ Nakagami(m_p, Ω_p).

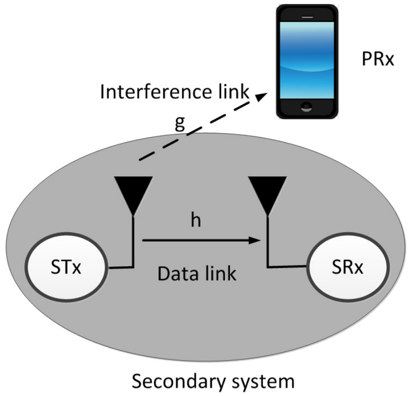

Fig. 1. System model showing a secondary system comprising of a STx that communicates with a SRx, and, in the process, interferes with a PRx.

A. Data Transmission

The STx transmits to the SRx a data symbol x that is chosen with equal probability from an MPSK or MQAM constellation. The signal y_s received by the SRx and the interference I_p seen by the PRx are given by

$$y_s = \sqrt{P}\sqrt{h}e^{j\theta_h}x + n_s + n_{ps}, \tag{1}$$

$$I_p = \sqrt{P}\sqrt{g}e^{j\theta_g}x, \tag{2}$$

where P is the power with which the STx transmits data symbols and $\mathbf{E}\left[|x|^2\right] = 1$. Also, θ_h and θ_g are the phases of the complex baseband channel gains of the data and the interference links, respectively, and n_s is circular symmetric complex additive white Gaussian noise (CAWGN) at the SRx. The interference at the SRx due to transmissions from the PTx is denoted by n_{ps} and is assumed to be Gaussian [1], [9], [23].[1] This is a worst case model for the interference from the PTx. It ensures analytical tractability and gives valuable insights. Thus, $n_s + n_{ps} \sim \mathcal{CN}(0, \sigma^2)$.

The STx is subject to the following two constraints:

1) *Interference outage probability constraint:* This constraint protects the primary link from the secondary transmissions by ensuring that the probability that the instantaneous interference power at the PRx exceeds the interference threshold τ is less than or equal to a target outage probability P_{out}. Hence, $\Pr(Pg > \tau) \le P_{\text{out}}$.
2) *Peak transmit power constraint:* This ensures that the instantaneous power of the STx does not exceed a peak value of $P_{\max}$, i.e., $P \le P_{\max}$.[2]

CSI Assumptions: We assume that the STx knows the channel power gains of its local data and interference links, i.e., h and g, as has also been assumed in [4], [8]–[10], [13], [14]. However, no knowledge of the phases of any of the channel gains is required at the STx. The SRx uses a coherent receiver and is assumed to know both h and θ_h, but neither g nor θ_g. As mentioned, the STx and the SRx also do not know the PTx-PRx channel power gain, unlike [4], [13], [14].

[1] The validity of this assumption depends on the channel fading statistics of the PTx-SRx link, the signal transmitted by the PTx, and the number of PTxs. It is also valid in a simpler scenario that has been assumed in the literature in which the PTx is far away from the SRx so that the interference that it causes to the SRx is negligible [9]–[12], [16], [20].

[2] The peak power constraint is more relevant to the secondary uplink because a secondary mobile is more likely to be peak power constrained than a secondary base station or access point.

B. Problem Statement

A power adaptation policy θ is a mapping $\theta : (\mathbb{R}^+)^2 \to \mathbb{R}^+$ that determines the STx transmit power P for every realization of h and g. We do not show explicitly the dependence of P on h and g in order to keep the notation simple. Let $\mathcal{S}$ denote the set of all feasible transmit power policies, where a feasible policy is defined as one that satisfies the peak transmit power constraint and the interference outage probability constraint. Our objective is to determine the optimal transmit power policy $\theta^* \in \mathcal{S}$ that minimizes the average SEP of the secondary system. Let $\text{SEP}(h, g)$ denote the resultant SEP given the channel power gains h and g. For MPSK, it is given by [24, (40)]

$$\text{SEP}^{\text{MPSK}}(h, g) = \frac{1}{\pi}\int_0^{\Lambda\pi} \exp\left(-\frac{Ph\sin^2\left(\frac{\pi}{M}\right)}{\sigma^2\sin^2\phi}\right) d\phi, \tag{3}$$

where $\Lambda = 1 - \frac{1}{M}$. Similarly, for square-MQAM, the SEP given h and g is equal to [24, (48)]

$$\begin{aligned}\text{SEP}^{\text{MQAM}}(h, g) = &\frac{4}{\pi}\left(1 - \frac{1}{\sqrt{M}}\right)\int_0^{\frac{\pi}{2}} \exp\left(\frac{-1.5Ph}{\Delta\sigma^2\sin^2\phi}\right) d\phi \\ &- \frac{4}{\pi}\left(1 - \frac{1}{\sqrt{M}}\right)^2 \int_0^{\frac{\pi}{4}} \exp\left(\frac{-1.5Ph}{\Delta\sigma^2\sin^2\phi}\right) d\phi,\end{aligned} \tag{4}$$

where $\Delta = M - 1$. Hence, our problem can be stated as:

$$\min_{\theta} \quad \mathbf{E}_{\text{h,g}}\left[\text{SEP}(h, g)\right], \tag{5}$$

$$\text{s. t.} \quad \Pr(Pg > \tau) \le P_{\text{out}}, \tag{6}$$

$$P \le P_{\max}. \tag{7}$$

III. Optimal Transmit Power Adaptation Policy

We now develop the optimal transmit power adaptation policy θ^* that minimizes the average SEP of the secondary system subject to the interference outage probability and peak transmit power constraints. We shall prove the following key result in this section.

Theorem *1:* Let α be the unique solution of the equation

$$1 - F_g(\alpha) = P_{\text{out}}, \tag{8}$$

where $F_g(\cdot)$ denotes the cumulative distribution function (CDF) of g. If $\frac{\tau}{P_{\max}} \ge \alpha$, then the optimal transmit power P^* is

$$P^* = P_{\max}, \ \forall\, g. \tag{9}$$

Else, the optimal transmit power is given by

$$P^* = \begin{cases} P_{\max}, & 0 \le g \le \frac{\tau}{P_{\max}} \text{ or } g > \alpha, \\ \frac{\tau}{g}, & \frac{\tau}{P_{\max}} < g \le \alpha. \end{cases} \tag{10}$$

■

The instantaneous transmit power and interference of the optimal transmit power policy are illustrated in Figures 2 and 3, respectively.

Before we delve into the proof, we highlight three interesting attributes of the optimal policy. First, it does not depend

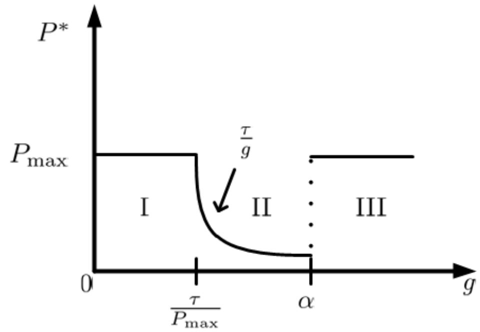

Fig. 2. Optimal policy: peak transmit power as a function of g.

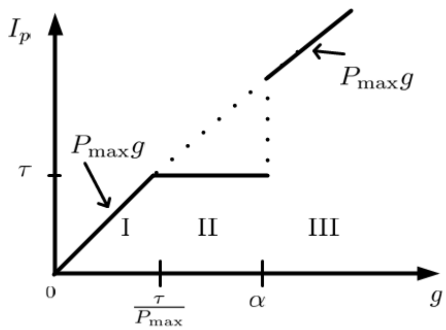

Fig. 3. Optimal policy: instantaneous interference power at the PRx as a function of g.

on the STx-SRx channel power gain h, as has also been seen in [8]–[10], [12]. Second, the STx transmits with peak power not only when the interference link is very weak but also when it is very strong. Intuitively, this is because the interference outage probability constraint penalizes the fraction of time the instantaneous interference exceeds the threshold τ, but not the extent to which it exceeds τ. This suggests that, in practice, one might impose an additional constraint on the maximum value that the interference can take. Third, the structure of the optimal policy is valid for all fading parameters; the fading statistics only affect α.

The proof is through the following sequence of claims that lead to the final result in Theorem 1, and show how the constraints influence the structure of the optimal policy.

Claim *1:* If $\frac{\tau}{P_{\max}} \geq \alpha$, then the max-power policy, i.e., $P^* = P_{\max}, \ \forall\, g$, is the optimal transmit power policy.

