Direct Link-Aware Optimal Relay Selection and a Low Feedback Variant for Underlay CR

Priyanka Das, *Student Member, IEEE*, and Neelesh B. Mehta, *Senior Member, IEEE*

***Abstract*—Cooperative relaying combined with selection has been extensively studied in the literature to improve the performance of interference-constrained secondary users in underlay cognitive radio (CR). We present a novel symbol error probability (SEP)-optimal amplify-and-forward relay selection rule for an average interference-constrained underlay CR system. A fundamental principle, which is unique to average interference-constrained underlay CR, that the proposed rule brings out is that the choice of the optimal relay is affected not just by the source-to-relay, relay-to-destination, and relay-to-primary receiver links, which are local to the relay, but also by the direct source-to-destination (SD) link, even though it is not local to any relay. We also propose a simpler, practically amenable variant of the optimal rule called the 1-bit rule, which requires just one bit of feedback about the SD link gain to the relays, and incurs a marginal performance loss relative to the optimal rule. We analyze its SEP and develop an insightful asymptotic SEP analysis. The proposed rules markedly outperform several ad hoc SD link-unaware rules proposed in the literature. They also generalize the interference-unconstrained and SD link-unaware optimal rules considered in the literature.**

***Index Terms*—Underlay cognitive radio, interference constraint, cooperative communications, relays, amplify-and-forward, selection, symbol error probability, feedback.**

I. Introduction

Cognitive radio (CR) promises to significantly improve the utilization of scarce wireless spectrum [1]. Different modes have been proposed to access licensed or primary users' (PUs) spectrum by the secondary users (SUs), such as interweave, overlay, and underlay [1]. In the underlay mode of CR, which is the focus of the paper, an SU can simultaneously transmit on the same band as a higher priority PU so long as the interference it causes to the PU is tightly controlled.

Cooperative relaying combined with selection, which is being standardized in next generation wireless systems [2], exploits spatial diversity to mitigate the impact of the interference constraint on underlay CR. In it, a single "best" relay is selected to forward a message from a secondary source (S) to a destination (D) based on the instantaneous channel conditions. Selection avoids the timing synchronization problems among spatially separated simultaneously transmitting relays. In underlay CR, selecting the relay that maximizes the signal-to-noise-ratio (SNR) at D, as has been done conventionally [3], may not be preferable if its transmissions cause excessive interference at the primary receiver (P_{Rx}). Therefore, the relay selection rule is now also a function of the links between the relays and P_{Rx}.

A. Literature on Relay Selection (RS) Rules in Underlay CR

Interference-aware amplify-and-forward (AF) RS has been extensively studied in the literature on underlay CR. We now explain these RS rules, which will motivate the model and results in this paper. We employ the following notation for a system with L relays. The complex baseband channel gain from S to D is denoted by h_{SD}, from S to relay i by h_{Si}, from relay i to D by h_{iD}, from relay i to P_{Rx} by h_{iP}, and from S to P_{Rx} by h_{SP}. Let β denote the selected relay.

Without any interference constraints, as mentioned, the conventional RS rule is simply $\beta = \arg\max_{i\in\{1,2,\dots,L\}} \gamma_i$, where γ_i is the SNR at D if relay i transmits [3]. In [4], the source and relay are subject to constraints on the peak interference power and the peak transmit power, and the RS rule used is $\beta = \arg\max_{i\in\{1,2,\dots,L\}} \min\{P_t|h_{Si}|^2, P_i|h_{iD}|^2\}$, where $P_t = \min\{I_p/|h_{SP}|^2, P_p\}$ and $P_i = \min\{I_p/|h_{iP}|^2, P_p\}$ are the transmit powers of S and relay i, respectively, and I_p and P_p are the interference power and transmit power thresholds, respectively. The RS rule in [5] is the same as that in [4] except that the source and relay are subject to the interference power constraint only. We shall refer to the RS rules in [4], [5] as *variable-power max-min rules*. In [6], [7], the AF relay with the maximum end-to-end SNR at D is selected. Another rule that selects the relay with the maximum source-to-relay (SR) link SNR is also proposed in [7]. A joint power allocation and RS algorithm is proposed in [8] in order to maximize the secondary system throughput. The RS rule in [9] also takes into account the interference at the relays due to transmissions by the primary transmitter (P_{Tx}), and selects the relay with the maximum signal-to-interference-plus-noise-ratio (SINR) at D.

Pruning-based RS rules are instead pursued in [10]–[12], where fixed-power relays that do not satisfy the peak interference constraint are first excluded. Among the remaining relays, the one that maximizes $\min\{P_t|h_{Si}|^2, P_r|h_{iD}|^2\}$ is selected in [10] and the one that maximizes $(\min\{P_t|h_{Si}|^2, P_r|h_{iD}|^2\} - P_r|h_{iP}|^2)$ is selected in [11], where P_r is the relay transmit power. Instead, in [12], even relays for whom the minimum

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The authors are with the Department of Electrical Communication Engineering (ECE), Indian Institute of Science (IISc), Bangalore 560012, India (e-mail: daspriyanka994@gmail.com; neeleshbmehta@gmail.com).

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of the SR and relay-to-destination (RD) link SNRs is below a threshold are excluded. Among the remaining relays, the one that maximizes $\min\{P_t|h_{Si}|^2, P_r|h_{iD}|^2\}/(P_r|h_{iP}|^2)$ is selected. We shall refer to the RS rules in [10]–[12] as *max-min*, *low-interference*, and *quotient rules*, respectively.

In [13], the transmit powers of the source and AF relays are adjusted such that the secondary outage probability is minimized subject to an average interference power constraint. Instead, [14] focuses on maximizing the average rate of the secondary network, and proposes a transmit power allocation strategy for the source and AF relays, which are subject to the average interference constraint. However, in both [13], [14], the conventional RS rule that selects the relay with the highest end-to-end SINR at D is employed.

