Discrete-Rate Adaptation and Selection in Energy Harvesting Wireless Systems

Parag S. Khairnar and Neelesh B. Mehta, *Senior Member, IEEE*

***Abstract*—In a system with energy harvesting (EH) nodes, the design focus shifts from minimizing energy consumption by infrequently transmitting less information to making the best use of available energy to efficiently deliver data while adhering to the fundamental energy neutrality constraint. We address the problem of maximizing the throughput of a system consisting of rate-adaptive EH nodes that transmit to a destination. Unlike related literature, we focus on the practically important discrete-rate adaptation model. First, for a single EH node, we propose a discrete-rate adaptation rule and prove its optimality for a general class of stationary and ergodic EH and fading processes. We then study a general system with multiple EH nodes in which one is opportunistically selected to transmit. We first derive a novel and throughput-optimal joint selection and rate adaptation rule (TOJSRA) when the nodes are subject to a weaker average power constraint. We then propose a novel rule for a multi-EH node system that is based on TOJSRA, and we prove its optimality for stationary and ergodic EH and fading processes. We also model the various energy overheads of the EH nodes and characterize their effect on the adaptation policy and the system throughput.**

***Index Terms*—Energy harvesting, fading channels, rate adaptation, power constraint, energy neutrality, opportunistic selection.**

I. Introduction

LIMITATIONS on the energy that can be stored in compact batteries have severely constrained the capabilities of wireless networks that operate using battery-powered nodes. Running cables to power the nodes is also often undesirable or simply infeasible. These constraints have, therefore, motivated several approaches that increase or trade-off the lifetime, reliability, and transmission coverage of such networks [1].

The capability of harvesting energy from the environment, using renewable sources of energy such as solar, wind, vibration, and thermoelectric effects, has the potential to solve this challenging problem [2]–[4]. Unlike a conventional battery-powered node that dies once the energy in its battery drains out, an energy harvesting (EH) node can harvest energy from the environment and become available for communication later. Thus, the EH capability offers the promise of perpetual, sustainable, and maintenance-free operation. However, an EH node needs to grapple with uncertainty in the amount of energy it can harvest at any time and the times at which this energy is available. This uncertainty depends on the EH source, and is abstracted in the form of an *energy profile*, which models the energy harvested as a stochastic process.

The operation of an EH node is fundamentally governed by the *energy neutrality constraint*, which mandates that, at any point of time, the total amount of energy utilized must be less than or equal to the sum of the initial energy in the battery and the total amount of energy harvested thus far [5]. Hence, the focus of the physical layer and multiple access (MAC) layer protocols shifts from minimizing energy consumption to judiciously utilizing the harvested energy and ensuring that the energy is available when required–to the extent possible.

The goal is now to maximize the capability of the system to sense and deliver possibly large amounts of data in a spectrally efficient manner at high data rates [6], [7]. Thus, EH has the potential to transform wireless sensor networks (WSNs), which have hitherto been constrained by rate and energy limitations, into perpetual networks that can even service multi-media applications [8].

EH nodes can achieve this by using link adaptation, which is a tried and tested way of exploiting fading in several wireless systems [9], [10]. Link adaptation policies in classical settings that maximize the average throughput when nodes are subject to an average power constraint or that maximize the average throughput per Joule have been extensively studied [9], [11]. However, link adaptation in EH wireless systems raises interesting new problems because of the aforementioned uncertainty in the amounts and times at which energy is harvested and the new energy neutrality constraint.

A. Related Literature on Link Adaptation in EH Nodes

Given their promising future, several recent papers have addressed the communication design aspects of systems with link-adaptive EH nodes. We discuss the most relevant literature on point-to-point single EH node and multipoint-to-point multi-EH node systems separately below.

1) Single EH Node System: A two-stage hierarchical rate and power adaptation scheme for an EH node assuming that the channel fading changes much faster than the rate at which energy is harvested is investigated in [12]. Optimal transmission policies that minimize the time by which packets are delivered to the destination are presented in [13]. Finite battery constraints and the related problem of maximizing short-term throughput over a finite time interval are treated in [14]. Channel fading is also included in the model in [15]. In [16], optimal transmission policies that take into account quality-of-service

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P. S. Khairnar is with Marvell India Private Ltd., Pune 411 014, India (e-mail: pskhairnar@gmail.com).

N. B. Mehta is with the Department of Electrical Communication Engineering, Faculty of Engineering, Indian Institute of Science, Bangalore 560 012, India (e-mail: nbmehta@ece.iisc.ernet.in).

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(QoS) constraints and finite battery size are characterized assuming non-causal information about EH and fading. Finite horizon online policies based on dynamic programming (DP) assuming static channel conditions, discrete power levels, and a Markovian EH process are developed in [17]. Scenarios with causal and non-causal channel state information (CSI) and EH are studied in [18], which models the channel fading and EH processes as first order Markov models. Throughput-optimal and delay-optimal policies for an EH node with a data queue, and sub-optimal extensions to tackle fading are developed in [19]. The capacity for an additive white Gaussian noise channel is derived in [20].

2) Multipoint-to-point Multiple EH Node Systems: Ad hoc MAC policies for EH nodes with fading channels are proposed in [21]. The performance of conventional MAC protocols that are based on time division multiple access or Aloha is analyzed for EH nodes in [22], but rate adaptation is not modeled. The optimal packet scheduling for a two-user MAC channel that minimizes the time required to deliver packets from both users to a destination is studied in [23].

B. Observations and Comments

Continuous rate adaptation, in which the transmitter has access to a continuum of rates, is assumed in several papers [11], [13]–[15], [18], [19], [21], [23]. In practice, however, the transmitter can only choose from a pre-determined, discrete set of rates. While the continuous rate adaptation model can serve as a good approximation when the number of available rates is large, the approach that we shall develop is applicable even when the number of rates is small. This is likely to be the case in WSNs due to hardware and software constraints. For example, in current IEEE 802.15.4-based WSNs, at most four rates have been considered [10].

Another common assumption is that the rate is a concave function of the transmit power or the product of the transmit power and the channel power gain when fading is modeled [13]–[15], [17]–[19]. This is motivated by the Shannon formula. This assumption begets the directional water-filling solutions developed in [13]–[15], [18] and is also required to prove the optimality of the queue stabilizing policy of [19]. However, this is not the case with discrete rate adaptation because an increase in the transmit power does not necessarily imply an increase in rate. Therefore, the problem of optimizing the throughput of EH nodes with discrete rate adaptation must be separately investigated, and is the focus of this paper.

C. Contributions

We address the fundamental problem of maximizing the long-term average throughput of a system consisting of one or more rate-adaptive EH nodes that transmit over a fading channel to a destination and meet a target bit error rate (BER). The average throughput is an important and widely studied performance measure in wireless systems. We focus on the practically relevant case of discrete rate adaptation. Now the transmission is affected not only by the channel gain, but also by the amount of energy available in the battery.

To gain intuition, we first consider a single EH node system, and propose an easy-to-implement, optimal discrete rate adaptation scheme. It is based on the adaptation scheme for an average power constrained node, which we shall henceforth refer to as a *non-EH* node. Our main contribution here is the generality of the optimality result, which we prove holds over the general class of stationary and ergodic EH and channel fading processes. This model encompasses several EH models considered in the literature, e.g., Bernoulli model [24] and time-correlated Markov model [18], [25], [26]. Also encompassed are a wide class of fading models such as line-of-sight (LoS) Rician fading, non-LoS Rayleigh fading, Nakagami-m fading, and Markovian time-correlated fading [9], [27].

We then study a more general system with multiple EH nodes that share a common channel to transmit individual data to a destination, and one of them is opportunistically selected to transmit. The goal here is to maximize the average throughput of the system. Since only one node transmits, our framework also applies to the scenario in which nodes have the same data to transmit to the sink. While allowing only one node to transmit need not be throughput-optimal in the general MAC setting [28], selection is practically appealing because it obviates the need for tight synchronization among the geographically separated transmitting nodes, and does not require exchange of CSI among the nodes. The problem is also theoretically interesting since the optimal transmission and selection rules even for conventional non-EH systems are not fully understood.

We make two important contributions here. Firstly, for a system with non-EH nodes, we first derive a throughput-optimal joint selection and rate adaptation rule (TOJSRA). To the best of our knowledge, such a joint selection and adaptation rule and a proof of its optimality has not been presented in the literature. Secondly, we present a novel selection and adaptation rule for a multi-EH node system that is based on TOJSRA, and prove its throughput-optimality using a coupling argument that generalizes the proof for the single EH node system. These results again hold over the general class of stationary and ergodic EH and channel fading processes, including the practical case where they are statistically non-identical across nodes. We also benchmark the performance of the proposed scheme with several known schemes. Among these contributions, the ones pertaining to the simpler single EH node case were presented in the conference version [29].