Proof: For the max-power policy, the outage probability P_{o} is given by

$$P_{\mathrm{o}} = \Pr(P_{\max} g > \tau) = 1 - F_g\left(\frac{\tau}{P_{\max}}\right). \tag{11}$$

If $\frac{\tau}{P_{\max}} \geq \alpha$, then it follows that $P_{\mathrm{o}} = 1 - F_g\left(\frac{\tau}{P_{\max}}\right) \leq 1 - F_g(\alpha)$ since the CDF is a monotonically non-decreasing function. However, from (8), $1 - F_g(\alpha) = P_{\mathrm{out}}$. Thus, the max-power policy is a feasible policy. Since it gives the lowest possible SEP among all possible policies, it is optimal. ■

Henceforth, we focus on the regime $\frac{\tau}{P_{\max}} < \alpha$, in which the max-power policy is not feasible. Next we characterize the optimal transmit power when the interference link is weak.

Claim *2:* For all $g \in \left[0, \frac{\tau}{P_{\max}}\right)$, $P^* = P_{\max}$ for the optimal transmit power policy.

Proof: When $g \in \left[0, \frac{\tau}{P_{\max}}\right)$, even when the secondary transmits at its peak power $P_{\max}$, the instantaneous interference that it causes at the PRx is less than or equal to τ, and does not contribute to the interference outage probability. Therefore, it is sub-optimal in terms of the SEP to transmit at a power below $P_{\max}$ in this interval. ■

Thus, when the interference link is weak, the STx should transmit data with peak power. The next claim shows that an optimal policy must satisfy the interference outage probability constraint with equality.

Claim *3:* Any feasible policy that satisfies Claim 2 and does not meet the interference outage probability constraint with equality is sub-optimal.

Proof: Let θ be a feasible policy that does not meet the interference outage probability constraint with equality. Since $\frac{\tau}{P_{\max}} < \alpha$ and the max-power policy is infeasible, there must be an interval $[g_1, g_2]$, where $g_2 > g_1 \geq \frac{\tau}{P_{\max}}$, in which $P < P_{\max}$. It follows that there exists an $\epsilon > 0$ such that the transmit power can be increased to $P + \epsilon \leq P_{\max}$ in this interval with the interference constraint still being satisfied. Doing so clearly reduces the SEP. Therefore, θ cannot be an optimal policy. ■

Next, we present a key property of the optimal policy for $g \geq \frac{\tau}{P_{\max}}$.

Claim *4:* For $g \geq \frac{\tau}{P_{\max}}$, a feasible policy in which I_p is neither τ nor $P_{\max} g$ (i.e., P is not $P_{\max}$) is sub-optimal.

Proof: Let θ be a feasible policy such that $I_p \neq \tau$ and $P < P_{\max}$, for all $g \in [g_1, g_2]$, where $g_2 > g_1 \geq \frac{\tau}{P_{\max}}$ and $g_2 - g_1$ is infinitesimally small. Then, one of the following two possibilities arises for all $g \in [g_1, g_2]$:

(i) $I_p < \tau$: In this case, one can strictly increase the transmit power of the STx for all $g \in [g_1, g_2]$ until either $P = P_{\max}$ or $I_p = \tau$, whichever happens earlier. Doing so yields another feasible policy whose interference outage probability is clearly the same as that of θ but whose SEP is lower than that of θ. Hence, θ is sub-optimal.

(ii) $\tau < I_p < P_{\max} g$: In this case, the primary link is already in outage. Hence, increasing the STx power to $P_{\max}$ for all $g \in [g_1, g_2]$ results in another feasible policy with the same interference outage probability but a lower SEP. Hence, again, θ is sub-optimal. ■

Let $\beth$ denote the subset of feasible policies that meet the interference outage probability constraint with equality and satisfy the conditions $I_p = P_{\max} g$, for $g < \frac{\tau}{P_{\max}}$, and $I_p = \tau$ or $P_{\max} g$, for all $g \geq \frac{\tau}{P_{\max}}$. From Claims 2, 3, and 4, we know that $\theta^* \in \beth$.

Claim *5:* For a policy $\theta \in \beth$ to be optimal, if there is an interval $(g_2, g_2 + \delta_2) \subset [\alpha, \infty)$ such that $I_p = \tau$, for $g \in (g_2, g_2 + \delta_2)$, then there must also exist another interval $(g_1, g_1 + \delta_1) \subset \left[\frac{\tau}{P_{\max}}, \alpha\right)$ in which $I_p = P_{\max} g$, for $g \in (g_1, g_1 + \delta_1)$.

Proof: We are given that there is an interval $(g_2, g_2 + \delta_2) \subset [\alpha, \infty)$ such that $I_p = \tau$, for all $g \in (g_2, g_2 + \delta_2)$. Since $I_p = \tau$, this interval does not contribute to the interference outage probability. Let there be no interval $(g_1, g_1 + \delta_1) \subset \left[\frac{\tau}{P_{\max}}, \alpha\right)$ in which $I_p = P_{\max} g$, for all $g \in (g_1, g_1 + \delta_1)$. Then, from Claim 4, since θ is optimal, we must have $I_p = \tau$, for $g \in \left[\frac{\tau}{P_{\max}}, \alpha\right)$. Thus, this region does not contribute to the interference outage probability. Therefore, the outage probability P_o' of θ can be upper bounded as follows:

$$P_o' \le \Pr(g > \alpha) - \Pr\left[g \in (g_2, g_2 + \delta_2)\right] < P_{\text{out}}. \quad (12)$$

From Claim 3, it follows that θ is sub-optimal, which is a contradiction. ■

The next claim shows that we can obtain a new transmit power policy in $\daleth$ that has a lower SEP by making the above variable g_2 equal to α itself.

Claim *6:* If a policy $\theta \in \daleth$ is optimal and there is an interval $(g_2, g_2 + \delta_2) \subset [\alpha, \infty)$ in which $I_p = \tau$, then $g_2 = \alpha$.

Proof: Without loss of generality, let δ_2 be infinitesimally small. Consider another policy θ' such that $I_p = \tau$, for $g \in (\alpha, \alpha + \delta_2')$, and $I_p = P_{\max} g$, for $g \in (g_2, g_2 + \delta_2)$, where δ_2' is chosen so that $\mathrm{p}(\alpha)\delta_2' = \mathrm{p}(g_2)\delta_2$, where $p(\cdot)$ denotes the probability density function (pdf) of g. This is allowed because $P = \frac{I_p}{g} = \frac{\tau}{\alpha} \le P_{\max}$. For all other values of $g \in \mathbb{R}^+$, I_p of θ' is the same as that for θ. It can be easily seen that the outage probability of θ is the same as that of θ', which is P_{out}. Hence, θ' is also feasible, and as required by Claim 3, it meets the interference outage probability constraint with equality. Thus, $\theta' \in \daleth$.

For θ, since $I_p = \tau$, we have $P = \frac{\tau}{g_2}$, for $g \in (g_2, g_2 + \delta_2)$. Similarly, for θ', $P = \frac{\tau}{\alpha}$, for $g \in (\alpha, \alpha + \delta_2')$. Now for MPSK, the SEP of θ', which we denote by $\text{SEP}_{\theta'}$, can be written in terms of the SEP of θ, denoted by SEP_θ, as follows:

$$\text{SEP}_{\theta'} = \text{SEP}_\theta - \frac{1}{\pi}\int_0^{\Lambda\pi} \frac{\mathrm{p}(g_2)\delta_2}{\left(1 + \frac{\tau \sin^2\left(\frac{\pi}{M}\right)}{\sigma^2 g_2 \sin^2 \phi}\right)^{m_s}} d\phi + \frac{1}{\pi}\int_0^{\Lambda\pi} \frac{\mathrm{p}(\alpha)\delta_2'}{\left(1 + \frac{\tau \sin^2\left(\frac{\pi}{M}\right)}{\sigma^2 \alpha \sin^2 \phi}\right)^{m_s}} d\phi, \quad (13)$$

where the term inside the integral arises because h is a Nakagami-m RV. Since $g_2 > \alpha$ and $\mathrm{p}(\alpha)\delta_2' = \mathrm{p}(g_2)\delta_2$, it follows that, $\frac{\mathrm{p}(g_2)\delta_2}{\left(1+\frac{\tau \sin^2\left(\frac{\pi}{M}\right)}{\sigma^2 g_2 \sin^2 \phi}\right)^{m_s}} > \frac{\mathrm{p}(\alpha)\delta_2'}{\left(1+\frac{\tau \sin^2\left(\frac{\pi}{M}\right)}{\sigma^2 \alpha \sin^2 \phi}\right)^{m_s}}$, for $0 \le \phi \le \Lambda\pi$. Hence, $\text{SEP}_{\theta'} < \text{SEP}_\theta$, which implies that θ is sub-optimal. The proof for MQAM is similar. ■

Intuitively, when $g_2 \in (\alpha, \infty)$ is reduced to α itself, the channel power gain of the interference link reduces. This enables the STx to transmit at a higher power, which lowers the SEP without affecting the interference outage probability.

Claim *7:* For an optimal transmit power policy in $\daleth$, the interval $(g_1, g_1 + \delta_1)$ over which $I_p = P_{\max} g$, and the interval $(\alpha, \alpha + \delta_2)$ over which $I_p = \tau$, where $\frac{\tau}{P_{\max}} < g_1 < \alpha$, must both be of zero length, i.e., $\delta_1 = \delta_2 = 0$.