In [15], the transmit powers of the source and the selected relay are adjusted such that the outage probability of the primary transmissions lies below a threshold. While such a constraint is best suited to protect the primary, it requires the channel state information (CSI) of the P_{Tx} to P_{Rx} link at S, which can be impractical for many CR systems. We, therefore, do not discuss this constraint further.

B. Focus and Contributions

An important principle that we bring out in this paper is that for the average interference constraint, the choice of the optimal relay is affected not just by the SR, RD, and relay-to-P_{Rx} (RP) links, which are local to the relays, but also by the state of the direct source-to-destination (SD) link, even though it is not local to any relay. We explore its implications on optimal RS, its performance, and its practicability. To the best of our knowledge, this important aspect has not been studied in the extensive literature on RS for underlay CR. We make the following specific contributions:

- We present a fully SD-aware optimal RS rule that minimizes the symbol error probability (SEP) of an average interference-constrained underlay secondary system with AF relays. Given its optimality, it serves as a fundamental benchmark for RS in underlay CR. It shows how the probability that no relay gets selected monotonically increases as the SINR of the SD link increases.
- Next, we address a practical challenge associated with the above optimal rule, which is that the SINR of the SD link needs to be broadcast to all the relays since it affects how suitable a relay is for selection. This has been referred to as the metric of a relay in the literature on distributed RS algorithms [16]. We then develop a simpler, novel 1-*bit rule*, which requires just one bit of feedback about the state of the SD link to all the relays. The bit informs them whether the SINR of the SD link is greater than or less than a threshold γ_{th}. The rule ensures low feedback, is tractable, and yet performs close to the optimal RS rule over a wide range of SINRs. We also optimize γ_{th}.
- Both the above rules differ from the aforementioned ad hoc rules [4], [5], [10]–[12], which do not consider the SD link for RS.[1] Moreover, both rules are generalizations of the conventional interference-unconstrained optimal RS rule proposed in [3] because in the absence of the interference constraint, they reduce to the latter. Furthermore, in the absence of the SD link, which happens, for example, when there is an obstruction between S and D, the optimal rule reduces to the rule proposed in [22].
- We present extensive numerical results to characterize the effect of various system parameters and benchmark the performance of the proposed rules. We show that the proposed rules reduce the SEP up to two orders of magnitude compared to the aforementioned rules [4], [5], [10]–[12], [22].
- We analyze the SEP of the 1-bit rule. In addition to verifying the simulation results, it provides valuable insights about the proposed scheme and begets an elegant asymptotic formulation that helps determine the optimal value of γ_{th}. Since the state of the SD link now affects the RS rule, the SEP analysis of the 1-bit rule is more involved than for the SD-unaware rules proposed in [5], [12], [22]. Further, the issues of optimizing γ_{th} and developing an analytical approach to easily determine it do not arise in [5], [12], [22].

We note that this paper presents a significant advancement over [22], which also considered optimal RS for underlay CR. Firstly, the important fact that the SD link affects the choice of the optimal relay is not considered in [22]. Consequently, the optimal rule proposed in this paper is a non-obvious generalization of that in [22]. Furthermore, the development of the 1-bit rule, its SEP analysis, and optimization of γ_{th} are new compared to [22], in which the issue of the feedback of the state of the SD link never arose. We note that the proof techniques used to arrive at the optimal RS rules in this paper and [22] bear similarities. However, this is obvious only in hindsight and does not undermine the novelty, analysis, and significance of the more involved proposed rules. As our benchmarking shows, the proposed rules reduce the SEPs up to two orders of magnitude compared to those in [22], which demonstrates the significance of our approach.

Outline: Section II develops the system model. The optimal rule and its low feedback variant are developed in Section III. The SEP analysis of the 1-bit rule and its asymptotic simplifications are presented in Section IV. Numerical results and benchmarking are presented in Section V. Our conclusions follow in Section VI.

Notation: The absolute value of a complex number x is denoted by $|x|$. The probability of an event A and the conditional

[1] Another instance where the SD link affects whether a relay transmits or not is incremental relaying (IR) [17]–[19]. Variants of IR that switch between relay-aided and direct transmission are proposed in [20], [21]. However, IR is motivated by the fact that using a relay requires two transmissions while a direct transmission to the destination requires only one transmission. On the other hand, in our set up, when the interference constraint becomes inactive, the dependence of the proposed rules on the SD link automatically disappears, which shows the unique influence of the interference constraint. Another difference is that in IR, no relay is selected if the SINR of the SD link is greater than a threshold, and the relay with the maximum end-to-end SINR is selected when the SINR of the SD link falls below the threshold. However, as we shall see, this is not the case for our proposed rules.

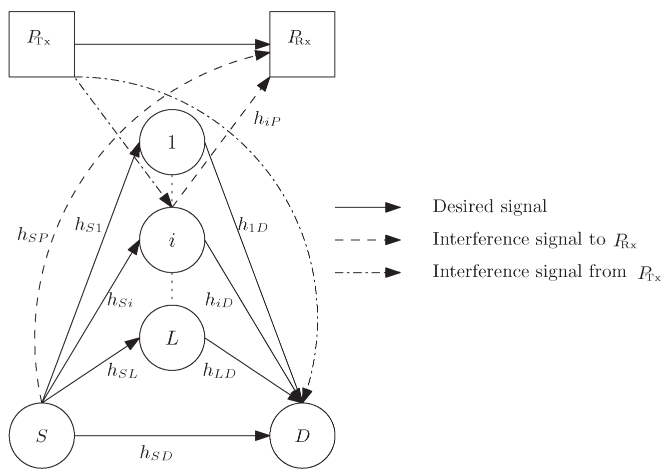

Fig. 1. An underlay CR system with a primary transmitter P_{Tx}, a primary receiver P_{Rx}, a secondary source S, a secondary destination D, and L secondary relays $1, \ldots, L$.

probability of A given B are denoted by $\Pr(A)$ and $\Pr(A|B)$, respectively. $\mathbb{E}_X[.]$ denotes the expectation with respect to a random variable (RV) X; the subscript is dropped if it is obvious from the context. $X \sim CN(0, \sigma^2)$ means that X is a circularly symmetric zero-mean complex Gaussian RV with variance σ^2, and $1_{\{a\}}$ denotes the indicator function; it is 1 if a is true and is 0 otherwise. $\text{E}_k(x) \triangleq \int_1^\infty e^{-xt}/t^k \, dt$ denotes the generalized exponential integral function [23, (5.1.4)], ${}_2F_1(a, b; c; z)$ denotes the Gauss hypergeometric function [23, (15.1)], $Q(x)$ denotes the Gaussian probability function [23, (26.2.3)], and $T(h, a) \triangleq \frac{1}{2\pi} \int_0^a \left(1 + x^2\right)^{-1} e^{-\frac{h^2}{2}(1+x^2)} \, dx$ denotes the Owen's T function [24], which arises in multi-variate statistics.