The paper also investigates how various energy overheads and non-idealities encountered by EH systems affect discrete rate adaptation. We show that they can be incorporated into the adaptation scheme in the form of a single parameter. Using a perturbation approach, we derive simple closed-form expressions for the adaptation policy and the system throughput for relatively small energy overheads. While energy overheads and non-idealities have been modeled in the literature [12], [30]–[32], the novelty and significance of our work lies in characterizing their impact on the optimal discrete rate adaptation scheme and its throughput for single and multi-EH node systems. This does not follow from [12], [30]–[32].

The paper is organized as follows. The system model is specified in Section II. Section III considers the single EH node case. Multi-EH node case is considered in Section IV.

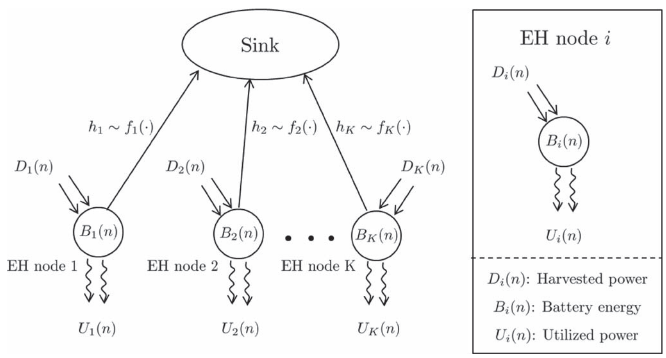

Fig. 1. Illustration of a multi-EH node system consisting of K EH nodes that have data to transmit to a common sink node.

Simulation results are presented in Section V, and are followed by our conclusions in Section VI.

II. Single and Multi-EH Node System Model

As shown in Fig. 1, we consider a system with K EH nodes that have data to transmit to a sink over a common frequency-flat block fading channel of bandwidth Ω. In order to determine the maximum throughput possible, we assume that each node always has data to send [14], [18]. It provides a theoretical limit on the maximum amount of data transmission by the EH network, which is a useful measure of its capabilities. A *time interval* consists of node selection and transmission by the selected node for a duration of T_t sec. The channel is assumed to remain unchanged during a time interval [33].

During the nth time interval, the baseband signal $y_i(n)$ at the receiver when EH node i transmits data symbol $x_i(n)$ is

$$y_i(n) = \sqrt{h_i(n)} e^{j\phi_i(n)} x_i(n) + w_i(n), \tag{1}$$

where $h_i(n)$ and $\phi_i(n)$ are the channel power gain and phase, respectively, of the channel between the ith node and the sink, and $w_i(n)$ is a circularly symmetric complex additive white Gaussian noise (CAWGN) with power spectral density $N_0/2$. The probability density function (PDF) and the cumulative distribution function (CDF) of $h_i(n)$, which shall henceforth be called the channel gain, are denoted by $f_i(\cdot)$ and $F_i(\cdot)$, respectively. The channel gains for different nodes are assumed to be independent, as is the case when the nodes are sufficiently spaced apart in a rich scattering environment, but they need not be statistically identical [9]. The random process $\{h_i(n); n \geq 1\}$ is assumed to be stationary and ergodic. No other limiting assumption is made about the channel gains.

The EH node is assumed to know its channel gain causally in order that it may adapt to it. In time division duplexing (TDD) systems, making the sink transmit a pilot periodically and exploiting reciprocity enables the nodes to acquire this information [12], [34], [35]. In frequency division duplexing (FDD) systems, this information needs to be fed back by the sink to each node [9]. Thus, in both TDD and FDD, acquiring CSI entails energy and bandwidth costs. We shall initially assume that these costs are negligible. Thereafter, in Section III-B, we incorporate them in our model.

A. Rate and Power Adaptation

The constellation size is chosen from the set $\mathcal{M} = \{m_1, m_2, \ldots, m_M\}$, with $m_1 = 1$ corresponding to no transmission and $m_1 < m_2 < \cdots < m_M < \infty$. The M transmission rates are then $\log_2(m_1) = 0, \log_2(m_2), \ldots, \log_2(m_M)$. The choice of $\mathcal{M}$ depends on the hardware complexity of the system. When a node transmits using a constellation of size m with power P and the channel gain is h, its BER is given by $c_1 \exp(-c_2 hP/(N_0\Omega(m^{c_3} - c_4)))$, where $c_1, \ldots, c_4$ are modulation-specific real constants [9]. For example, for M-ary quadrature amplitude modulation (M-QAM), $c_1 = 2$, $c_2 = 1.5$, $c_3 = 1$, and $c_4 = 1$.

Given the channel gain, an EH node that is selected for transmission, adapts its constellation size and transmit power to ensure a BER of P_b. Equating the above BER formula to P_b, we see that a node i that transmits in time interval n with a constellation of size $\mu_i(n) = m_j$ needs to set its transmit power $P_{req}(h_i(n), m_j)$ as

$$P_{req}\left(h_i(n), m_j\right) = \frac{d_j}{h_i(n)}. \tag{2}$$

Here, $h_i(n)$ is the channel gain in time interval n, $d_1 = 0$,

$$d_j = \kappa N_0 \Omega \left(m_j^{c_3} - c_4\right), \quad \text{for } 2 \leq j \leq M, \tag{3}$$

$\kappa = \log_e(c_1/P_b)/c_2$, and Ω is the bandwidth. Note that $d_1 = 0$ implies that when the channel is in a deep fade, the node sets its rate and power to zero as it cannot transmit reliably.

We note that this framework can be extended to include adaptation using coded modulation schemes using the coding gain model of [9, Chapter 9.3]. It can be shown that the expression for d_j now has in its denominator a new constant G_j, which is the coding gain of the jth coded modulation scheme. This approach is motivated by Shannon's capacity formula and has been shown to work for coset codes, which includes trellis codes and lattice codes.

B. Energy Harvesting, Storage, and Consumption

The node stores its harvested energy in a buffer such as a rechargeable battery or a super-capacitor, both of which shall be referred to as a battery henceforth. Initially, the battery is assumed to be ideal, i.e., it has infinite capacity and has 100% storage efficiency [12], [19], [20], [34]. Generalizations that discard these assumptions are addressed in Sections III-B and IV-C for single and multiple EH nodes, respectively.

Let $B_i(n)$ denote the energy in the battery of the ith node at the beginning of the nth time interval. Let $D_i(n)$ and $U_i(n)$ denote the harvested and utilized power, respectively, during the nth time interval, with $U_i(n)T_t \leq B_i(n)$. Thus,

$$B_i(n+1) = B_i(n) + T_t D_i(n) - T_t U_i(n). \tag{4}$$

The energy neutrality constraint implies that during the nth time interval, the ith node can transmit a data symbol drawn from a constellation of size $\mu_i(n)$ only if it has sufficient energy stored in its battery, i.e., $B_i(n) \geq T_t P_{req}(h_i(n), \mu_i(n))$. The EH process $\{D_i(n); n \geq 1\}$ is assumed to be a stationary and ergodic random process with a finite mean of $\bar{P}_{EH_i}$. We make no other limiting assumptions about the process.

Notation: For a sequence $\{s(n); n \geq 0\}$ of random variables (RVs), $\lim_{n\to\infty} s(n) \overset{\text{as}}{>} z$ shall denote greater than in the almost sure sense, i.e., $\Pr(\lim_{n\to\infty} s(n) > z) = 1$ [36]. Here, $z \in \mathbb{R}$. Similarly, $\overset{\text{as}}{=}$ and $\overset{\text{as}}{<}$ denote equality and less than, respectively, almost surely. $\mathbb{E}[\cdot]$ denotes expectation and $\Pr(\cdot)$ denotes probability. Unless required to avoid confusion, we shall drop the time index n from the notation. When $K = 1$ or when the nodes are statistically similar, the node index i is dropped from variables such as h_i, μ_i, B_i, D_i, U_i, and $\bar{P}_{EH_i}$.