Proof: The proof is given in Appendix A. ■

Combining the above results leads to Theorem 1.

IV. SEP Analysis of the Secondary System

We now derive the fading-averaged SEP of the secondary system of the optimal transmit power policy for both MPSK and MQAM. We first derive the SEP when $\frac{\tau}{P_{\max}} < \alpha$. The nature of the result depends on whether m_s is an integer or not. In the latter case, a simplification is achieved by means of an upper bound.

Theorem *2:* For $\frac{\tau}{P_{\max}} < \alpha$, the SEP for MPSK is

$$\text{SEP}^{\text{MPSK}} = \left(1 + \frac{\gamma\left(m_p, \frac{m_p \tau}{\Omega_p P_{\max}}\right) - \gamma\left(m_p, \frac{m_p \alpha}{\Omega_p}\right)}{\Gamma(m_p)}\right)\psi\left(c_1^{\text{MPSK}}\right) + \left(\frac{m_p}{\Omega_p}\right)^{m_p} \int_{\frac{\tau}{P_{\max}}}^{\alpha} \frac{g^{m_p - 1} e^{-m_p g/\Omega_p}\, \psi\left(c_2^{\text{MPSK}}(g)\right)}{\Gamma(m_p)} dg, \quad (14)$$

where $\gamma(\cdot,\cdot)$ is the lower incomplete gamma function [25, (8.350.1)] and

$$\psi(x_0) = \frac{1}{\pi}\int_0^{\Lambda\pi} \left(\frac{\sin^2\phi}{\sin^2\phi + x_0}\right)^{m_s} d\phi. \quad (15)$$

This equation holds for both integer as well as non-integer m_s. For integer m_s, $\psi(x_0)$ is given in closed-form by

$$\psi(x_0) = \Lambda - \frac{1}{\pi}\sqrt{\frac{x_0}{1+x_0}} \times \left(\left(\frac{\pi}{2} + \tan^{-1}\beta\right)\left[\sum_{k=0}^{m_s-1}\binom{2k}{k}\frac{1}{4^k(1+x_0)^k}\right] + \frac{\beta}{\sqrt{1+\beta^2}}\sum_{k=1}^{m_s-1}\sum_{j=1}^{k}\frac{T_{jk}}{(1+x_0)^k(1+\beta^2)^{k-j+\frac{1}{2}}}\right), \quad (16)$$

and for non-integer m_s, $\psi(x_0)$ is upper bounded by

$$\psi(x_0) \le \frac{\Lambda}{(1+x_0)^{m_s}}. \quad (17)$$

Here, $\beta \triangleq \sqrt{\frac{x_0}{1+x_0}}\cot\left(\frac{\pi}{M}\right)$, $T_{jk} \triangleq \frac{\binom{2k}{k}}{\binom{2(k-j)}{k-j}4^j[2(k-j)+1]}$, $c_1^{\text{MPSK}} = \frac{\Omega_s \sin^2\left(\frac{\pi}{M}\right)P_{\max}}{\sigma^2 m_s}$, and $c_2^{\text{MPSK}}(g) = \frac{\Omega_s \sin^2\left(\frac{\pi}{M}\right)\tau}{\sigma^2 m_s g}$.

Proof: The proof is given in Appendix B. ■

Theorem *3:* For $\frac{\tau}{P_{\max}} < \alpha$, the SEP for MQAM is

$$\text{SEP}^{\text{MQAM}} = \left(1 + \frac{\gamma\left(m_p, \frac{m_p \tau}{\Omega_p P_{\max}}\right) - \gamma\left(m_p, \frac{m_p \alpha}{\Omega_p}\right)}{\Gamma(m_p)}\right)\chi\left(c_1^{\text{MQAM}}\right) + \left(\frac{m_p}{\Omega_p}\right)^{m_p} \int_{\frac{\tau}{P_{\max}}}^{\alpha} \frac{g^{m_p - 1} e^{-m_p g/\Omega_p}\, \chi\left(c_2^{\text{MQAM}}(g)\right)}{\Gamma(m_p)} dg, \quad (18)$$

where

$$\chi(y_0) = \frac{4}{\pi}\left(1 - \frac{1}{\sqrt{M}}\right)\int_0^{\frac{\pi}{2}}\left(\frac{\sin^2\phi}{\sin^2\phi + y_0}\right)^{m_s} d\phi - \frac{4}{\pi}\left(1 - \frac{1}{\sqrt{M}}\right)^2\int_0^{\frac{\pi}{4}}\left(\frac{\sin^2\phi}{\sin^2\phi + y_0}\right)^{m_s} d\phi. \quad (19)$$

For integer m_s, $\chi(y_0)$ is given in closed-form as

$$\chi(y_0) = 2\left(1 - \frac{1}{\sqrt{M}}\right)\left(1 - \sqrt{\frac{y_0}{1+y_0}}\sum_{k=0}^{m_s-1}\frac{\binom{2k}{k}}{4^k(1+y_0)^k}\right) - \left(1 - \frac{1}{\sqrt{M}}\right)^2\left(1 - \frac{4}{\pi}\sqrt{\frac{y_0}{1+y_0}}\left[\left(\frac{\pi}{2} - \tan^{-1}\nu\right)\sum_{k=0}^{m_s-1}\frac{\binom{2k}{k}}{4^k(1+y_0)^k} + \frac{\nu}{\sqrt{1+\nu^2}}\sum_{k=1}^{m_s-1}\sum_{j=1}^{k}\frac{T_{jk}}{(1+y_0)^k(1+\nu^2)^{k-j+\frac{1}{2}}}\right]\right), \quad (20)$$

and for non-integer m_s, $\chi(y_0)$ is upper bounded by

$$\chi(y_0) \le \frac{1}{\sqrt{M}}\left(1-\frac{1}{\sqrt{M}}\right)\frac{1}{(1+2y_0)^{m_s}} + \left(1-\frac{1}{\sqrt{M}}\right)\frac{1}{(1+y_0)^{m_s}}. \quad (21)$$

Here, $\nu \triangleq \sqrt{\frac{y_0}{1+y_0}}$, $c_1^{\text{MQAM}} = \frac{1.5\Omega_s P_{\max}}{\sigma^2 \Delta m_s}$, and $c_2^{\text{MQAM}}(g) = \frac{1.5\Omega_s \tau}{\sigma^2 \Delta m_s g}$.

Proof: The proof is given in Appendix C. ■

The SEP expressions in (14) and (18) are in the form of a single integral in g and cannot be simplified any further. They are easily evaluated numerically.

Next we derive the SEP for the case when $\frac{\tau}{P_{\max}} \ge \alpha$. In this case, the STx always transmits at the peak transmit power $P_{\max}$.

Theorem *4:* When $\frac{\tau}{P_{\max}} \ge \alpha$, the SEP for MPSK is given by $\text{SEP}^{\text{MPSK}} = \psi(c_1^{\text{MPSK}})$. For MQAM, the SEP is given by $\text{SEP}^{\text{MQAM}} = \chi(c_1^{\text{MQAM}})$.

Proof: The proof is given in Appendix D. ■

A. Asymptotic Scenarios

To gain more insights about the optimal policy, we now analyze its SEP in the asymptotic regimes in which the STx is either not peak power constrained or not interference-limited.

Theorem *5:* When $P_{\max} \to \infty$, the SEP for MPSK is

$$\text{SEP}^{\text{MPSK}} = \left(\frac{m_p}{\Omega_p}\right)^{m_p} \int_0^{\alpha} \frac{g^{m_p-1} e^{-m_p g/\Omega_p}\, \psi(c_2^{\text{MPSK}}(g))}{\Gamma(m_p)}\, dg. \quad (22)$$

For MQAM, the SEP expression is the same as (22) except that $\psi(c_2^{\text{MPSK}}(g))$ is replaced by $\chi(c_2^{\text{MQAM}}(g))$.

Proof: The proof is given in Appendix E. ■

When $\tau \to \infty$, the STx is no longer interference-limited, and the results given in Theorem 4 apply.

V. Impact of Imperfect Channel Estimation of the Interference Link

The optimal transmit power policy requires the STx to know the instantaneous channel power gain g of the STx-PRx link. In this section, we investigate how robust the scheme is to imperfect estimates of g. We develop expressions for the SEP and the interference outage probability, which will no longer be P_{out}. Knowledge of the STx-SRx channel power gain h is assumed, as before, since this is a link that is internal to the secondary system. For analytical tractability, we focus on Rayleigh fading. With Nakagami-m fading, the problem becomes intractable as it involves dealing with the joint pdf of correlated Nakagami-m distributed RVs.