II. System Model and Problem Statement

As shown in Fig. 1, our system comprises of a primary network, in which a primary transmitter P_{Tx} sends data to a primary receiver P_{Rx}, and an underlay secondary network, in which a source S transmits to a destination D using L relays $1, 2, \ldots, L$. Each node is equipped with a single antenna. The complex baseband channel gain from S to P_{Rx} is h_{SP}, from S to D is h_{SD}, from S to relay i is h_{Si}, from relay i to D is h_{iD}, and from relay i to P_{Rx} is h_{iP}. Let $\mathbf{h}_S \triangleq [h_{S1}, h_{S2}, \ldots, h_{SL}]$, $\mathbf{h}_D \triangleq [h_{1D}, h_{2D}, \ldots, h_{LD}]$, $\mathbf{h}_P \triangleq [h_{1P}, h_{2P}, \ldots, h_{LP}]$, and $\mathbf{h} \triangleq [\mathbf{h}_S, \mathbf{h}_D, \mathbf{h}_P]$. All channels undergo frequency-flat, block fading, and remain constant over the duration of at least two transmissions.

A. Relay Selection

An RS rule selects one out of the L relays or decides that no relay should transmit depending on the instantaneous channel conditions. For notational simplicity, we denote the case where no relay transmits by a virtual relay 0 with $h_{S0} = h_{0D} = h_{0P} \triangleq 0$. Therefore, a fully SD-aware RS rule ϕ is a mapping:

$$\phi : \mathbb{R}^+ \times \left(\mathbb{R}^+\right)^L \times \left(\mathbb{R}^+\right)^L \times \left(\mathbb{R}^+\right)^L \to \{0, 1, \ldots, L\}, \quad (1)$$

that selects one out of the $L + 1$ relays for every realization of $|h_{SD}|^2$, $\left\{|h_{Si}|^2\right\}_{i=1}^L$, $\left\{|h_{iD}|^2\right\}_{i=1}^L$, and $\left\{|h_{iP}|^2\right\}_{i=1}^L$.

B. Data Transmission: Preliminaries

The data transmission occurs over two time slots. In the first time slot, S transmits a data symbol x that is drawn with equal probability from a constellation of size M. The received signals y_{Si} at the relay i and y_{SD} at D are given by

$$y_{Si} = \sqrt{P_t} h_{Si} x + n_i + w_i, \quad 1 \le i \le L, \quad (2)$$

$$y_{SD} = \sqrt{P_t} h_{SD} x + n_D + w_D, \quad (3)$$

where P_t is the source transmit power and $\mathbb{E}\left[|x|^2\right] = 1$. The noises at relay i and D are $n_i \sim CN\left(0, \sigma_0^2\right)$ and $n_D \sim CN\left(0, \sigma_0^2\right)$, respectively. The interferences at relay i and D due to transmissions by P_{Tx} are w_i and w_D, respectively. These are assumed to be Gaussian, as has also been assumed in [22], [25]. Therefore, $w_i \sim CN\left(0, \sigma_1^2\right)$ and $w_D \sim CN\left(0, \sigma_2^2\right)$. The assumption is justified with one primary transmitter when it transmits a constant amplitude signal over a Rayleigh fading link [25] and with many primary transmitters, on the basis of the central limit theorem. In general, the assumption corresponds to a worst case model for the interference and ensures mathematical tractability.

In the second time slot, if relay β is selected, it amplifies the signal $y_{S\beta}$ by a factor $\alpha_\beta = \sqrt{\frac{P_r}{P_t|h_{S\beta}|^2 + \sigma_0^2 + \sigma_1^2}}$ [12], so that its transmit power is P_r, and forwards it to D. The source does not transmit in the second time slot. Therefore, the received signal $y_{\beta D}$ at D in the second time slot is given by

$$y_{\beta D} = y_{S\beta} \alpha_\beta h_{\beta D} + n'_D + w'_D, \quad (4)$$

where $n'_D \sim CN\left(0, \sigma_0^2\right)$ is the noise at D and $w'_D \sim CN\left(0, \sigma_2^2\right)$ is the interference from P_{Tx} at D in the second time slot.

The instantaneous end-to-end SINR at D after maximal ratio combining, which is the ML-optimal rule given the above Gaussian interference assumption, is given by [10]

$$\gamma_\beta = \frac{\gamma_{S\beta} \gamma_{\beta D}}{\gamma_{S\beta} + \gamma_{\beta D} + 1}, \quad (5)$$

where $\gamma_{S\beta} = \frac{P_t|h_{S\beta}|^2}{\sigma_0^2 + \sigma_1^2}$ and $\gamma_{\beta D} = \frac{P_r|h_{\beta D}|^2}{\sigma_0^2 + \sigma_2^2}$ are the SINRs of the first and second hops, respectively.