III. Single EH Node: Transmission Rule

At the beginning of time interval n, depending on the current battery energy $B(n)$ and channel gain $h(n)$, the node must decide its data rate $R(n)$ in the time interval, which corresponds to a constellation of size $\mu(n)$. In general, $R(n)$ depends on both $h(n)$ and $B(n)$. The latter, in turn, depends on $\{D(k); k < n\}$ and $\{U(k); k < n\}$. Our goal is to find a causal scheme that maximizes the cumulative average throughput $\overline{R}$, which is defined as $\lim_{n\to\infty}(\sum_{k=1}^{n} R(k))/n$. The optimization problem can be stated as

$$\begin{aligned}
&\max_{R(1),R(2),\ldots} \quad \overline{R} \\
&\text{s.t.} \quad B(k) \geq 0, \quad \text{for } k \geq 1, \\
&\qquad c_1 \exp\left(\frac{-c_2 h(k) P(k)}{N_0 \Omega\left[(\mu(k))^{c_3} - c_4\right]}\right) \leq P_b, \ \forall P(k) > 0.
\end{aligned} \tag{5}$$

Let $\overline{R}^*_{EH}(\bar{P}_{EH})$ and $\Psi(\bar{P}_{EH})$ respectively denote the maximum average throughputs achieved by an EH node that harvests energy at a rate $\bar{P}_{EH}$ and a non-EH node with an average transmit power constraint of $\bar{P}_{EH}$.

Consider first the optimal transmission scheme for a non-EH node that is subject to an average power constraint of $\bar{P}$. This is derived in [9, Chap. 9]. The scheme is defined in terms of a set of thresholds on the channel gain h, which determine the power and rate. It is as follows. The non-EH node transmits with a constellation of size m_j and with power $P_{req}(h, m_j) = d_j/h$ (cf. (2)) if $h \in [\eta\zeta_j, \eta\zeta_{j+1})$, where $\zeta_1 = 0$,

$$\zeta_j = \begin{cases} \frac{m_2^{c_3} - c_4}{\log_2(m_2)}\kappa, & j = 2, \\ \frac{m_j^{c_3} - m_{j-1}^{c_3}}{\log_2(m_j) - \log_2(m_{j-1})}\kappa, & 3 \leq j \leq M, \\ \infty, & j = M+1. \end{cases} \tag{6}$$

The constant η, which we shall refer to as the *power control parameter*, is the unique solution of

$$\sum_{j=2}^{M} \int_{\eta\zeta_j}^{\eta\zeta_{j+1}} P_{req}(h, m_j) f(h)\, dh = \bar{P}. \tag{7}$$

η needs to be computed numerically only once.

Let $\Psi(\bar{P}_{EH})$ denote the average throughput achieved by this system. It can be seen that $\Psi(\bar{P}_{EH})$ upper bounds the average throughput achievable by *any* transmission rule for the EH system since the energy neutrality constraint is stricter than the average power constraint.

A. EH Transmission Rule and Its Optimality

We propose the following transmission rule for an EH node that harvests energy at an average rate of $\bar{P}_{EH}$: At time n, transmit with a constellation of size m_j and with power $P_{req}(h, m_j)$ given by (2) if $h \in [\eta\zeta_j, \eta\zeta_{j+1})$ and $P_{req}(h, m_j) T_t \leq B(n)$. Else, do not transmit. Here, η is the solution of (7) for $\bar{P} = \bar{P}_{EH} - \delta$, where $\delta > 0$ is an arbitrarily small constant.

Notice that whether the EH node transmits is now also affected by its battery state $B(n)$. Since the node transmits only when $P_{req}(h, m_j) T_t \leq B(n)$, the energy neutrality constraint is satisfied. Let $R_{EH}(\bar{P}_{EH}, k)$ denote the throughput of the above transmission rule in time interval k. Then, its cumulative average throughput $\overline{R}_{EH}(\bar{P}_{EH}, n)$ until time n equals

$$\overline{R}_{EH}(\bar{P}_{EH}, n) = \frac{1}{n}\sum_{k=1}^{n} R_{EH}(\bar{P}_{EH}, k). \tag{8}$$

Theorem 1: When the channel fading process $\{h(n); n \geq 1\}$ and the EH process $\{D(n); n \geq 1\}$ are stationary and ergodic, then for the proposed policy, for any given $\epsilon > 0$, there exists a $\delta > 0$ such that

$$\Psi(\bar{P}_{EH}) - \epsilon \overset{\text{as}}{<} \lim_{n\to\infty} \overline{R}_{EH}(\bar{P}_{EH} - \delta, n) < \Psi(\bar{P}_{EH}). \tag{9}$$

Proof: The proof is given in Appendix A. ∎

The above result establishes that the proposed scheme's average throughput can be made arbitrarily close to the optimal throughput achievable over the set of all EH transmission schemes that adhere to the energy neutrality constraint. This is because the upper bound applies to any EH transmission scheme. Intuitively, by targeting an average power consumption that is marginally less than $\bar{P}_{EH}$, the scheme lets the energy stored in the battery increase with time. This effectively removes the randomness in the energy harvested.

We make the following comments about the above result: (i) We note that the battery energy is also allowed to increase in [19] and [20]. However, as mentioned, the system model in [19] is different because it assumes continuous rate adaptation and a concave power-to-rate mapping. The system model in [20] does not consider fading or rate adaptation. Furthermore, both do not consider multi-EH node systems, which we consider next. (ii) While Result 1 implies that δ can be an arbitrarily small but positive constant, the following trade-off occurs in practice. The smaller δ is, the closer the average throughput is to the optimal value. However, this also increases the time for the battery energy to build up and the system to reach the optimal regime. (iii) For finite battery capacity, the proposed scheme need not be optimal. We study this in Section V-B1.

B. Effect of Energy Overheads and Non-Idealities

In practice, inefficiencies in energy storage and energy losses exist. Specifically, if an energy Z is harvested, then only a fraction $\beta_c Z$, where $0 < \beta_c \leq 1$, gets stored in the battery. Furthermore, if an energy Z needs to be utilized by the node, then the energy withdrawn from the battery is (Z/β_d), where $0 < \beta_d \leq 1$.[1] In an interval, let the energy loss in the battery due to an internal leakage current be E_l and let the energy required by the node for sensing, processing, and acquiring channel gain information be E_p. Therefore, the energy drawn from the battery for processing is (E_p/β_d) and the energy consumed for transmission is $U(n)T_t/\beta_d$. Thus,

$$B(n+1) = \left(B(n) - \frac{E_p}{\beta_d} - E_l\right)^+ + \beta_c D(n) T_t - \frac{U(n)T_t}{\beta_d}, \tag{10}$$

where $z^+ \triangleq \max\{z, 0\}$.

A sufficient condition for the energy neutrality constraint to be satisfied is obtained if we assume that $(E_p/\beta_d) + E_l$ are subtracted from the battery energy before considering whether the node transmits even if its battery is running low on energy.[2] In such a case, the energy neutrality condition becomes $T_t(\sum_{k=1}^n U(k))/(n\beta_d) \leq \beta_c T_t(\sum_{k=1}^n D(k))/n - E_p/\beta_d - E_l$. This is equivalent to an average transmit power constraint of $\bar{P}'_{EH} = \bar{P}_{EH} - \alpha$, where

$$\alpha = (1 - \beta_c\beta_d)\bar{P}_{EH} + E_p + \beta_d E_l. \tag{11}$$

From Theorem 1, it then follows that the average throughput of an EH node with energy overheads can be made arbitrarily close to $\Psi(\bar{P}'_{EH})$. The effect of the energy overheads on the average throughput is as follows.

Corollary 1: Let η and η' denote the power control parameters for average power constraints $\bar{P}_{EH}$ and $\bar{P}_{EH} - \alpha$, respectively. Let $\Delta = \eta/(\sum_{j=2}^M f(\zeta_j\eta)(d_j - d_{j-1}))$. If $\Delta\alpha \ll 1$, then $\eta' \approx \eta + \Delta\alpha$. The corresponding average throughputs are related as

$$\Psi(\bar{P}_{EH} - \alpha) \approx \Psi(\bar{P}_{EH}) - \frac{\eta\alpha}{N_0\Omega}. \tag{12}$$

Proof: The proof is relegated to Appendix B. ∎

Thus, the effect of all the energy overheads on the power control parameter, which determines the transmission scheme, and the average throughput is conveniently captured by the single parameter α. The above corollary enables us to easily determine the power control parameter and the average throughput in terms of the power control parameter and throughput of an EH system without any energy overheads.