Channel estimation: Let x_p denote the pilot symbol transmitted by the PRx. The baseband signal y_b received at the STx is then given by

$$y_b = \sqrt{E_b}\omega x_p + n_p, \quad (23)$$

where E_b is the power with which the PRx sends out pilot symbol, ω is the baseband channel gain of the STx-PRx link, $|x_p|^2 = 1$, and $n_p \sim \mathcal{CN}(0, \sigma_p^2)$. Furthermore, $\omega \sim \mathcal{CN}(0, \Omega_p)$ and is independent of n_p. Now, given the observable y_b at the STx, the minimum mean square error (MMSE) estimate of the STx-PRx link $\widehat{\omega}$ can be written as

$$\widehat{\omega} = \frac{\sqrt{E_b}\Omega_p x_p^* y_b}{E_b \Omega_p + \sigma_p^2} = \frac{E_b \Omega_p \omega}{E_b \Omega_p + \sigma_p^2} + e, \quad (24)$$

where $e \sim \mathcal{CN}\left(0, \frac{E_b \sigma_p^2 \Omega_p^2}{(E_b\Omega_p+\sigma_p^2)^2}\right)$ is the noise-induced channel estimation error, $\widehat{\omega} \sim \mathcal{CN}\left(0, \frac{E_b \Omega_p^2}{E_b \Omega_p + \sigma_p^2}\right)$, and e is independent of ω. Note that similar channel estimation models have also been considered in the literature. For example, in [4], [13], [20], [26], [27], the channel estimate is given as $\widehat{\omega} = \zeta_1 \omega + \epsilon$, where ζ_1 is a scaling constant and ϵ is the channel estimation error. Let the pilot SNR be given by $\Upsilon_p = \frac{E_b}{\sigma_p^2}$. Note that $g = |\omega|^2$, and let $\hat{g} = |\widehat{\omega}|^2$. The SEP of the secondary system is then given as follows.

Theorem *6:* When $\frac{\tau}{P_{\max}} < \alpha$, the SEP for MPSK with imperfect g is given by

$$\begin{aligned}\text{SEP} = &\left(1 + e^{-\alpha\left(\frac{1+\Upsilon_p\Omega_p}{\Upsilon_p\Omega_p^2}\right)} - e^{-\frac{\tau(1+\Upsilon_p\Omega_p)}{\Upsilon_p\Omega_p^2 P_{\max}}}\right) \\ &\times \left(\Lambda - \frac{1}{\pi}\sqrt{\frac{c_5}{1+c_5}}\left[\frac{\pi}{2} + \tan^{-1}\left(\sqrt{\frac{c_5}{1+c_5}}\cot\left(\frac{\pi}{M}\right)\right)\right]\right) \\ &+ \int_{\frac{\tau}{P_{\max}}}^{\alpha} \left(\frac{1+\Upsilon_p\Omega_p}{\Upsilon_p\Omega_p^2}\right) e^{-\hat{g}\left(\frac{1+\Upsilon_p\Omega_p}{\Upsilon_p\Omega_p^2}\right)} \left(\Lambda - \frac{1}{\pi}\sqrt{\frac{c_6(\hat{g})}{1+c_6(\hat{g})}} \right.\\ &\left.\times\left[\frac{\pi}{2} + \tan^{-1}\left(\sqrt{\frac{c_6(\hat{g})}{1+c_6(\hat{g})}}\cot\left(\frac{\pi}{M}\right)\right)\right]\right) d\hat{g}, \end{aligned} \quad (25)$$

where $c_5 = \frac{P_{\max}\sin^2\left(\frac{\pi}{M}\right)\Omega_s}{\sigma^2}$ and $c_6(\hat{g}) = \frac{\tau \sin^2\left(\frac{\pi}{M}\right)\Omega_s}{\sigma^2 \hat{g}}$.

When $\frac{\tau}{P_{\max}} \ge \alpha$, the SEP for MPSK is given by

$$\text{SEP} = \Lambda - \frac{1}{\pi}\sqrt{\frac{c_5}{1+c_5}}\left[\frac{\pi}{2} + \tan^{-1}\left(\sqrt{\frac{c_5}{1+c_5}}\cot\left(\frac{\pi}{M}\right)\right)\right]. \quad (26)$$

Proof: The proof involves averaging the instantaneous SEP over the channel realizations of h and $\hat{g}$ and is not shown here. ■

Notice that the SEP with imperfect channel estimates in (25) is now a function of Υ_p. However, this is not so when $\frac{\tau}{P_{\max}} \ge \alpha$, as can be seen from (26). The SEP for MQAM can be derived in a similar manner.

Next, we find out the impact of imperfect estimation on interference outage probability, which we denote by $\widehat{P}_{\text{out}}$.

Theorem *7:* When $\frac{\tau}{P_{\max}} \ge \alpha$, $\widehat{P}_{\text{out}}$ is given by

$$\widehat{P}_{\text{out}} = e^{-\frac{\tau}{\Omega_p P_{\max}}}. \quad (27)$$

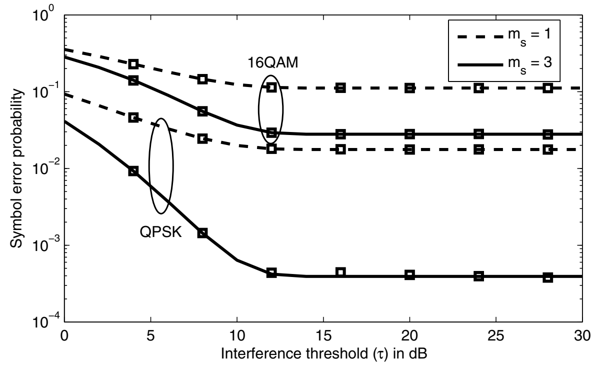

Fig. 4. Impact of the STx-SRx fading parameter m_s and constellation on the SEP of the secondary system ($m_p = 1$, $\Omega_s = 5$, $\Omega_p = 1$, $P_{\text{out}} = 10\%$, and $P_{\max} = 10$ dB). Simulations are shown using the marker □.

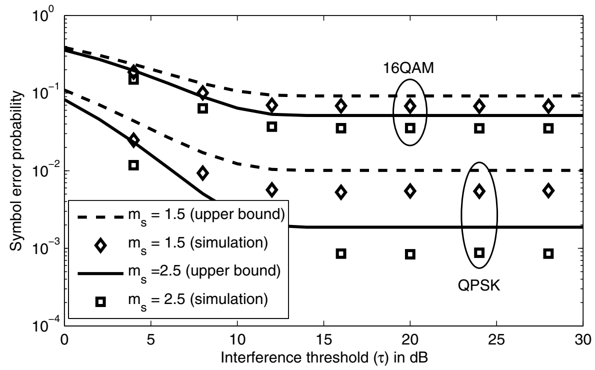

Fig. 5. Upper bound for non-integer m_s ($m_p = 1$, $\Omega_s = 5$, $\Omega_p = 1$, $P_{\text{out}} = 10\%$, and $P_{\max} = 10$ dB).

When $\frac{\tau}{P_{\max}} < \alpha$, $\widehat{P}_{\text{out}}$ is given by

$$\widehat{P}_{\text{out}} = \frac{2(1+\Upsilon_p\Omega_p)}{\Upsilon_p\Omega_p^2 q^2}\left[e^{-\frac{q^2b^2}{2(a^2+q^2)}}Q_1\left(c\sqrt{a^2+q^2}, \frac{ab}{\sqrt{a^2+q^2}}\right)\right.$$
$$\left. - e^{-\frac{q^2c^2}{2}}Q_1\left(ac, b\right) + q^2\int_{\sqrt{\frac{\tau}{P_{\max}}}}^{\sqrt{\alpha}} x_2 e^{-\frac{q^2x_2^2}{2}}Q_1\left(ax_2, b_1x_2\right)dx_2\right]$$
$$+ \frac{2(1+\Upsilon_p\Omega_p)}{\Upsilon_p\Omega_p^2}\left[\frac{e^{-\frac{q^2\alpha}{2}}Q_1\left(a\sqrt{\alpha}, b\right)}{q^2} + e^{-\frac{q^2b^2}{2(a^2+q^2)}}\right.$$
$$\left.\times\left[1 - Q_1\left(\sqrt{\alpha}\sqrt{a^2+q^2}, \frac{ab}{\sqrt{a^2+q^2}}\right)\right]\right], \quad (28)$$

where $q^2 = \frac{2}{K_2} - \frac{K_1K_3^2}{2}$, $a = K_3\sqrt{\frac{K_1}{2}}$, $b = \sqrt{\frac{2\tau}{K_1P_{\max}}}$, $c = \sqrt{\frac{\tau}{P_{\max}}}$, $b_1 = \frac{2}{K_1}$, $K_1 = \frac{\Omega_p}{1+\Upsilon_p\Omega_p}$, $K_2 = \frac{\Upsilon_p\Omega_p^2}{(1+\Upsilon_p\Omega_p)^2}$, $K_3 = \frac{2(1+\Upsilon_p\Omega_p)}{\Omega_p}$, $I_0(\cdot)$ is the modified Bessel function of the zeroth order, and $Q_1(\cdot,\cdot)$ is the Marcum-Q function [22, (4.34)].