C. CSI Assumptions and Acquisition

Relay i is assumed to know the instantaneous channel power gains of its local links, i.e., $|h_{Si}|^2$, $|h_{iD}|^2$, and $|h_{iP}|^2$. The relay can use a band manager [26] or can exploit channel reciprocity to estimate $|h_{iP}|^2$ by periodically sensing the transmitted signal from P_{Rx} whenever it is engaged in a two-way communication with P_{Tx}; see [27] and the references therein for a discussion of various channel estimation techniques. Similarly, D is assumed to know the baseband channel gains h_{SD}, $h_{S\beta}$, and $h_{\beta D}$ to enable coherent demodulation [28], [29]. This CSI can be acquired by a training protocol [30] and by exploiting reciprocity, and has been assumed in [4], [5], [9]–[12]. The channel statistics-based parameters σ_1^2 and σ_2^2, which change over a time scale that is several orders of magnitude larger than the instantaneous channel gains, can also be estimated by the selected relay and the destination, respectively [15].

D. Comments About Model and Alternate Formulations

In the following, we discuss our modeling assumptions and present possible alternate formulations, which are interesting avenues for future work. (i) We focus on the classical AF relaying protocol, because it has attracted considerable interest in both conventional cooperation [3], [31] and underlay CR [9], [10], [12]. Other cooperative protocols such as non-orthogonal relaying [32] and incremental relaying [17], which achieve higher spectral efficiencies at the expense of a more involved receiver, are beyond the scope of this paper. (ii) We use the fixed-power relay model given its extensive use in the literature [10]–[12] and because it enables the use of energy-efficient transmit power amplifiers at the relays. (iii) We study the average interference constraint because it is less restrictive than the conservative peak interference constraint, and is well motivated when the packet transmitted by P_{Tx} spans multiple channel coherence times. It has also been studied in the CR literature [13], [14], [22]. (iv) We assume that each node has one antenna because reducing the hardware complexity and cost of multiple-input multiple-output (MIMO) systems is one of the motivations for cooperative relaying. As we shall see, our problem formulation and solution are novel even for the widely studied single antenna model [4], [5], [9]–[12]. (v) We focus on the SEP because it has attracted considerable interest in the relaying literature [5], [31] and leads to a theoretically rich problem and an insightful solution. Alternate problem formulations that maximize the rate or minimize the outage probability are also possible.

E. Optimal RS Rule: Problem Statement

Our goal is to find an optimal RS rule ϕ^* that minimizes the SEP of the secondary system while ensuring that the average interference caused to P_{Rx} is below a threshold I_{avg}.[2] We focus on MPSK. Corresponding SEP-optimal RS rules can be developed for several other constellations such as M-PAM and MQAM [29, (6.1)], and M-DPSK and MFSK [28, (8.1)] whose SEP upper bound is an exponentially decaying function of the SINR. The instantaneous SEP for MPSK at D when relay β is selected is given by [28, (8.23)]

$$\text{SEP}\left(|h_{SD}|^2, |h_{S\beta}|^2, |h_{\beta D}|^2\right) = \frac{1}{\pi}\int_0^{m\pi} e^{-\frac{q(\gamma_{SD}+\gamma_\beta)}{\sin^2\theta}}\, d\theta, \quad (6)$$

where $q = \sin^2(\pi/M)$, $m = 1 - (1/M)$, and $\gamma_{SD} = \frac{P_t|h_{SD}|^2}{\sigma_0^2+\sigma_2^2}$ is the SINR of the SD link.

[2]The interference caused to P_{Rx} due to transmissions by S can also be accounted for in our model as follows. In the *per slot constraint* [14], the average interference in slot 1 due to transmissions by S and that in slot 2 due to transmissions by the selected relay β are considered separately. Here, $P_t\mathbb{E}_{h_{SP}}[|h_{SP}|^2] \le I_{\text{avg}}$ and $\mathbb{E}_{h_{SD},\mathbf{h}}[P_\beta|h_{\beta P}|^2] \le I_{\text{avg}}$. The above formulation then requires $P_t \le \frac{I_{\text{avg}}}{\mathbb{E}_{h_{SP}}[|h_{SP}|^2]}$. In the *slot-averaged constraint*, the average interference resulting from both slots is constrained, i.e., $(P_t\mathbb{E}_{h_{SP}}[|h_{SP}|^2] + \mathbb{E}_{h_{SD},\mathbf{h}}[P_\beta|h_{\beta P}|^2])\,/2 \le I'_{\text{avg}}$. This is equivalent to $\mathbb{E}_{h_{SD},\mathbf{h}}[P_\beta|h_{\beta P}|^2] \le I_{\text{avg}}$, where $I_{\text{avg}} = 2I'_{\text{avg}} - P_t\mathbb{E}_{h_{SP}}[|h_{SP}|^2]$.

Therefore, our problem can be mathematically stated as the following stochastic, constrained optimization problem:

$$\min_{\phi}\ \mathbb{E}_{h_{SD},\mathbf{h}}\left[\text{SEP}\left(|h_{SD}|^2, |h_{S\beta}|^2, |h_{\beta D}|^2\right)\right], \quad (7)$$

$$\text{s.t.}\ \mathbb{E}_{h_{SD},\mathbf{h}}\left[P_\beta|h_{\beta P}|^2\right] \le I_{\text{avg}}, \quad (8)$$

$$\beta = \phi(h_{SD}, \mathbf{h}) \in \{0, 1, \ldots, L\}, \quad (9)$$

where $P_\beta = 0$, for $\beta = 0$, and $P_\beta = P_r$, for $1 \le \beta \le L$. Henceforth, we shall refer to any RS rule that satisfies the constraints in (8) and (9) as a *feasible rule*.

III. Fully SD-Aware Optimal Rule

Let us first consider the conventional RS rule that minimizes the SEP at D when the average interference constraint in (8) is not active. From (6), it selects the relay with the highest end-to-end SINR [3]:

$$\beta = \underset{i\in\{1,2,\ldots,L\}}{\arg\max}\ \{\gamma_i\}. \quad (10)$$

We shall refer to this as the *unconstrained rule*. Let I_{un} denote the average interference caused to P_{Rx} due to the selected relay's transmission by the unconstrained rule. It is given by $I_{\text{un}} = P_r\mathbb{E}_{\mathbf{h}}[|h_{\beta P}|^2]$; note that β also gets averaged over since it is a function of $\mathbf{h}$. However, when $I_{\text{un}} > I_{\text{avg}}$, the unconstrained rule is not feasible, and, thus, cannot be optimal. In general, the optimal RS rule for MPSK is as follows.