IV. Multiple EH Nodes With Opportunistic Selection

We now consider the general case with $K \geq 2$ EH nodes and selection. Maximizing the average throughput now involves determining the following two related quantities: (i) which node to opportunistically select, and (ii) the selected node's rate and transmit power.[3]

Let $s(k) \in \{1, \ldots, K\}$ denote the node selected during the kth time interval. Let its transmit power be $P_{s(k)}(k)$ and its rate be $R_{s(k)}(k)$, which corresponds to a constellation of size $\mu_{s(k)}(k)$. Our goal is again to find a causal scheme that maximizes the cumulative average throughput $\overline{R}_{EH}(\bar{P}_{EH})$, which is defined as $\lim_{n\to\infty}(\sum_{k=1}^n R_{s(k)}(k))/n$. The optimization problem can be stated as

$$\begin{aligned} &\max_{s(1),R_{s(1)}(1),s(2),R_{s(2)}(2),\ldots} \overline{R}_{EH}(\bar{P}_{EH}), \\ \text{s.t.}\quad & B_i(k) \geq 0, \ \text{for } k \geq 1 \text{ and } 1 \leq i \leq K \\ & c_1 \exp\left(\frac{-c_2 h_{s(k)}(k) P_{s(k)}(k)}{N_0\Omega\left[\left(\mu_{s(k)}(k)\right)^{c_3} - c_4\right]}\right) \leq P_b, \ \forall P_{s(k)}(k) > 0. \end{aligned} \tag{13}$$

A. Multiple Non-EH Nodes: Throughput-Optimal Joint Selection and Rate Adaptation

As before, we first consider the non-EH nodes case. This is then generalized to EH nodes in Section IV-B. Also, we first assume that the average power constraint is the same for all the nodes. This is subsequently generalized in Section IV-B.

Theorem 2: For a system with K non-EH nodes, each of which is subject to an average power constraint $\bar{P}$, the throughput-optimal selection and transmission rules at time n are

- *Selection rule:* $s(n) = \arg\max_{1\leq i\leq K}\{h_i(n)/\eta_i\}$.
- *Transmission rule:* The selected node $s(n)$ transmits with a constellation size of m_j with power $d_j/h_{s(n)}(n)$ if

$$\eta_{s(n)}\zeta_j \leq h_{s(n)}(n) < \eta_{s(n)}\zeta_{j+1}. \tag{14}$$

Here, $\eta_1, \ldots, \eta_K$ are the solution of

$$\bar{P} = \sum_{j=2}^{M} d_j \int_{\eta_i\zeta_j}^{\eta_i\zeta_{j+1}} \frac{f_i(h)}{h}\left[\prod_{l=1,\ldots,K,\ l\neq i} F_l\left(\frac{h\eta_l}{\eta_i}\right)\right] dh, 1 \leq i \leq K. \tag{15}$$

The optimal average throughput $\Psi(\bar{P})$ is equal to

$$\Psi(\bar{P}) = \sum_{i=1}^{K}\sum_{j=2}^{M} \log_2(m_j) \times \int_{\eta_i\zeta_j}^{\eta_i\zeta_{j+1}} f_i(h)\left[\prod_{l=1,\ldots,K,\ l\neq i} F_l\left(\frac{h\eta_l}{\eta_i}\right)\right] dh. \tag{16}$$

Proof: The proof is given in Appendix C.

As before, the constants $\eta_1, \ldots, \eta_K$ are computed numerically once. The selection rule in Theorem 2 implies that

[1]We assume here that the harvested energy is always stored in the battery first and then utilized as required. An alternate model, which we do not consider, is one where some of the energy harvested is directly used with 100% efficiency while the remaining unused energy is stored with a lower efficiency [12], [19].

[2]In practice, the node would shut down in case its battery is running low on energy so that it has sufficient energy to be booted up later.

[3]We assume that the time and energy overhead of selection are negligible, as is often assumed in the literature on opportunistic selection.

selecting the node with the highest channel gain is suboptimal when the nodes see statistically different channels. However, in the special case of symmetric nodes where $f_i(h) = f(h)$, for all i, we have $\eta_i = \eta$ and the selection rule does simplify to highest channel gain-based selection: $s(n) = \arg\max_{1\le i\le K}\{h_i(n)\}$. For the symmetric case, the average power constraint in (15), which determines η, simplifies to $\bar{P} = \sum_{j=2}^{M} d_j \int_{\eta\zeta_j}^{\eta\zeta_{j+1}} f(h)h^{-1}F^{K-1}(h)dh$. And,

$$\Psi(\bar{P})=\log_2(m_M) - \sum_{j=2}^{M} F^K(\eta\zeta_j)\left(\log_2(m_j) - \log_2(m_{j-1})\right). \tag{17}$$

We note that the above selection rule can be implemented using scalable, distributed selection schemes, which require limited time and energy [37], [38]. For example, using the timer scheme [37], node i locally sets a timer that is a monotonically non-increasing function of h_i/η_i. It transmits its packet when its timer expires. Thus, the first node to transmit is the one with the highest h_i/η_i. Consequently, the sink does not need to centrally acquire the CSI of every node.

B. Multiple EH Nodes: Throughput-Optimal Selection and Adaptation Rule

Motivated by the non-EH case, we propose the following multi-EH node transmission scheme when each EH node harvests energy at an average rate of $\bar{P}_{EH}$:

- Selection rule: $s(n) = \arg\max_{1\le i\le K}\{h_i(n)/\eta_i\}$.
- Transmission rule: The selected node $s(n)$ transmits with a constellation of size m_j, if $\eta_{s(n)}\zeta_j \le h_{s(n)}(n) < \eta_{s(n)}\zeta_{j+1}$ and $P_{req}(h_{s(n)}(n), m_j)T_t \le B_{s(n)}(n)$, where $P_{req}(h_{s(n)}(n), m_j)$ is given by (2). Else, it does not transmit.

Here, $\eta_1, \ldots, \eta_K$ are the solution of the system of K equations in (15) and target an average transmit power of $\bar{P}_{EH} - \delta$ for each node, where $\delta > 0$ is an arbitrarily small constant.

As in Section III-A, the transmission by the selected node depends on not just its channel condition but also its battery state, and the energy neutrality condition is satisfied. Let the cumulative average throughput until time n of the above scheme, when it is designed for an average power consumption of $\bar{P}_{EH}$ per node, be denoted by $\overline{R}_{EH}(\bar{P}_{EH}, n)$. We now prove that $\overline{R}_{EH}(\bar{P}_{EH}, n)$ is arbitrarily close to the optimal throughput.

Theorem 3: When the channel fading processes of the K nodes, which we denote by $\{h_1(n), \ldots, h_K(n); n \ge 1\}$, and EH processes $\{D_1(n), \ldots, D_K(n); n \ge 1\}$ are stationary and ergodic, then for the proposed scheme, for any given $\epsilon > 0$, there exists a $\delta > 0$ such that

$$\Psi(\bar{P}_{EH}) - \epsilon \overset{\text{as}}{<} \lim_{n\to\infty} \overline{R}_{EH}(\bar{P}_{EH} - \delta, n) < \Psi(\bar{P}_{EH}). \tag{18}$$

Proof: The proof is given in Appendix D. ■

Since the proposed EH transmission scheme's average throughput can be made arbitrarily close to an upper bound on the average throughput achievable by *any scheme* that satisfies the energy neutrality constraint of each node, it implies it is arbitrarily close to the optimal throughput.

The following result shows how the scheme can be designed for the practically relevant scenario in which the nodes harvest energy at different average rates. This occurs, for example, in solar-based EH when the intensity of solar radiation reaching the different EH nodes is different, and vibration-based EH in which the amplitude of vibration differs across the EH nodes.

Corollary 2: Let $\eta_1, \eta_2, \ldots, \eta_K$ be the power control parameters of a system in which all the K nodes harvest energy with same rate $\bar{P}_{EH}$ and have channel gains $\{h_1(n), \ldots, h_K(n); n \ge 1\}$. Then, the optimal throughput when the K nodes harvest energy with rates $\bar{P}_{EH}, \nu_2\bar{P}_{EH}, \ldots, \nu_K\bar{P}_{EH}$ and have channel gains $\{h_1(n), h_2(n), \ldots, h_K(n); n \ge 1\}$ is the same as that of a system in which each node harvests with power $\bar{P}_{EH}$, has power control parameters $\eta_1, \nu_2\eta_2, \ldots, \nu_K\eta_K$, and channel gains $\{h_1(n), \nu_2 h_2(n), \ldots, \nu_K h_K(n); n \ge 1\}$.

Proof: The proof is relegated to Appendix E. ■

C. Effect of Energy Overheads and Non-Idealities

For the ith node, let β_{c_i}, β_{d_i}, E_{p_i}, and E_{l_i} denote the charging efficiency, discharging efficiency, processing energy per time interval, and leakage energy per time interval, respectively, as defined in Section III-B. In addition, let the circuit energy consumption when the node is idle be E_{I_i}. Following an approach similar to that in Section III-B, we conclude that the average throughput of this system can be made arbitrarily close to that of a system consisting of K non-EH nodes with no energy overheads, where the ith non-EH node satisfies the average power constraint of $\bar{P}'_{EH_i} = \bar{P}_{EH_i} - \alpha_i$ and $\alpha_i = (1 - \beta_{c_i}\beta_{d_i})\bar{P}_{EH_i} + \beta_{d_i}E_{l_i} + E_{p_i} + E_{I_i}$.