Proof: The proof is given in Appendix F. ■

When $\frac{\tau}{P_{\max}} \geq \alpha$, we see that the outage probability decays exponentially with the interference threshold for both perfect and imperfect CSI (cf. (27)), since the STx transmits at $P_{\max}$ regardless of the state of the STx-PRx link. However, when $\frac{\tau}{P_{\max}} < \alpha$, the outage probability does not remain constant at P_{out} with imperfect CSI unlike the perfect CSI case, and is given by (28). The integral over x_2 in (28) can be written in closed-form using [28, (B.47)]. We do not show it here as it is lengthy and does not provide additional insights.

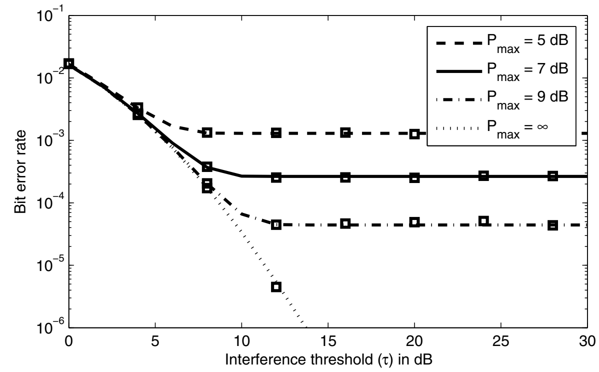

Fig. 6. Impact of the peak transmit power constraint $P_{\max}$ on the BER of the secondary system ($m_s = 5$, $m_p = 1$, $\Omega_s = 5$, $\Omega_p = 1$, $P_{\text{out}} = 10\%$, and QPSK). Simulations are shown using the marker □.

VI. Numerical Results and Discussion

We now present Monte Carlo simulation results, which use 10^6 samples, to verify the analytical results and understand the impact of system parameters such as τ, $P_{\max}$, P_{out}, m_s, and m_p on the SEP. We first consider the perfect CSI case and then the imperfect CSI case.

A. Perfect CSI

Figure 4 plots the SEP of QPSK and 16QAM as a function of the interference threshold τ for $P_{\max} = 10$ dB and $P_{\text{out}} = 10\%$. Results are shown for different values of m_s. We observe that the analysis results match well with the simulation results. Furthermore, for both the constellations and all m_s, as τ increases, the SEP first reduces and eventually reaches an error floor because of the peak transmit power constraint. As m_s increases, the severity of the fading over the data link decreases. Hence, the SEP improves and the error floor decreases. Figure 5 plots the SEP upper bounds for MPSK (cf. (17)) and MQAM (cf. (21)) for non-integer values of m_s. Also plotted are the corresponding simulation results. Trends similar to Figure 4 are observed when m_s increases. We observe that upper bound for both QPSK and 16QAM is within 2 dB of the exact SEP and tracks it well.

In order to evaluate the impact on the bit error rate (BER), which is another oft-used performance metric, Figure 6 plots the bit error rate (BER) of QPSK as a function of τ for different $P_{\max}$. As expected, an error floor again arises, which decreases as $P_{\max}$ increases. It disappears when $P_{\max} \to \infty$. Note that the BER and SEP are closely related. Specifically, BER $\approx \frac{\text{SEP}}{\log_2(M)}$, where M is size of the constellation and Gray coding is used [7].

Figure 7 investigates the effect of P_{out} on the SEP of 8-PSK. It uses the same set of parameters as in Figure 6, but with $P_{\max} = 10$ dB. For smaller $\tau\,(< 15$ dB$)$, as P_{out} increases, the SEP decreases because the interference constraint becomes more lax. However, for $\tau\,(> 15$ dB$)$, P_{out} does not affect the SEP, which is now driven by the peak transmit power constraint. It also plots the SEP when

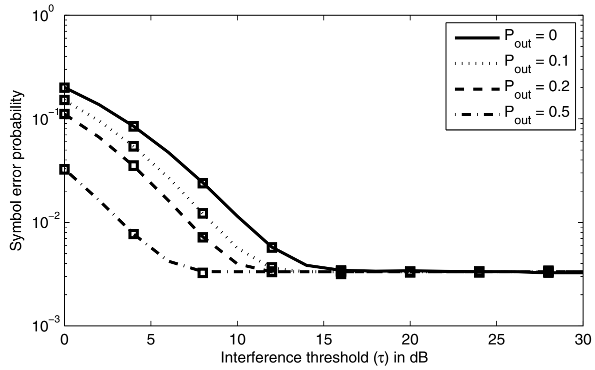

Fig. 7. Impact of the interference outage probability constraint P_{out} on the SEP of the secondary system ($m_s = 5$, $m_p = 1$, $\Omega_s = 5$, $\Omega_p = 1$, $P_{\max} = 10$ dB, and 8-PSK). Simulations are shown using the marker □.

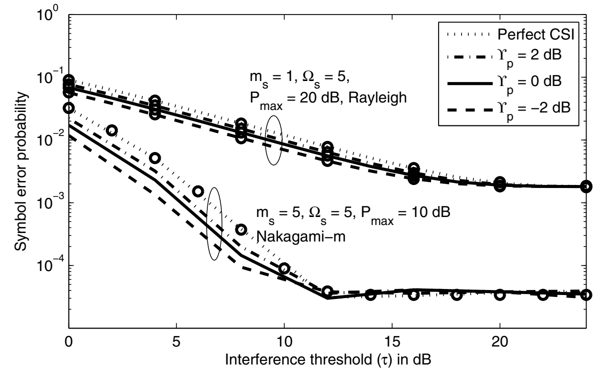

Fig. 9. Impact of imperfect channel estimates on the SEP of the secondary system ($m_p = 1$, $\Omega_p = 1$, $P_{\text{out}} = 10\%$, and QPSK). Analytical results are shown using the marker ∘.

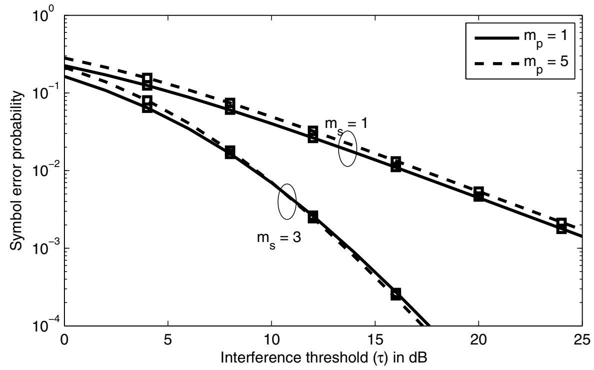

Fig. 8. Asymptotic regime ($P_{\max} \to \infty$): Impact of STx-SRx fading parameter m_s and STx-PRx fading parameter m_p on the SEP of the secondary system ($\Omega_s = 5$, $\Omega_p = 1$, $P_{\text{out}} = 10\%$, and 8-PSK). Simulations are shown using the marker □.

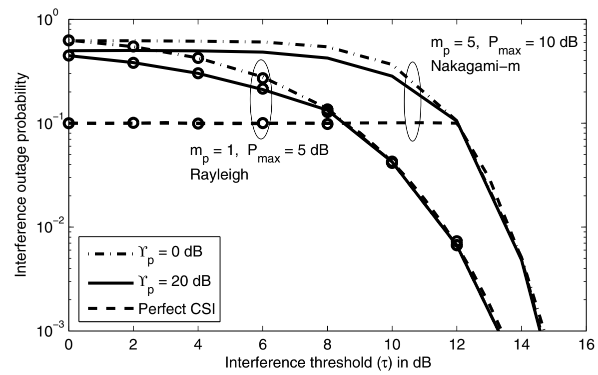

Fig. 10. Impact of imperfect channel estimates on the interference outage probability ($m_s = 1$, $\Omega_s = 5$, $\Omega_p = 1$, $P_{\text{out}} = 10\%$, and QPSK). Analytical results are shown using the marker ∘.

$P_{\text{out}} = 0$. Under this condition, the STx must ensure that its instantaneous interference at the PRx never exceeds τ. Thus, its instantaneous power is $\min\left\{P_{\max}, \frac{\tau}{g}\right\}$, which is akin to the rule used in [9], [12]. Clearly, the SEP is the highest for this case.

Figure 8 plots the SEP as a function of τ for different values of m_s and m_p in the asymptotic regime of large $P_{\max}$ for 8-PSK. Now, no error floors occur because the STx is power unconstrained. Also, as m_s increases, the SEP improves because the severity of the fading over the data link reduces. As m_p increases, the SEP marginally increases. This is because the STx transmits at peak power relatively less often as m_p increases.