Result 1: The selected relay $\beta^* = \phi^*(h_{SD}, \mathbf{h})$, where ϕ^* is an optimal rule, is given by

$$\beta^* = \begin{cases} \arg\max_{i\in\{1,2,\ldots,L\}}\{\gamma_i\}, & I_{\text{un}} \le I_{\text{avg}}, \\ \arg\min_{i\in\{0,1,\ldots,L\}}\left\{\frac{1}{\pi}\int_0^{m\pi} e^{-\frac{q(\gamma_i+\gamma_{SD})}{\sin^2\theta}}\, d\theta + \lambda P_i|h_{iP}|^2\right\}, & \\ & I_{\text{un}} > I_{\text{avg}}, \end{cases} \quad (11)$$

where $P_i = 0$, for $i = 0$, and $P_i = P_r$, for $1 \le i \le L$. Here, λ is a strictly positive constant that arises only if $I_{\text{un}} > I_{\text{avg}}$. In this case, it is chosen such that the average interference constraint is satisfied with equality, and such a choice exists.

Proof: The proof is relegated to Appendix A. □

As the optimal RS rule is a function of γ_{SD}, we shall call it the *fully SD-aware optimal rule*. The constant λ is computed numerically, as is typical in several constrained optimization problems in wireless communications, e.g., optimal rate and power adaption and water-filling [29]. It is a function of the mean channel power gains, and it needs to be computed only once. In general, as I_{avg} decreases, λ increases. We, therefore, treat λ as a system parameter henceforth.

When the RS rule is mandated to not depend on the SD link CSI, i.e., β is independent of γ_{SD}, the fully SD-aware optimal rule reduces to the following *SD-unaware optimal rule* [22]:

$$\beta^* = \begin{cases} \arg\max_{i\in\{1,2,\ldots,L\}}\{\gamma_i\}, & I_{\text{un}} \le I_{\text{avg}}, \\ \arg\min_{i\in\{0,1,\ldots,L\}}\left\{\frac{1}{\pi}\int_0^{m\pi} M_{\gamma_{SD}}\left(\frac{q}{\sin^2\theta}\right) e^{-\frac{q\gamma_i}{\sin^2\theta}}\, d\theta \right. & \\ \left. \qquad + \lambda P_i|h_{iP}|^2\right\}, & I_{\text{un}} > I_{\text{avg}}, \end{cases} \quad (12)$$

where $M_{\gamma_{SD}}(.)$ denotes the moment generating function (MGF) of the RV γ_{SD}. Here, only the statistics of γ_{SD} matter but not its instantaneous value.

A. Simpler, Low Feedback 1-Bit Rule

For the fully SD-aware optimal rule, D needs to broadcast the SINR of the SD link γ_{SD} to all the relays so that the relays can determine their metrics and participate in a distributed selection algorithm that selects the best relay [16]. Another challenge is the presence of the integral in (11), which can be cumbersome to implement in practice.

To reduce the feedback burden, we propose a simpler variant of the fully SD-aware optimal rule called the *1-bit rule*. In it, the selected relay depends only on one bit f, which is 0 if $\gamma_{SD} \leq \gamma_{\text{th}}$ and 1 if $\gamma_{SD} > \gamma_{\text{th}}$. Here, the threshold γ_{th} is a system parameter that we shall optimize later. Therefore, the 1-bit rule $\phi_{1-\text{bit}}$ is a mapping:

$$\phi_{\text{1-bit}}: \{0,1\} \times (\mathbb{R}^+)^L \times (\mathbb{R}^+)^L \times (\mathbb{R}^+)^L \to \{0, 1, \ldots, L\}, \quad (13)$$

that selects one out of the $L+1$ relays for every realization of $f \in \{0,1\}$, $\{|h_{Si}|^2\}_{i=1}^L$, $\{|h_{iD}|^2\}_{i=1}^L$, and $\{|h_{iP}|^2\}_{i=1}^L$.

Starting from the fully SD-aware optimal rule in (11), we develop the 1-bit rule as follows:

- Substituting $\theta = \pi/2$ in the integrand in (6), we get
$$\text{SEP}\left(|h_{SD}|^2, |h_{S\beta}|^2, |h_{\beta D}|^2\right) \leq m e^{-q(\gamma_{SD}+\gamma_\beta)}. \quad (14)$$
- Using the inequality $e^{-x} \leq 1/(1+x)$, for $x \geq 0$, yields
$$\text{SEP}\left(|h_{SD}|^2, |h_{S\beta}|^2, |h_{\beta D}|^2\right) \leq \frac{m e^{-q\gamma_{SD}}}{1+q\gamma_\beta}. \quad (15)$$
- We replace the integral term $\text{SEP}\left(|h_{SD}|^2, |h_{Si}|^2, |h_{iD}|^2\right)$ in (11) with the bound in (15). We also replace γ_{SD} with its expected value conditioned on the feedback, i.e., with $\mathbb{E}[\gamma_{SD}|\gamma_{SD} \leq \gamma_{\text{th}}]$ when $f=0$ and with $\mathbb{E}[\gamma_{SD}|\gamma_{SD} > \gamma_{\text{th}}]$ when $f=1$.