The analytical characterization of the effect of this energy overhead on the average throughput for the general case is quite involved. However, as shown below, it is insightful when the nodes are statistically identical, i.e., $f_i(h) = f(h)$ and $\alpha_i = \alpha$, for $1 \le i \le K$.

Corollary 3: Let η and η' be the power control parameters for average EH rates $\bar{P}_{EH}$ and $\bar{P}_{EH} - \alpha$, respectively. Let $\Delta = \eta/(\sum_{j=2}^{M} f(\eta\zeta_j)F^{K-1}(\eta\zeta_j)(d_j - d_{j-1}))$. If $\Delta\alpha \ll 1$, then $\eta' \approx \eta + \Delta\alpha$. The corresponding average throughputs are related as

$$\Psi(\bar{P}_{EH}) \approx \Psi(\bar{P}_{EH} - \alpha) - \frac{K\eta\alpha}{N_0\Omega}. \tag{19}$$

Proof: The proof is along lines similar to that for Corollary 1, except that $f(h)$ is replaced by $f(h)F^{K-1}(h)$. The details are skipped to conserve space. ■

Thus, as before, the energy overheads are encapsulated in a single parameter α that drives the selection and rate adaptation rules and determines the average throughput. Furthermore, Δ captures relevant information about the discrete rate adaptation scheme and channel statistics.

V. Numerical Results

We illustrate our results using adaptation over the following three M-QAM constellations: 4-QAM ($m_2 = 4$), 16-QAM ($m_3 = 16$), 64-QAM ($m_4 = 64$), and $m_1 = 1$ (no transmission), with $P_b = 0.001$ and Rayleigh fading. For the symmetric case, we set $\mathbb{E}[h_i] = 1$. We also verify our analysis using Monte

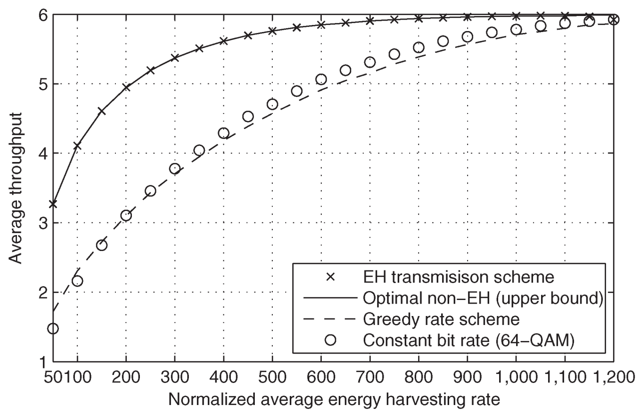

Fig. 2. Average throughput in bits/symbol/Hz and comparisons as a function of the normalized average EH rate.

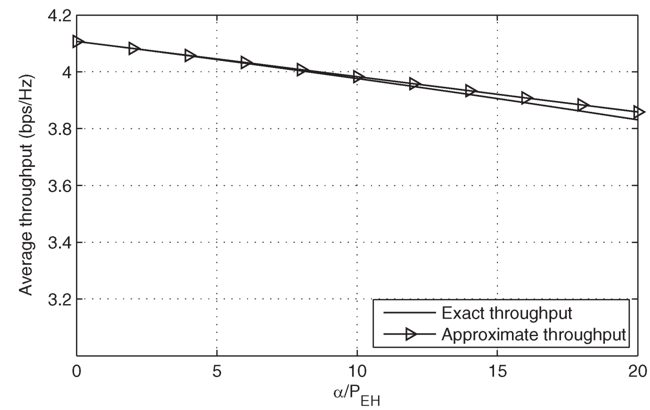

Fig. 3. Zoomed-in view of average throughput as a function of the energy overhead relative to average EH rate ($\bar{P}_{EH}/(N_0\Omega) = 100$).

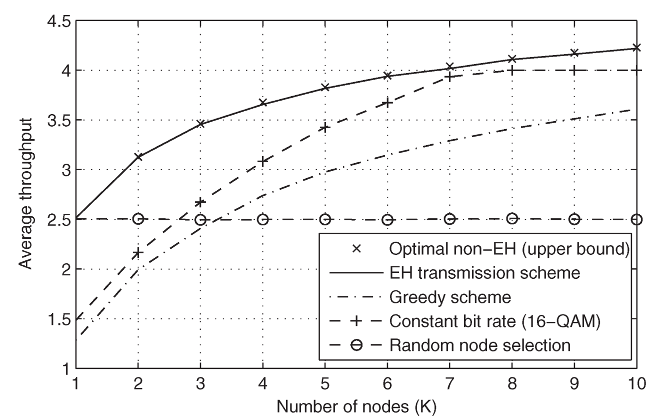

Fig. 4. Multiple EH nodes: Average throughput in bits/symbol/Hz as a function of number of nodes when the total harvested energy by all the nodes is kept fixed ($K\bar{P}_{EH}/(N_0\Omega) = 25$).

Carlo simulations that are run over 10^6 time intervals. The Bernoulli energy injection model is simulated, in which every node harvests an energy E every T_t sec with probability ρ [24]. Hence, $\bar{P}_{EH} = \rho E/T_t$. Unless mentioned otherwise, $\rho = 0.1$, $\delta = 0.01$, and $B_i(1) = 0$, for $1 \leq i \leq K$.

A. Single EH Node: Throughput and Benchmarking

The average throughput of the proposed EH transmission scheme as a function of the normalized average EH rate $\bar{P}_{EH}/(N_0\Omega)$, which also corresponds to the fading-averaged signal-to-noise-ratio (SNR) at the receiver, is plotted in Fig. 2 in linear scale. Also plotted are the average throughputs of: (i) *Greedy rate scheme*, in which an EH node transmits data at the highest rate that can be supported by its current battery and channel states. It uses a constellation of size m_j if $P_{req}(h, m_j)T_t \leq B(n) < P_{req}(h, m_{j+1})T_t$, and (ii) *Constant bit rate scheme*, in which a node transmits only with 64-QAM and only if sufficient energy is available. Also plotted is the upper bound, which corresponds to the average throughput of a non-EH node with an average power constraint of $\bar{P}_{EH}$. We observe that the average throughput of the proposed EH transmission scheme coincides with its upper bound. It is greater than or equal to that of the greedy and constant bit rate schemes, which are both overly aggressive in their choice of the rate and empty the battery more when the channel is in a deep fade, especially for small $\bar{P}_{EH}$.

Fig. 3 plots the average throughput of the EH transmission rule as a function of $(\alpha/\bar{P}_{EH})$. Also plotted is the approximate expression in (12), which turns out to be quite accurate. For example, it is off by only 0.7% even when α is 20% of $\bar{P}_{EH}$.

B. Multiple EH Nodes: Throughput and Benchmarking

Fig. 4 plots the average throughput as a function of the number of EH nodes K when the channel gains of the users and the energy harvested by them are statistically identical. As before, we compare the average throughput of the proposed multi-EH node transmission scheme with the following three benchmarks: (i) Non-EH multi-node scheme (cf. Theorem 2), in which each node is only subject to an average power constraint of $\bar{P}_{EH}$, (ii) Greedy rate scheme, in which the selected node, which has the highest channel gain among all nodes, transmits at the highest possible rate supported by its current battery state and channel gain, and (iii) Constant bit rate scheme, in which the selected node transmits only using 16-QAM if sufficient energy is available in its battery. In the above benchmarking schemes, the node with the highest channel gain is selected.

To understand the impact of opportunistic selection, the average throughput of random selection, in which a node is selected with probability $1/K$ irrespective of its channel and battery states, is also plotted. Since a node only transmits one out of K times on average, the adaptation thresholds are as given in (6) except that η in (7) is chosen to meet an average power constraint of $K\bar{P}_{EH}$. Since $K\bar{P}_{EH}$ is kept constant in the figure, the throughput of random selection does not change with K. The average throughput of the proposed scheme is greater than or equal to the other benchmark schemes. It increases as K increases because it exploits multi-user diversity. This is also why a smaller amount of energy needs to be harvested than in Fig. 2 to achieve the same throughput.