B. Imperfect CSI

Figure 9 plots the SEP as a function of τ for different values of the pilot SNR Υ_p and for both Rayleigh and Nakagami-m fading channels. The parameters have been chosen so as to avoid clutter. As Υ_p decreases, the SEP decreases relative to the perfect CSI case for both the channel models. This is because as Υ_p decreases, the power of the estimated channel gain of the interference link decreases. Therefore, the STx transmits at the peak power $P_{\max}$ more often. This increases the outage probability beyond the stipulated P_{out}, as we shall see in the next figure. As expected, the SEP decays at a faster rate for $m_s = 5$, when compared to $m_s = 1$, until it reaches an error floor.

Figure 10 plots the outage probability as a function of τ for different values of Υ_p for Rayleigh and Nakagami-m fading. With imperfect CSI, as τ increases, the outage probability decreases. This is unlike the perfect CSI case, in which the outage probability remains at P_{out} as long as $\frac{\tau}{P_{\max}} < \alpha$ and then gradually reduces when $\frac{\tau}{P_{\max}} \geq \alpha$. As Υ_p decreases, the outage probability increases because the estimate of g gets noisier. For $\tau > 8$ dB for Rayleigh fading and $\tau > 13$ dB for Nakagami-m fading, $\hat{P}_{\text{out}}$ decays exponentially, as can be seen from (27). It is insensitive to Υ_p because the STx transmits at $P_{\max}$ more often, regardless of $\hat{g}$.

VII. Conclusions and Extensions

We developed an SEP-optimal transmit power adaptation policy for an underlay CR system that is subject to two practically motivated constraints, namely, a peak transmit power constraint and an interference outage probability constraint. We saw that the STx transmits at peak power not only when the interference link is very weak but also when it is very strong. Otherwise, the STx adjusts its transmit power so that the interference at the PRx equals the threshold τ. To implement it, the STx only needs to know the channel gain of its link to the PRx. It does not require the knowledge of the channel gain of the PTx-PRx link or the STx-SRx link.

We also analyzed the SEP of the optimal transmit power policy for both MPSK and MQAM. We saw that imperfect channel knowledge of the STx-PRx link leads to an increased interference at the PRx and lowers the SEP of the SU link. An interesting avenue for future work is developing the optimal power allocation policy for a CR system with multiple PRxs, with an interference outage probability constraint to be satisfied for each PRx.

APPENDIX

A. Proof of Claim 7

Without loss of generality, let δ_1 and δ_2 be infinitesimally small. Consider a policy θ'', in which $I_p = P_{\max} g$, for $g \in (g_1, g_1 + \delta_1)$, and $I_p = \tau$, for $g \in (\alpha, \alpha + \delta_2)$. To ensure that the outage probability of θ'' and θ^* is P_{out}, we set $\mathrm{p}(g_1)\,\delta_1 = \mathrm{p}(\alpha)\delta_2$. Recall that θ^* is the power policy given in (10). Also, it can be seen that

$$\mathrm{SEP}_{\theta''} = \mathrm{SEP}_{\theta^*} - \frac{1}{\pi}\int_0^{\Lambda\pi} \frac{\mathrm{p}(g_1)\delta_1}{\left(1 + \frac{\tau \sin^2\left(\frac{\pi}{M}\right)}{\sigma^2 g_1 \sin^2\phi}\right)^{m_s}}\, d\phi + \frac{1}{\pi}\int_0^{\Lambda\pi} \frac{\mathrm{p}(\alpha)\delta_2}{\left(1 + \frac{\tau \sin^2\left(\frac{\pi}{M}\right)}{\sigma^2 \alpha \sin^2\phi}\right)^{m_s}}\, d\phi. \quad (29)$$

Since $\alpha > g_1$ and $\mathrm{p}(g_1)\,\delta_1 = \mathrm{p}(\alpha)\delta_2$, we get $\frac{\mathrm{p}(g_1)\delta_1}{\left(1+\frac{\tau\sin^2\left(\frac{\pi}{M}\right)}{\sigma^2 g_1 \sin^2\phi}\right)^{m_s}} < \frac{\mathrm{p}(\alpha)\delta_2}{\left(1+\frac{\tau\sin^2\left(\frac{\pi}{M}\right)}{\sigma^2 \alpha \sin^2\phi}\right)^{m_s}}$, for $0 \le \phi \le \Lambda\pi$. Hence, $\mathrm{SEP}_{\theta^*} < \mathrm{SEP}_{\theta''}$. Therefore, θ'' is sub-optimal.

B. Proof of Theorem 2

From (3), the SEP of MPSK for the optimal transmit power policy is given by

$$\mathrm{SEP}^{\text{MPSK}} = \mathbf{E}_{\mathrm{h,g}}\left[\frac{1}{\pi}\int_0^{\Lambda\pi} \exp\left(-\frac{P^* h \sin^2\left(\frac{\pi}{M}\right)}{\sigma^2 \sin^2\phi}\right) d\phi\right]. \quad (30)$$

As per (10), the optimal transmit power is $P_{\max}$, for $0 < g \le \frac{\tau}{P_{\max}}$ or $g > \alpha$, and is $\frac{\tau}{g}$, for $\frac{\tau}{P_{\max}} < g \le \alpha$. Hence, integrating over the above three regions of g separately, we get

$$\mathrm{SEP}^{\text{MPSK}} = T_1 + T_2 + T_3, \quad (31)$$

where

$$T_1 = \frac{1}{\pi}\int_0^\infty \int_0^{\frac{\tau}{P_{\max}}} \int_0^{\Lambda\pi} \exp\left(-\frac{P_{\max} h \sin^2\left(\frac{\pi}{M}\right)}{\sigma^2 \sin^2\phi}\right) \times \mathrm{p}(h)\mathrm{p}(g)\, d\phi\, dg\, dh, \quad (32)$$

$$T_2 = \frac{1}{\pi}\int_0^\infty \int_{\frac{\tau}{P_{\max}}}^{\alpha} \int_0^{\Lambda\pi} \exp\left(-\frac{h\tau \sin^2\left(\frac{\pi}{M}\right)}{g\sigma^2 \sin^2\phi}\right) \times \mathrm{p}(h)\mathrm{p}(g)\, d\phi\, dg\, dh, \quad (33)$$

$$T_3 = \frac{1}{\pi}\int_0^\infty \int_{\alpha}^{\infty} \int_0^{\Lambda\pi} \exp\left(-\frac{P_{\max} h \sin^2\left(\frac{\pi}{M}\right)}{\sigma^2 \sin^2\phi}\right) \times \mathrm{p}(h)\mathrm{p}(g)\, d\phi\, dg\, dh. \quad (34)$$

We evaluate the above three terms separately below.

Evaluating T_1: Substituting the Nakagami-m pdfs of h and g in (32) and integrating over h, we get

$$T_1 = \frac{m_p^{m_p} m_s^{m_s}}{\pi\Gamma(m_p)\Omega_p^{m_p}\Omega_s^{m_s}} \int_0^{\frac{\tau}{P_{\max}}} g^{m_p - 1} e^{-m_p g/\Omega_p} \times \int_0^{\Lambda\pi} \left(\frac{\sigma^2 \Omega_s \sin^2\phi}{\sigma^2 m_s \sin^2\phi + \Omega_s \sin^2\left(\frac{\pi}{M}\right) P_{\max}}\right)^{m_s} d\phi\, dg. \quad (35)$$

Writing the integral over g in terms of the incomplete gamma function, we get

$$T_1 = \frac{\gamma\left(m_p, \frac{m_p \tau}{\Omega_p P_{\max}}\right) \psi(c_1^{\text{MPSK}})}{\Gamma(m_p)}, \quad (36)$$

where $c_1^{\text{MPSK}} = \frac{\Omega_s \sin^2\left(\frac{\pi}{M}\right) P_{\max}}{\sigma^2 m_s}$ and $\psi(\cdot)$ is defined in the theorem statement.

Evaluating T_2: Substituting the pdfs of h and g in (33), and integrating over h, we get

$$T_2 = \left(\frac{m_p}{\Omega_p}\right)^{m_p} \int_{\frac{\tau}{P_{\max}}}^{\alpha} \frac{g^{m_p - 1} e^{-m_p g/\Omega_p}\, \psi(c_2^{\text{MPSK}}(g))}{\Gamma(m_p)}\, dg, \quad (37)$$

where $c_2^{\text{MPSK}}(g) = \frac{\Omega_s \sin^2\left(\frac{\pi}{M}\right)\tau}{\sigma^2 m_s g}$.

Evaluating T_3: Similarly, we can show that $T_3 = \left(1 - \frac{1}{\Gamma(m_p)}\gamma\left(m_p, \frac{m_p \alpha}{\Omega_p}\right)\right)\psi(c_1^{\text{MPSK}})$. Substituting the above expressions for T_1, T_2, and T_3 in (31) yields (14).