Finally, removing the common constant m, we get the following simpler 1-bit rule:

$$\beta = \begin{cases} \arg\max_{i\in\{1,2,\ldots,L\}}\{\gamma_i\}, & I_{\text{un}} \leq I_{\text{avg}}, \\ \arg\min_{i\in\{0,1,\ldots,L\}}\left\{\frac{e^{-q\mathbb{E}[\gamma_{SD}|\gamma_{SD}\leq\gamma_{\text{th}}]}}{1+q\gamma_i} + \lambda P_i|h_{iP}|^2\right\}, & I_{\text{un}} > I_{\text{avg}} \text{ and } \gamma_{SD} \leq \gamma_{\text{th}}, \\ \arg\min_{i\in\{0,1,\ldots,L\}}\left\{\frac{e^{-q\mathbb{E}[\gamma_{SD}|\gamma_{SD}>\gamma_{\text{th}}]}}{1+q\gamma_i} + \lambda P_i|h_{iP}|^2\right\}, & I_{\text{un}} > I_{\text{avg}} \text{ and } \gamma_{SD} > \gamma_{\text{th}}. \end{cases} \quad (16)$$

For Rayleigh fading, γ_{SD} is an exponential RV with mean $\overline{\gamma}_{SD}$. Therefore, $\mathbb{E}\left[\gamma_{SD}|\gamma_{SD} > \gamma_{\text{th}}\right] = \int_{\gamma_{\text{th}}}^{\infty} \frac{\gamma e^{-\frac{\gamma}{\overline{\gamma}_{SD}}}}{\overline{\gamma}_{SD} e^{-\frac{\gamma_{\text{th}}}{\overline{\gamma}_{SD}}}} d\gamma = \gamma_{\text{th}} + \overline{\gamma}_{SD}$ and $\mathbb{E}\left[\gamma_{SD}|\gamma_{SD} \leq \gamma_{\text{th}}\right] = \int_0^{\gamma_{\text{th}}} \frac{\gamma e^{-\frac{\gamma}{\overline{\gamma}_{SD}}}}{\overline{\gamma}_{SD}\left(1-e^{-\frac{\gamma_{\text{th}}}{\overline{\gamma}_{SD}}}\right)} d\gamma = \left(\overline{\gamma}_{SD} - (\gamma_{\text{th}} + \overline{\gamma}_{SD}) e^{-\frac{\gamma_{\text{th}}}{\overline{\gamma}_{SD}}}\right) \Big/ \left(1 - e^{-\frac{\gamma_{\text{th}}}{\overline{\gamma}_{SD}}}\right)$. As before, $\lambda > 0$ is chosen such that the average interference constraint is satisfied with equality.

The 1-bit rule is easier to implement due to its simpler, integral-free form. Only one bit f needs to be broadcast by D to all the relays instead of γ_{SD}. This enables relay selection to be implemented using a distributed, scalable, low feedback selection scheme such as the timer scheme [16] with D serving as the coordinating node. In it, each relay sets a timer whose value is a monotone non-decreasing function of its metric $\left[e^{-q\mathbb{E}[\gamma_{SD}|\gamma_{SD}\leq\gamma_{\text{th}}]}/(1+q\gamma_i)\right] + \lambda P_i|h_{iP}|^2$, for $f=0$, and $\left[e^{-q\mathbb{E}[\gamma_{SD}|\gamma_{SD}>\gamma_{\text{th}}]}/(1+q\gamma_i)\right] + \lambda P_i|h_{iP}|^2$, for $f=1$. The relay transmits a packet to D containing its identity upon expiry of the timer. The first relay to transmit is automatically the desired best relay to forward the data.

Comments: We note that SD awareness does not matter for RS rules under the peak interference constraint, where it is beneficial to always select a relay having the highest end-to-end SINR. The transmitting relay simply adjusts its transmit power to meet the peak interference constraint [4]–[7], [9]. However, with the average interference constraint, this is not the case. Intuitively, this is because if the SD link is strong, transmissions by a relay will only improve the secondary system's performance incrementally. Then, it is preferable to not select any relay in order to avoid interfering with P_{Rx}. This facilitates subsequent transmissions by a relay when the SD link is weak, which is when the relay is needed the most.

When no SD link is present due to path loss or severe shadowing (i.e., $\gamma_{SD} = 0$), for $I_{\text{un}} > I_{\text{avg}}$, the fully SD-aware optimal rule in (11) reduces to $\beta^* = \arg\min_{i\in\{0,1,\ldots,L\}}\left\{\frac{1}{\pi}\int_0^{m\pi} e^{-\frac{q\gamma_i}{\sin^2\theta}} d\theta + \lambda P_i|h_{iP}|^2\right\}$. Note that the SD-unaware optimal rule in (12) also reduces to the same form. Furthermore, for $I_{\text{un}} > I_{\text{avg}}$, the 1-bit rule in (16) simplifies to $\beta = \arg\min_{i\in\{0,1,\ldots,L\}}\left\{\frac{1}{1+q\gamma_i} + \lambda P_i|h_{iP}|^2\right\}$. All three rules become identical for $I_{\text{un}} \leq I_{\text{avg}}$.

IV. SEP Analysis, Computing λ, and Optimization of Threshold of 1-Bit Rule

We now analyze the SEP of the 1-bit rule and optimize γ_{th}. The SEP analysis of the optimal rule in (11) is intractable due to its involved form.

When $\lambda = 0$, the problem reduces to analyzing the conventional interference-unconstrained RS rule, which is available in [3]. We, therefore, focus on $\lambda > 0$ henceforth, i.e., $I_{\text{un}} > I_{\text{avg}}$. We shall refer to this as the *interference-constrained* regime. In the analysis that follows, we assume that the various links undergo Rayleigh fading and are mutually independent. Thus, for $i = 1, 2, \ldots, L$, $h_{Si} \sim CN(0, \mu_{SR})$, $h_{iD} \sim CN(0, \mu_{RD})$, $h_{iP} \sim CN(0, \mu_{RP})$, and $h_{SD} \sim CN(0, \mu_{SD})$. The 1-bit rule in (16) for $I_{\text{un}} > I_{\text{avg}}$ reduces to

$$\beta = \begin{cases} \arg\min_{i\in\{0,1,\ldots,L\}}\left\{\frac{c_0}{1+q\gamma_i} + \lambda P_i|h_{iP}|^2\right\}, & \gamma_{SD} \leq \gamma_{\text{th}}, \\ \arg\min_{i\in\{0,1,\ldots,L\}}\left\{\frac{c_1}{1+q\gamma_i} + \lambda P_i|h_{iP}|^2\right\}, & \gamma_{SD} > \gamma_{\text{th}}, \end{cases} \quad (17)$$