1) Effect of Finite Battery Capacity: Fig. 5 plots a zoomed-in view of the average throughput as a function of η when the battery capacity of every EH node is limited to $B_{\max}$. In order to identify key trends, $B_{\max}$ is characterized in terms of the energy required to transmit a given number of bits using

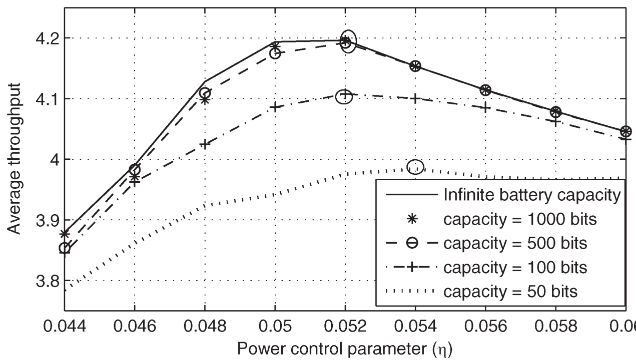

Fig. 5. Zoomed-in view of the average throughput in bits/symbol/Hz as a function of power control parameter η ($K = 10$, $T_t = 1$, $E/(N_0\Omega) = 200$, $\rho = 0.01$, and $\bar{P}_{EH}/(N_0\Omega) = 2$).

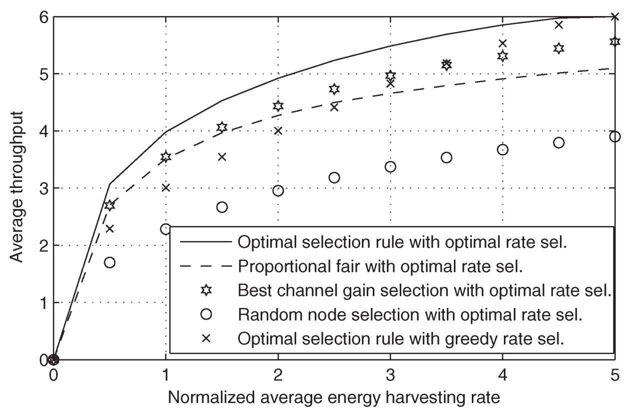

Fig. 6. Statistically non-identical channels: Average throughput in bits/symbol/Hz as a function of the average EH rate ($K = 6$ nodes with mean channel gains 0.5, 1, 2, 4, 8, and 16).

64-QAM when the channel gain is unity.[4] In order to stress test our approach, we study a low probability of injection of $\rho = 0.01$. For this, the energy when harvested $E = \bar{P}_{EH}/\rho$ is large, which increases the odds that the energy in the battery overflows. Even with a battery that can store enough energy for transmitting $B_{\max} = 100$ bits, the loss in average throughput compared to the infinite battery capacity model is less than 3.5%. Further, the optimal value of η that maximizes the average throughput shifts marginally as $B_{\max}$ decreases, except when $B_{\max}$ is very small. Thus, the results from the infinite battery capacity model are good, tractable approximations to handle finite battery capacities.

2) Statistically Non-Identical Channels and Fairness: The average throughput of the proposed multi-EH node transmission scheme when the nodes see statistically non-identical channels is shown in Fig. 6 as a function of the normalized average EH rate. To study the role of the selection rule, also plotted are the average throughputs of: (i) Proportional fair (PF) selection, in which the node with the highest $h_i/\mathbb{E}[h_i]$ is selected [39], (ii) highest channel gain-based selection, and (iii) random selection (sel.). As before, in each of the above schemes, the rate adaptation is optimized for the channel statistics seen by the selected node and to satisfy the corresponding average power constraint. To illustrate the impact of sub-optimal rate selection, the greedy rate scheme that uses the optimal selection rule but transmits with the highest possible rate that its current battery and channel state can support is also plotted. We see that neither PF nor highest channel gain-based selection are optimal. For example, at $\bar{P}_{EH}/(N_0\Omega) = 2$, the optimal scheme outperforms them by 10.3% and 11.4%, respectively, and random selection by 40.3%.

[4]Given the distance between the transmitter and receiver, path loss model, bandwidth, and operating voltage, this can be translated into energy and storage capacity. For example, using the path loss model in [9, Ch. 2.6], for $\Omega =$ 1 MHz, $P_b = 0.001$, carrier frequency of 2.4 GHz, path loss exponent of 3.7, room temperature of 300 K, noise figure of 10 dB, voltage of 3 V, reference distance $d_0 = 10$ m, and distance between the EH node and sink of 100 m, transmitting 1000 bits using 64-QAM over a channel with unit gain consumes 66.9 μJ of energy and requires a battery capacity of 14.9 μF.

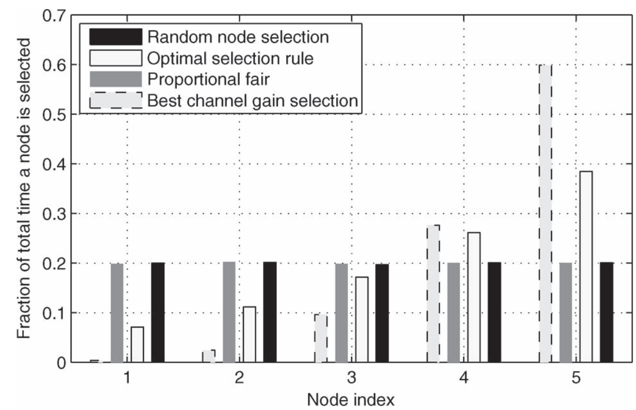

Fig. 7. Fairness comparison of different policies ($K = 5$ nodes with mean channel gains 0.5, 1, 2, 4, and 8).

Since our focus has been on maximizing the throughput, the proposed scheme need not be fair in terms of granting access to the different EH nodes. In order to delve deeper into this aspect, Fig. 7 plots the fraction of time different EH nodes get to transmit for the proposed, highest channel gain-based selection, PF, and random selection schemes. We see that the proposed policy is not as fair as PF or random node selection, but it is not as unfair as highest channel gain-based selection.

VI. Conclusion

We proposed rate and power adaptation policies for single and multi-EH node systems with selection, and proved them to be throughput-optimal for a general class of stationary and ergodic energy harvesting and channel fading processes. The results hold even when the time scales of the EH and channel fading processes are different. We also saw that the optimal selection rule selects a node not just on the basis of its channel condition but also its power control parameter. Consequently, both the highest channel gain-based and PF selection rules can be sub-optimal. We also saw that effect of energy overheads and non-idealities such as battery storage inefficiencies, idle circuit energy consumption, and battery leakage currents can be captured by a single parameter that modifies the transmission policy.

Several interesting avenues for future work exist given the importance of discrete rate adaptation in wireless communications. These include modeling imperfect CSI, considering multi-hop networks, incorporating fairness constraints, and allowing for more advanced receiver designs. Another interesting extension is to allow simultaneous transmissions over N orthogonal channels.

Appendix

A. Proof of Theorem 1

It is easy to see that the average throughput achieved by an EH node that harvests energy at an average rate of $\bar{P}_{EH}$ is upper bounded by $\Psi(\bar{P}_{EH})$, since the latter is the optimal average throughput of a system that is subject to a weaker constraint. Hence, $\lim_{n\to\infty} \overline{R}_{EH}(\bar{P}_{EH}-\delta,n) \leq \Psi(\bar{P}_{EH}-\delta) < \Psi(\bar{P}_{EH})$, which proves the right side of the inequality in (9).

The proof of the left side inequality of (9) formalizes the following intuition using the Birkhoff-Khinchin ergodic theorem [36]. By targeting an average power consumption of $\bar{P}_{EH}-\delta$, which is marginally less than $\bar{P}_{EH}$, the node's battery will eventually have sufficient energy to support transmission at any rate that is required by the transmission rule. Hence, for a sufficiently large n, the EH node's average throughput depends only on the channel gain, and will approach that of a non-EH node that is subject to an average power constraint of $\bar{P}_{EH}-\delta$. Most importantly, as we shall show below, the reduction of power consumption is such that it causes a correspondingly negligible reduction in average throughput.

Let $R_{nonEH}(\bar{P}_{EH},k)$ denote the rate in time interval k of a non-EH node subject to an average power constraint of $\bar{P}_{EH}$. Thus, its cumulative average throughput $\overline{R}_{nonEH}(\bar{P}_{EH},n)$ is $\overline{R}_{nonEH}(\bar{P}_{EH},n) = (\sum_{k=1}^{n} R_{nonEH}(\bar{P}_{EH},k))/n$.