Simplifying $\psi(\cdot)$: For integer m_s, using [22, (5A.17)], it can be shown that $\psi(x_0)$ reduces to (16). For non-integer m_s, such a closed-form solution is not possible. However, a closed-form upper bound is obtained using the inequality $\sin\phi^2 \le 1$, which yields (17).

C. Proof of Theorem 3

For MQAM, the SEP of the optimal transmit power policy is

$$\mathrm{SEP}^{\text{MQAM}} = \mathbf{E}_{\mathrm{h,g}}\left[\frac{4}{\pi}\left(1 - \frac{1}{\sqrt{M}}\right)\int_0^{\frac{\pi}{2}} \exp\left(\frac{-1.5 P^* h}{\Delta\sigma^2 \sin^2\phi}\right) d\phi - \frac{4}{\pi}\left(1 - \frac{1}{\sqrt{M}}\right)^2 \int_0^{\frac{\pi}{4}} \exp\left(\frac{-1.5 P^* h}{\Delta\sigma^2 \sin^2\phi}\right) d\phi\right]. \quad (38)$$

Using (10), and carefully integrating over the three regions of g defined in Theorem 1, we get (18), where

$$\chi(y_0) = \frac{4}{\pi}\left(1 - \frac{1}{\sqrt{M}}\right)\int_0^{\frac{\pi}{2}} \left(\frac{\sin^2\phi}{\sin^2\phi + y_0}\right)^{m_s} d\phi - \frac{4}{\pi}\left(1 - \frac{1}{\sqrt{M}}\right)^2 \int_0^{\frac{\pi}{4}} \left(\frac{\sin^2\phi}{\sin^2\phi + y_0}\right)^{m_s} d\phi. \quad (39)$$

Using the identities [22, (5A.21)] and [22, (5A.24)], (39) reduces to (20) for integer m_s. For non-integer m_s, $\chi(y_0)$

can be upper bounded as follows. By rearranging (39), $\chi(y_0)$ can be written as

$$\chi(y_0) = \frac{4}{\pi\sqrt{M}}\left(1-\frac{1}{\sqrt{M}}\right)\int_0^{\frac{\pi}{4}}\left(\frac{\sin^2\phi}{\sin^2\phi+y_0}\right)^{m_s}d\phi + \frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\left(\frac{\sin^2\phi}{\sin^2\phi+y_0}\right)^{m_s}d\phi. \quad (40)$$

The first integral can be upper bounded by substituting $\phi = \frac{\pi}{4}$ in its integrand and the second integral by substituting $\phi = \frac{\pi}{2}$. This yields the bound in (21).

D. Proof of Theorem 4

When $\frac{\tau}{P_{\max}} \geq \alpha$, from Claim 1, we have $P^* = P_{\max}$, $\forall g$. Substituting this in (30), we get

$$\text{SEP}^{\text{MPSK}} = \frac{1}{\pi}\int_0^\infty\int_0^\infty\int_0^{\Lambda\pi}\exp\left(\frac{-P_{\max}h\sin^2\left(\frac{\pi}{M}\right)}{\sin^2\phi}\right)\left(\frac{m_p}{\Omega_p}\right)^{m_p} \times \frac{g^{m_p-1}e^{-m_pg/\Omega_p}}{\Gamma(m_p)}\left(\frac{m_s}{\Omega_s}\right)^{m_s}\frac{h^{m_s-1}e^{-m_sh/\Omega_s}}{\Gamma(m_s)}\,d\phi\,dg\,dh. \quad (41)$$

Integrating over h and g yields

$$\text{SEP}^{\text{MPSK}} = \frac{1}{\pi}\int_0^{\Lambda\pi}\left(\frac{\sin^2\phi}{\sin^2\phi+c_1^{\text{MPSK}}}\right)^{m_s}d\phi = \psi(c_1^{\text{MPSK}}). \quad (42)$$

In a similar manner, substituting $P^* = P_{\max}$ in (38) and integrating over h and g yields the expression for MQAM.

E. Proof of Theorem 5

For $g \leq \alpha$, $P^* = \frac{\tau}{g}$, and the STx transmits with infinite power otherwise and does not contribute to the SEP. Substituting this in (30), we get

$$\text{SEP}^{\text{MPSK}} = \frac{1}{\pi}\int_0^\infty\int_0^\alpha\int_0^{\Lambda\pi}\exp\left(\frac{-h\tau\sin^2\left(\frac{\pi}{M}\right)}{g\sin^2\phi}\right)\left(\frac{m_p}{\Omega_p}\right)^{m_p} \times \frac{g^{m_p-1}e^{-m_pg/\Omega_p}}{\Gamma(m_p)}\left(\frac{m_s}{\Omega_s}\right)^{m_s}\frac{h^{m_s-1}e^{-m_sh/\Omega_s}}{\Gamma(m_s)}\,d\phi\,dg\,dh. \quad (43)$$

Simplifying the integral over h yields (22). The derivation is similar for MQAM.

F. Proof of Theorem 7

When $\frac{\tau}{P_{\max}} \geq \alpha$, the STx transmits at $P_{\max}$ irrespective of $\hat{g}$. Therefore, $\widehat{P}_{\text{out}} = \Pr\left(g > \frac{\tau}{P_{\max}}\right)$. Upon simplification this yields (27).

When $\frac{\tau}{P_{\max}} < \alpha$, $\widehat{P}_{\text{out}}$ can be written as

$$\widehat{P}_{\text{out}} = \Pr\left(P(\hat{g})g > \tau\right) = \Pr\left(\sqrt{P(\hat{g})g} > \sqrt{\tau}\right). \quad (44)$$

Let $X_1 = \sqrt{g}$ and $X_2 = \sqrt{\hat{g}}$. Their joint pdf $\mathrm{p}_{X_1,X_2}(x_1,x_2)$ is given by [22, (6.2)]

$$\mathrm{p}_{X_1,X_2}(x_1,x_2) = \frac{4x_1x_2}{\Omega_1\Omega_2(1-\rho)}\exp\left(\frac{-1}{1-\rho}\left(\frac{x_1^2}{\Omega_1}+\frac{x_2^2}{\Omega_2}\right)\right) \times I_0\left(\frac{2\sqrt{\rho}x_1x_2}{(1-\rho)\sqrt{\Omega_1\Omega_2}}\right),\ x_1 \geq 0,\ x_2 > 0, \quad (45)$$

where $\Omega_1 = \mathbf{E}[g] = \Omega_p$, $\Omega_2 = \mathbf{E}[\hat{g}] = \frac{\Upsilon_p\Omega_p^2}{1+\Upsilon_p\Omega_p}$, $\rho = \frac{\Upsilon_p\Omega_p}{1+\Upsilon_p\Omega_p}$, $I_0(\cdot)$ is the modified Bessel function of the zeroth order, and $Q_1(\cdot,\cdot)$ is the Marcum-Q function [22, (4.34)].

As in Appendix B, $\widehat{P}_{\text{out}}$ can be written as

$$\widehat{P}_{\text{out}} = \hat{T}_1 + \hat{T}_2 + \hat{T}_3, \quad (46)$$

where

$$\hat{T}_1 = \int_0^{\sqrt{\tau/P_{\max}}}\int_{\sqrt{\tau/P_{\max}}}^\infty \mathrm{p}_{X_1,X_2}(x_1,x_2)\,dx_1\,dx_2. \quad (47)$$

This corresponds to the interval $\hat{g} \in \left[0, \frac{\tau}{P_{\max}}\right)$. The term $\hat{T}_2$ corresponds to the interval $\hat{g} \in \left[\frac{\tau}{P_{\max}}, \alpha\right)$ and is given by $\hat{T}_2 = \int_{\sqrt{\tau/P_{\max}}}^{\alpha}\int_{x_2}^\infty \mathrm{p}_{X_1,X_2}(x_1,x_2)\,dx_1\,dx_2$. Similarly, $\hat{T}_3 = \int_{\sqrt{\alpha}}^\infty\int_{\sqrt{\tau/P_{\max}}}^\infty \mathrm{p}_{X_1,X_2}(x_1,x_2)\,dx_1\,dx_2$ corresponds to the interval $\hat{g} \in [\alpha, \infty)$.