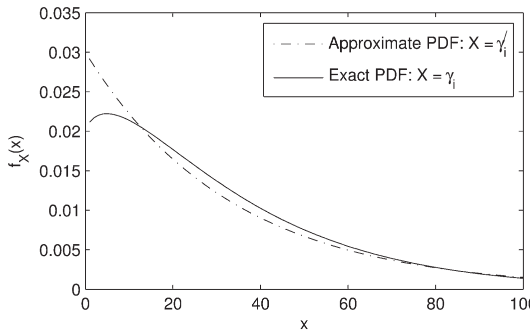

Fig. 2. Comparison of the PDFs of the end-to-end SINR γ_i and its exponential approximation γ_i' ($\overline{\gamma}' = 15.23$ dB) .

where $c_0=\exp\left[-q\left(\overline{\gamma}_{SD}-(\gamma_{\text{th}}+\overline{\gamma}_{SD})e^{-\frac{\gamma_{\text{th}}}{\overline{\gamma}_{SD}}}\right)\Big/\left(1-e^{-\frac{\gamma_{\text{th}}}{\overline{\gamma}_{SD}}}\right)\right]$, $c_1=\exp[-q(\gamma_{\text{th}}+\overline{\gamma}_{SD})]$, and $\overline{\gamma}_{SD}=\mathbb{E}[\gamma_{SD}]=P_t\mu_{SD}\big/\left(\sigma_0^2+\sigma_2^2\right)$.

From (6), the fading-averaged SEP is given by $\text{SEP}=\frac{1}{\pi}\int_0^{m\pi}\mathbb{E}_{h_{SD},\mathbf{h}}\left[e^{-q(\gamma_{SD}+\gamma_\beta)/\sin^2\theta}\right]d\theta$. After considerable simplification, the exact expression for the SEP can at best be reduced to an involved four integral form. We present below an alternate approach that addresses this challenge [22]. It is based on the observation that γ_i can be bounded on both sides by scaling the same exponential RV [31]:

$$\frac{1}{2}\min\{\gamma_{Si},\gamma_{iD}\}\le\gamma_i\le\min\{\gamma_{Si},\gamma_{iD}\}. \quad (18)$$

This motivates us to *approximate* γ_i as an *exponential* RV γ_i'. By the moment-matching method, its mean $\overline{\gamma}'=\mathbb{E}\left[\gamma_i'\right]$ is equal to $\mathbb{E}[\gamma_i]\approx\mathbb{E}\left[\frac{\gamma_{Si}\gamma_{iD}}{\gamma_{Si}+\gamma_{iD}}\right]$. Using the PDF of $\frac{\gamma_{Si}\gamma_{iD}}{\gamma_{Si}+\gamma_{iD}}$, which is derived in [31], it can be shown that

$$\overline{\gamma}'\approx\mathbb{E}\left[\frac{\gamma_{Si}\gamma_{iD}}{\gamma_{Si}+\gamma_{iD}}\right]=\frac{2\sqrt{p}}{15}{}_2F_1\left(3,3;\frac{7}{2};\frac{1}{2}-\frac{\sigma'}{4\sqrt{p}}\right)+\frac{\sigma'}{10}{}_2F_1\left(4,2;\frac{7}{2};\frac{1}{2}-\frac{\sigma'}{4\sqrt{p}}\right), \quad (19)$$

where $p\triangleq\frac{P_tP_r\mu_{SR}\mu_{RD}}{(\sigma_0^2+\sigma_1^2)(\sigma_0^2+\sigma_2^2)}$ and $\sigma'\triangleq\frac{P_t\mu_{SR}}{\sigma_0^2+\sigma_1^2}+\frac{P_r\mu_{RD}}{\sigma_0^2+\sigma_2^2}$.

The PDFs of γ_i and its exponential approximation γ_i' are compared in Fig. 2. We see that while there is a minor mismatch for small values of γ_i (<10), the approximation is accurate for larger γ_i. The veracity of this approximation for the purpose of the SEP analysis will be evaluated in Section V-A.[3] Using the above exponential approximation of γ_i, we get the following expression for the SEP.

Result 2: Given λ, the SEP of the 1-bit rule is given by

$$\text{SEP}^{1\text{-bit}}\approx T_{11}+T_{12}+LT_{21}+LT_{22}, \quad (20)$$

[3]We note that such an approximation has also been used in [22]. However, the SEP analysis in this paper differs from the corresponding analysis in [22] because of the more involved SD-aware nature of the RS rules considered in this paper.

where

$$T_{11}=\phi(c_1,\lambda)\psi(q\gamma_{\text{th}},q\overline{\gamma}_{SD}), \quad (21)$$

$$T_{12}=\phi(c_0,\lambda)\left[\text{SEP}_0-\psi(q\gamma_{\text{th}},q\overline{\gamma}_{SD})\right], \quad (22)$$

$$T_{21}=\frac{c_1e^{\frac{1}{q\overline{\gamma}'}}}{q\overline{\gamma}'}\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{1}{\lambda P_r\mu_{RP}}\right)^{k+1}\left[\sum_{i=1}^{2}b_{i,k}(c_1,\lambda,\theta_i)\times a_i(q\gamma_{\text{th}}-1,q\overline{\gamma}_{SD},\theta_{i-1},\theta_i)+\sum_{i=3}^{N}b_{i-1,k}(c_1,\lambda,\theta_i)\times a_i(q\gamma_{\text{th}}-1,q\overline{\gamma}_{SD},\theta_{i-1},\theta_i)1_{\{M>2\}}\right], \quad (23)$$