Proposition 1: For the proposed transmission scheme,

$$\lim_{n\to\infty} \overline{R}_{EH}(\bar{P}_{EH}-\delta,n) \overset{\text{as}}{=} \lim_{n\to\infty} \overline{R}_{nonEH}(\bar{P}_{EH}-\delta,n). \quad (20)$$

Proof: Let $X(k) \triangleq D(k) - U(k)$ and $Y(k) \triangleq D(k) - P_{req}(h(k),\mu(k))$, where $\mu(k) = m_j$, if $\eta\zeta_j \leq h(k) < \eta\zeta_{j+1}$, and η is chosen as per (7) to meet an average power constraint of $\bar{P}_{EH}-\delta$. The EH transmission policy is such that $U(k) = P_{req}(h(k),\mu(k))$, if $B(k) \geq T_t P_{req}(h(k),\mu(k))$, and is zero, otherwise. Therefore, $X(k) \geq Y(k)$. Substituting this in (4), we get

$$B(n+1) = B(1) + T_t \sum_{k=1}^{n} X(k) \geq B(1) + nT_t \left(\frac{1}{n} \sum_{k=1}^{n} Y(k) \right).$$

For a fixed η, $P_{req}(h(k),\mu(k))$ is a function of $h(k)$ only and is an integrable function of $h(k)$, which is stationary and ergodic. Therefore, from the Birkhoff-Khinchin ergodic theorem, it follows that

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} P_{req}(h(k),\mu(k)) \overset{\text{as}}{=} \mathbb{E}[P_{req}(h(n),\mu(n))] = \bar{P}_{EH}-\delta. \quad (21)$$

Since $\{D(k); k \geq 1\}$ is ergodic and stationary with mean $\bar{P}_{EH}$, it follows from (21) and the definition of $Y(k)$ that $\lim_{n\to\infty}(\sum_{k=1}^{n} Y(k))/n \overset{\text{as}}{=} \delta$.

Hence, from (21), $\lim_{n\to\infty} B(n+1) \overset{\text{as}}{\geq} B(1) + \lim_{n\to\infty} nT_t\delta$. We also know from (2) that $P_{req}(h(k),\mu(k))$ is finite. Consequently, $\lim_{k\to\infty}[B(k) - P_{req}(h(k),\mu(k))] \overset{\text{as}}{>} 0$. Hence, for a large enough k, the battery has enough energy to support transmission at the same rate as a non-EH node almost surely. Thus,

$$\lim_{k\to\infty} R_{EH}(\bar{P}_{EH}-\delta,k) \overset{\text{as}}{=} \lim_{k\to\infty} R_{nonEH}(\bar{P}_{EH}-\delta,k). \quad (22)$$

The desired result in (20), which deals with the cumulative averages of the above terms $R_{EH}(\bar{P}_{EH}-\delta,k)$ and $R_{nonEH}(\bar{P}_{EH}-\delta,k)$, then follows easily. ■

Since $R_{nonEH}(\bar{P}_{EH}-\delta,k)$ is an integrable function of $h(k)$, which is a stationary and ergodic random process, it follows from the Birkhoff-Khinchin theorem that the cumulative average of $R_{nonEH}(\bar{P}_{EH}-\delta,k)$ tends to its ensemble average $\Psi(\bar{P}_{EH}-\delta)$ almost surely, i.e.,

$$\lim_{n\to\infty} \overline{R}_{nonEH}(\bar{P}_{EH}-\delta,n) \overset{\text{as}}{=} \Psi(\bar{P}_{EH}-\delta). \quad (23)$$

From (7), it can be shown that $\bar{P}_{EH}$ is a continuous and monotonic function of η. Thus, its inverse is a continuous function, i.e., η is a continuous function of $\bar{P}_{EH}$. The optimal average throughput can be written in terms of η as $\Psi(\bar{P}_{EH}) = \sum_{j=2}^{M} r_j[F(\eta\zeta_{j+1}) - F(\eta\zeta_j)]$. Since the CDF of the channel gain is a continuous function, $\Psi(\bar{P}_{EH})$ is a continuous function in η, which, in turn, is a continuous function of $\bar{P}_{EH}$. Hence, $\Psi(\bar{P}_{EH})$ is a continuous function of $\bar{P}_{EH}$. Therefore, given an $\epsilon > 0$, there exists a $\delta > 0$ such that $\Psi(\bar{P}_{EH}-\delta) > \Psi(\bar{P}_{EH}) - \epsilon$. Combining this with (23) and Prop. 1 yields the desired result.

B. Proof of Corollary 1

For small ϵ, let η and $\eta+\epsilon$ correspond to average power constraints of $\bar{P}_{EH}$ and $\bar{P}_{EH}-\alpha$, respectively. Thus, the power consumed changes by $\alpha \approx -\epsilon(d\bar{P}_{EH}(\eta)/d\eta)$ and the average throughput changes by $\epsilon(dR(\eta)/d\eta)$. Differentiating (7), we get

$$\alpha = -\epsilon \frac{d\bar{P}_{EH}(\eta)}{d\eta} = -\frac{\epsilon}{\eta} \sum_{j=2}^{M} f_H(\eta\zeta_j)(d_j - d_{j-1}). \quad (24)$$

Furthermore, we can show that

$$\Psi(\bar{P}_{EH}) = \log_2(m_M) - \sum_{j=2}^{M} F(\eta\zeta_j) \log_2\left(\frac{m_j}{m_{j-1}}\right). \quad (25)$$

Therefore, $\epsilon(dR(\eta)/d\eta) = -\epsilon\sum_{j=2}^{M}\zeta_j f_H(\eta\zeta_j)\log_2(m_j/m_{j-1})$. Combining this with (24) and the fact that $\zeta_j \log_2(m_j/m_{j-1})N_0\Omega = d_j - d_{j-1}$, for $j = 2,\ldots,M$ (from (3) and (6)), yields (12). It is valid for $\epsilon \ll 1$, which, from (24), is equivalent to $\eta\alpha/(\sum_{j=2}^{M} d_j[f_H(\eta\zeta_j) - f_H(\eta\zeta_{j+1})]) \ll 1$.

C. Proof of Theorem 2

Let $\boldsymbol{\omega} = (\omega_1,\omega_2,\ldots,\omega_K)$ denote a rate vector, where $\omega_j \in \{r_1,\ldots,r_M\}$ is the rate of node j. In our problem, since only one node is selected for transmission, only one entry in $\boldsymbol{\omega}$, which corresponds to the selected node, can be non-zero at any time. Let the ith entry of $\boldsymbol{\omega}$, which is ω_i, be non-zero and let it be equal to r_j. Therefore, the constellation size used is $m_j = 2^{\omega_i} = 2^{r_j}$. From (2), we have $P_{req}(h_i, 2^{\omega_i}) = d_j/h_i = N_0\Omega\kappa((2^{r_j})^{c_3} - c_4)/h_i$.

Hence, all the possible rate vectors are $\boldsymbol{\omega}_{11},\ldots,\boldsymbol{\omega}_{1M}$, $\boldsymbol{\omega}_{21},\ldots,\boldsymbol{\omega}_{2M},\ldots,\boldsymbol{\omega}_{K1},\ldots,\boldsymbol{\omega}_{KM}$, where $\boldsymbol{\omega}_{ij} = (0,\ldots,0,r_j,0,\ldots,0)$. Here, i is the position (node index) of the non-zero element in the rate vector and r_j is its value (rate). The set of all channel gain vectors seen by the K nodes is $(\mathbb{R}^+)^K$. Let Υ_{ij} denote the set of all channel gain vectors in $(\mathbb{R}^+)^K$ in which the rate vector $\boldsymbol{\omega}_{ij}$ is used. Therefore, the optimization problem in (13) reduces to choosing the disjoint regions Υ_{ij}, for $1 \leq i \leq K$ and $1 \leq j \leq M$. It can be stated in terms of Υ_{ij} and the PDF $p(\mathbf{h})$ of the channel gain vector $\mathbf{h} = (h_1, \ldots, h_K)$ as follows:

$$\max_{\substack{\Upsilon_{ij} \\ 1\leq i\leq K,\ 1\leq j\leq M}} \quad \sum_{j=1}^{M} r_j \sum_{i=1}^{K} \int_{\Upsilon_{ij}} p(\mathbf{h})\, d\mathbf{h}, \tag{26}$$

$$\text{s.t.} \quad \sum_{j=1}^{M} \int_{\Upsilon_{ij}} \frac{d_j}{h_i} p(\mathbf{h})\, d\mathbf{h} = \bar{P},\ 1 \leq i \leq K, \tag{27}$$

$$\Upsilon_{i_1 j_1} \cap \Upsilon_{i_2 j_2} = \phi,\ \text{if } i_1 \neq i_2 \text{ or } j_1 \neq j_2, \tag{28}$$

$$\cup_{i,j}\ \Upsilon_{ij} = (\mathbb{R}^{+})^{K}, \tag{29}$$

where ϕ denotes the null set. Here, (26) is the expression for the average throughput written in terms of the probabilities of the disjoint regions. And, the left hand side of (27) is the expression for the average power consumed by node i. This follows from (2) since when node i transmits with rate r_j, it does so with a power of d_j/h_i. The constraints in (28) and (29) ensure that the rate regions are disjoint and that their union covers the entire space of channel realizations.