Using [28, (B.19)], $\hat{T}_1$ can be simplified to

$$\hat{T}_1 = \frac{2(1+\Upsilon_p\Omega_p)}{\Upsilon_p\Omega_p^2q^2}\left[e^{-\frac{q^2b^2}{2(a^2+q^2)}}Q_1\left(c\sqrt{a^2+q^2}, \frac{ab}{\sqrt{a^2+q^2}}\right) - e^{-\frac{q^2c^2}{2}}Q_1(ac,b)\right], \quad (48)$$

where $q^2 = \frac{2}{K_2} - \frac{K_1K_3^2}{2}$, $a = K_3\sqrt{\frac{K_1}{2}}$, $b = \sqrt{\frac{2\tau}{K_1P_{\max}}}$, $c = \sqrt{\frac{\tau}{P_{\max}}}$, $K_1 = \frac{\Omega_p}{1+\Upsilon_p\Omega_p}$, $K_2 = \frac{\Upsilon_p\Omega_p^2}{(1+\Upsilon_p\Omega_p)^2}$, and $K_3 = \frac{2(1+\Upsilon_p\Omega_p)}{\Omega_p}$. Using the definition of Marcum-Q function, $\hat{T}_2$ simplifies to

$$\hat{T}_2 = \frac{2(1+\Upsilon_p\Omega_p)}{\Upsilon_p\Omega_p^2}\int_{\sqrt{\frac{\tau}{P_{\max}}}}^{\sqrt{\alpha}} x_2e^{-\frac{q^2x_2^2}{2}}Q_1\left(ax_2, b_1x_2\right)\,dx_2, \quad (49)$$

where $b_1 = \sqrt{\frac{2}{K_1}}$. Similarly, using the identity in [28, (B.18)], $\hat{T}_3$ can be written as

$$\hat{T}_3 = \frac{2(1+\Upsilon_p\Omega_p)}{\Upsilon_p\Omega_p^2}\left(\frac{1}{q^2}e^{-\frac{q^2\alpha}{2}}Q_1\left(a\sqrt{\alpha}, b\right) + e^{-\frac{q^2b^2}{2(a^2+q^2)}}\left[1 - Q_1\left(\sqrt{\alpha}\sqrt{a^2+q^2}, \frac{ab}{\sqrt{a^2+q^2}}\right)\right]\right). \quad (50)$$

Substituting $\hat{T}_1$, $\hat{T}_2$, and $\hat{T}_3$ in (46) yields (28).

References

[1] M. Vu, N. Devroye, and V. Tarokh, "On the primary exclusive region of cognitive networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 7, pp. 3380–3385, Jul. 2009.

[2] A. Giorgetti, M. Varrella, and M. Chiani, "Analysis and performance comparison of different cognitive radio algorithms," in *Proc. 2009 Int. Conf. Cognitive Radio and Advanced Spectrum Management*, pp. 127–131.

[3] Q. Zhao and B. M. Sadler, "A survey of dynamic spectrum access," *IEEE Signal Process. Mag.*, vol. 24, no. 3, pp. 79–89, May 2007.

[4] Y. Chen, G. Yu, Z. Zhang, H. H. Chen, and P. Qiu, "On cognitive radio networks with opportunistic power control strategies in fading channels," *IEEE Trans. Wireless Commun.*, vol. 7, no. 7, pp. 2752–2761, Jul. 2008.

[5] A. Jovicic and P. Viswanath, "Cognitive radio: an information-theoretic perspective," *IEEE Trans. Inf. Theory*, vol. 55, no. 9, pp. 3945–3958, Sep. 2009.

[6] A. Goldsmith, S. A. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: an information theoretic perspective," *Proc. IEEE*, vol. 97, no. 5, pp. 894–914, May 2009.

[7] A. J. Goldsmith, *Wireless Communications*, 2nd ed. Cambridge University Press, 2005.

[8] A. Ghasemi and E. S. Sousa, "Fundamental limits of spectrum-sharing in fading environments," *IEEE Trans. Wireless Commun.*, vol. 6, no. 2, pp. 649–658, Feb. 2007.

[9] X. Kang, Y.-C. Liang, A. Nallanathan, H. K. Garg, and R. Zhang, "Optimal power allocation for fading channels in cognitive radio networks: ergodic capacity and outage capacity," *IEEE Trans. Wireless Commun.*, vol. 8, no. 2, pp. 940–950, Feb. 2009.

[10] R. Zhang, "On peak versus average interference power constraints for protecting primary users in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 4, pp. 2112–2120, Apr. 2009.

[11] L. Li and M. Pesavento, "Link reliability of underlay cognitive radio: symbol error rate analysis and optimal power allocation," in *Proc. 2011 Int. Conf. Cognitive Radio and Advanced Spectrum Management*, pp. 1–5.

[12] L. Li, P. Derwin, and M. Pesavento, "Symbol error rate analysis in multiuser underlay cognitive radio systems," in *Proc. 2011 PIMRC*, pp. 681–684.

[13] X. Kang, R. Zhang, Y.-C. Liang, and H. K. Garg, "Optimal power allocation strategies for fading cognitive radio channels with primary user outage constraint," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 2, pp. 374–383, Feb. 2011.

[14] P. Popovski, Z. Utkovski, and R. Di Taranto, "Outage margin and power constraints in cognitive radio with multiple antennas," in *Proc. 2009 Signal Process. Advances in Wireless Commun.*, pp. 111–115.

[15] M. Gastpar, "On capacity under receive and spatial spectrum-sharing constraints," *IEEE Trans. Inf. Theory*, vol. 53, no. 2, pp. 471–487, Feb. 2007.

[16] L. Musavian and S. Aissa, "Capacity and power allocation for spectrum-sharing communications in fading channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 1, pp. 148–156, Jan. 2009.

[17] H. A. Suraweera, P. J. Smith, and M. Shafi, "Capacity limits and performance analysis of cognitive radio with imperfect channel knowledge," *IEEE Trans. Veh. Technol.*, vol. 59, no. 4, pp. 1811–1822, 2010.

[18] R. Zhang, "Optimal power control over fading cognitive radio channel by exploiting primary user CSI," in *Proc. 2008 IEEE Globecom*, pp. 1–5.

[19] H. Cho and J. Andrews, "Upper bound on the capacity of cognitive radio without cooperation," *IEEE Trans. Wireless Commun.*, vol. 8, no. 9, pp. 4380–4385, Sept. 2009.

[20] Z. Rezki and M. S. Alouini, "Ergodic capacity of cognitive radio under imperfect channel-state information," *IEEE Trans. Veh. Technol.*, vol. 61, no. 5, pp. 2108–2119, Jun. 2012.

[21] A. Bagayoko, P. Tortelier, and I. Fijalkow, "Impact of shadowing on the primary exclusive region in cognitive networks," in *Proc. 2010 Eur. Wireless Conf.*, pp. 105–110.

[22] M. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, 2nd ed. Wiley-Interscience, 2005.

[23] R. Zhang and Y. C. Liang, "Exploiting multi-antennas for opportunistic spectrum sharing in cognitive radio networks," *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 88–102, Feb. 2008.

[24] M. S. Alouini and A. Goldsmith, "A unified approach for calculating error rates of linearly modulated signals over generalized fading channels," *IEEE Trans. Commun.*, vol. 47, no. 9, pp. 1324-1334, Sep. 1999.

[25] I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series and Products*. Academic Press, 2000.

[26] A. Kuhne and A. Klein, "Throughput analysis of multi-user OFDMA systems using imperfect CQI feedback and diversity techniques," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 8, pp. 1440–1450, 2008.

[27] Y. Huang and B. D. Rao, "Performance analysis of heterogeneous feedback design in an OFDMA downlink with partial and imperfect feedback," *IEEE Trans. Signal Process.*, vol. 61, no. 4, pp. 1033–1046, 2013.

[28] M. Simon, *Probability Distributions Involving Gaussian Random Variables: A Handbook for Engineers, Scientists and Mathematicians*. Springer-Verlag, Inc., 2006.

Salil Kashyap received his Bachelor of Technology degree in Electronics and Communication Eng. from NERIST, Itanagar, India in 2007 and Master of Technology degree in Digital Signal Processing from the Indian Institute of Technology (IIT), Guwahati in 2009. He is currently working towards his Ph.D. in the Dept. of Electrical Communication Eng., at the Indian Institute of Science (IISc), Bangalore. His research interests include cognitive radio and performance analysis of antenna selection and resource allocation algorithms in MIMO-OFDMA systems.

Neelesh B. Mehta (S'98-M'01-SM'06) received his Bachelor of Technology degree in Electronics and Communications Eng. from the Indian Institute of Technology (IIT), Madras in 1996, and his M.S. and Ph.D. degrees in Electrical Eng. from the California Institute of Technology, Pasadena, USA in 1997 and 2001, respectively. He is now an Associate Professor in the Dept. of Electrical Communication Eng., Indian Institute of Science (IISc), Bangalore. Before joining IISc in 2007, he was a research scientist in USA from 2001 to 2007 in AT&T Laboratories, NJ, Broadcom Corp., NJ, and Mitsubishi Electric Research Laboratories (MERL), MA.

His research includes work on link adaptation, multiple access protocols, cellular system design, MIMO and antenna selection, cooperative communications, energy harvesting networks, and cognitive radio. He was also actively involved in the Radio Access Network (RAN1) standardization activities in 3GPP from 2003 to 2007. He is an Editor of IEEE WIRELESS COMMUNICATIONS LETTERS, IEEE TRANSACTIONS ON COMMUNICATIONS, and the *Journal of Communications and Networks*. He currently serves as the Director of Conference Publications on the Board of Governors of IEEE ComSoc.