Solving this problem can be shown to be equivalent to

$$\max_{\substack{\Upsilon_{ij} \\ 1\leq i\leq K,\ 1\leq j\leq M}} \sum_{i=1}^{K}\sum_{j=1}^{M} \int_{\Upsilon_{ij}} \left(r_j - \lambda_i \frac{d_j}{h_i} \right) p(\mathbf{h})\, d\mathbf{h}, \tag{30}$$

subject to the constraints in (28) and (29), where the constants $\lambda_1, \ldots, \lambda_K$ are chosen to satisfy the K equality constraints in (27). The integrand in (30) is maximized when the regions are chosen such that the integrand $r_j - \lambda_i(d_j/h_i)$ is maximized. Therefore, using (3), the optimal values of i and j, which are denoted by i^* and j^*, respectively, are jointly given by

$$(i^*, j^*) = \underset{1\leq i\leq K,\ 1\leq j\leq M}{\arg\max} \left\{ r_j - \frac{\eta_i \kappa \left[(2^{r_j})^{c_3} - c_4 \right]}{h_i} \right\}, \tag{31}$$

where $\eta_i = N_0 \Omega \lambda_i$.

Notice that for any given j, the term $r_j - \eta_i\kappa[(2^{r_j})^{c_3} - c_4]/h_i$ in (31) is maximized by choosing the node with the largest (h_i/η_i). Therefore, the optimal node that should be selected is $i^* = \arg\max_{1\leq i\leq K}\{h_i/\eta_i\}$.

Finally, to maximize (31), the selected node, i^*, should transmit with rate r_{j^*} if

$$r_{j^*} - \frac{\kappa\eta_{i^*} \left[(2^{r_{j^*}})^{c_3} - c_4 \right]}{h_{i^*}} \geq r_k - \frac{\kappa\eta_{i^*} \left[(2^{r_k})^{c_3} - c_4 \right]}{h_i^*},\ \forall\, k \neq j.$$

Rearranging terms, this can be shown to be equivalent to

$$\frac{\kappa\eta_{i^*} \left[(2^{r_{j^*}})^{c_3} - (2^{r_k})^{c_3} \right]}{r_{j^*} - r_k} \leq h_{i^*} \leq \frac{\kappa\eta_{i^*} \left[(2^{r_{j^*}})^{c_3} - (2^{r_l})^{c_3} \right]}{r_{j^*} - r_l},\ \forall k < j^*, l > j^*, \tag{32}$$

which is equivalent to the condition $\eta_{i^*}\zeta_{j^*} \leq h_{i^*} < \eta_{i^*}\zeta_{j^*+1}$, where ζ_js are defined in (6). Hence, the result follows.

D. Brief Proof of Theorem 3

As in Appendix A, the right side inequality in (18) follows easily. To prove the left side inequality, consider an EH system with K EH nodes and another non-EH system with K non-EH nodes that operate in parallel such that an EH node i sees exactly the same sequence of channel gains as its counterpart non-EH node i. In the proposed EH scheme, the selection rule is the same as that for the non-EH system. Therefore, at time k, EH node $s(k)$ gets selected if and only if its counterpart non-EH node, which also has the index $s(k)$, is selected.

Further, notice that the EH transmission rule depends on the battery state of the selected EH node only to the extent the node does not transmit if its current battery state cannot support the transmission power required. Therefore, as in Theorem 1, targeting an average power consumption of $\bar{P}_{EH} - \delta < \bar{P}_{EH}$ can be shown to ensure that an EH node has enough energy in its battery to transmit as its non-EH counterpart almost surely. These imply that

$$\lim_{k\to\infty} R_{EH}(\bar{P}_{EH} - \delta, k) \overset{\text{as}}{=} \lim_{k\to\infty} R_{nonEH}(\bar{P}_{EH} - \delta, k). \tag{33}$$

Therefore, the corresponding cumulative averages, which are defined in Section III, satisfy

$$\lim_{n\to\infty} \overline{R}_{EH}(\bar{P}_{EH} - \delta, n) \overset{\text{as}}{=} \lim_{n\to\infty} \bar{P}_{nonEH}(\bar{P}_{EH} - \delta, n) \overset{\text{as}}{=} \Psi(\bar{P}_{EH} - \delta), \tag{34}$$

where the last equality follows because the channel gains are stationary and ergodic.

It can again be shown that $\Psi(\bar{P}_{EH})$, which is given in (16), is a continuous function in $\bar{P}_{EH}$. Therefore, $\Psi(\bar{P}_{EH} - \delta) > \Psi(\bar{P}_{EH}) - \epsilon$. This combined with (34) yields (18).

E. Proof of Corollary 2

For $1 \leq i \leq K$, we are given that

$$\nu_i \bar{P}_{EH} = \sum_{j=2}^{M} d_j \int_{\zeta_j\eta_i}^{\zeta_{j+1}\eta_i} \frac{f_i(h_i)}{h_i} \left[\prod_{l\neq i} F_l\left(\frac{h_i\eta_l}{\eta_i} \right) \right] dh_i. \tag{35}$$

Let $\tilde{f}(\cdot)$ and $\tilde{F}(\cdot)$ denote the PDF and CDF, respectively, of $\tilde{h}_i \triangleq h_i\nu_i$. Since $f_i(h_i)dh_i = \tilde{f}_i(\tilde{h}_i)d\tilde{h}_i$ and $F_l(h_i\eta_l/\eta_i) = \tilde{F}_l(\tilde{h}_i\tilde{\eta}_l/\tilde{\eta}_i)$, where $\nu_i\eta_i = \tilde{\eta}_i, \forall i$, (35) is equivalent to

$$\bar{P}_{EH} = \sum_{j=2}^{M} d_j \int_{\zeta_j\tilde{\eta}_i}^{\zeta_{j+1}\tilde{\eta}_i} \frac{\tilde{f}_i(\tilde{h}_i)}{\tilde{h}_i} \left[\prod_{l\neq i} \tilde{F}_l\left(\frac{\tilde{h}_i\tilde{\eta}_l}{\tilde{\eta}_i} \right) \right] d\tilde{h}_i.$$

This is nothing but the power constraint equation in a system in which a node i harvests at rate $\bar{P}_{EH}$, has power control parameter $\tilde{\eta}_i = \nu_i\eta_i$, and channel gain of $\tilde{h}_i = h_i\eta_i$.

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Parag S. Khairnar received the bachelor's degree in electronics and telecommunications engineering from Vishwakarma Institute of Technology, Pune, India, in 2007 and the master's degree in electrical communication engineering from the Indian Institute of Science, Bangalore, India, in 2011. He has worked with Oneirix Engineering Research Laboratories in 2007–2008, with Brocade in 2011–2012, and with Broadcom in 2012–2014. He is currently a Senior Digital Signal Processing Engineer with Marvell India Private Ltd., Pune, India. His research interests include rate adaptation and multiple access algorithms for energy harvesting sensor networks.

Neelesh B. Mehta (S'98–M'01–SM'06) received the Bachelor of Technology degree in electronics and communications engineering from the Indian Institute of Technology Madras, Chennai, India, and the M.S. and Ph.D. degrees in electrical engineering from the California Institute of Technology, Pasadena, CA, USA, in 1996, 1997, and 2001, respectively. He is currently an Associate Professor with the Department of Electrical Communication Engineering, Indian Institute of Science (IISc), Bangalore, India. Before joining IISc in 2007, he was a Research Scientist with AT&T Laboratories, Middletown, NJ, USA; Broadcom Corporation, Matawan, NJ, USA; and Mitsubishi Electric Research Laboratories, Cambridge, MA, USA from 2001 to 2007.

His research includes work on multiple access protocols, cellular systems, multiple-input–multiple-output, cooperative communications, energy harvesting networks, and cognitive radio. He was also actively involved in Third Generation Partnership Program (3GPP) Radio Access Network 1 standardization activities from 2003 to 2007. Dr. Mehta is an Editor of the IEEE Wireless Communications Letters and the IEEE Transactions on Communications, and he is an Executive Editor of the IEEE Transactions on Wireless Communications. In 2012–2013, he served as the Director of Conference Publications on the Board of Governors of the IEEE Communications Society, where he currently serves as a Member-at-Large.