$$T_{22}=\frac{c_0e^{\frac{1}{q\overline{\gamma}'}}}{q\overline{\gamma}'}\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{1}{\lambda P_r\mu_{RP}}\right)^{k+1}\left[\sum_{i=1}^{2}b_{i,k}(c_0,\lambda,\theta_i)\times\left(a_i'(q\overline{\gamma}_{SD},\theta_{i-1},\theta_i)-a_i(q\gamma_{\text{th}}-1,q\overline{\gamma}_{SD},\theta_{i-1},\theta_i)\right)+\sum_{i=3}^{N}1_{\{M>2\}}b_{i-1,k}(c_0,\lambda,\theta_i)\left(a_i'(q\overline{\gamma}_{SD},\theta_{i-1},\theta_i)-a_i(q\gamma_{\text{th}}-1,q\overline{\gamma}_{SD},\theta_{i-1},\theta_i)\right)\right], \quad (24)$$

with $\theta_0=0,\theta_1=\pi/4,\theta_2=\pi/2,\theta_3=3\pi/4,\theta_4=m\pi,N=3$ for $M=4$, and $N=4$ for $M>4$,

$$\text{SEP}_0=m-\left(1+\frac{1}{q\overline{\gamma}_{SD}}\right)^{-\frac{1}{2}}\left[\frac{1}{2}+\frac{1}{\pi}\tan^{-1}\left(\sqrt{\frac{1-q}{q+\overline{\gamma}_{SD}^{-1}}}\right)\right],$$

$$\phi(c,\lambda)=\left[\frac{e^{\frac{1}{q\overline{\gamma}'}-\frac{c}{\lambda P_r\mu_{RP}}}}{q\overline{\gamma}'}\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{c}{\lambda P_r\mu_{RP}}\right)^k\mathrm{E}_k\left(\frac{1}{q\overline{\gamma}'}\right)\right]^L,$$

$$\psi(x,y)=e^{-\frac{x}{y}}\left[2T\left(\sqrt{2x},\cot\left(\frac{\pi}{M}\right)\right)+Q\left(\sqrt{2x}\right)\right]-\sqrt{\frac{y}{1+y}}\times\left[2T\left(\sqrt{\frac{2x(1+y)}{y}},\sqrt{\frac{y}{1+y}}\cot\left(\frac{\pi}{M}\right)\right)+Q\left(\sqrt{\frac{2x(1+y)}{y}}\right)\right],$$

$$a_i'(x,\theta_{i-1},\theta_i)=\frac{\theta_i-\theta_{i-1}}{\pi}+\frac{1}{\pi}\sqrt{\frac{x}{1+x}}\times\left[\cot^{-1}\left(\sqrt{\frac{1+x}{x}}\tan\theta_i\right)-\cot^{-1}\left(\sqrt{\frac{1+x}{x}}\tan\theta_{i-1}\right)\right],$$

$$a_i(x,y,\theta_{i-1},\theta_i)=2e^{-\frac{x+1}{y}}\left|T\left(\sqrt{2x},|\cot\theta_{i-1}|\right)-T\left(\sqrt{2x},|\cot\theta_i|\right)\right|-2e^{-\frac{1}{y}}\sqrt{\frac{y}{1+y}}\times\left|T\left(\sqrt{\frac{2x(1+y)}{y}},\sqrt{\frac{y}{1+y}}|\cot\theta_{i-1}|\right)-T\left(\sqrt{\frac{2x(1+y)}{y}},\sqrt{\frac{y}{1+y}}|\cot\theta_i|\right)\right|,$$

and

$$b_{i,k}(c,\lambda,\theta_i) \approx \frac{c}{2}\sum_{l=1}^{W} w_l z_l^{k-1} \mathrm{E}_k\left(\frac{c}{z_l}\left(\csc^2\theta_i + \frac{1}{q\overline{\gamma}'}\right)\right) \times \left[1 - e^{-\frac{1}{q\overline{\gamma}'}\left(\frac{c}{z_l}-1\right)} + \frac{c e^{\frac{1}{q\overline{\gamma}'} - \frac{z_l}{\lambda P_r \mu_{RP}}}}{q\overline{\gamma}'} \times \sum_{k=0}^{\infty} \frac{1}{k!}\frac{z_l^{k-1}}{(\lambda P_r \mu_{RP})^k} \mathrm{E}_k\left(\frac{c}{q\overline{\gamma}' z_l}\right)\right]^{L-1} e^{-\frac{z_l}{\lambda P_r \mu_{RP}}}.$$

Here, $z_l \triangleq c(1+x_l)/2$, and x_l and w_l, $1 \le l \le W$, are the Gauss-Legendre abscissas and weights, respectively [23].

Proof: The proof is relegated to Appendix B. □

In (20), $T_{11}+T_{12}$ is due to the contribution from the direct SD link, and $L(T_{21}+T_{22})$ is due to the contributions from the L relay links. The series in k can be truncated to have $K+1$ terms, where K depends on $1/(\lambda P_r \mu_{RP}) \triangleq \nu$ in order to ensure numerical accuracy. We have found that $K=5$, for $\nu \le 1$, $K = \lfloor \nu + 5 \rfloor$, for $1 < \nu \le 5$, and $K = \lfloor \nu + 10 \rfloor$, for $\nu > 5$ suffice. More terms are needed as ν increases in order to ensure that $\sum_{k=0}^{K} \nu^k/k!$ accurately approximates e^ν. Furthermore, $W=6$ terms are sufficient to accurately compute the SEP over four orders of magnitude. As we shall see in Section V, the above SEP approximation is accurate to within 0.6 dB of the actual SEP. We also note that it is in closed-form and is considerably simpler than evaluating an involved four-dimensional integral.

1) Computing λ: The average interference caused to P_{Rx} due to relay transmissions for the 1-bit rule is given by $I^{1-\mathrm{bit}} = \mathbb{E}_{h_{SD},\mathbf{h}}\left[P_\beta |h_{\beta P}|^2\right] = L\mathbb{E}_{h_{SD},\mathbf{h}}\left[P_r |h_{1P}|^2 \Pr(\beta = 1|h_{SD},\mathbf{h})\right]$. Using the law of total probability and conditioning on whether γ_{SD} exceeds γ_{th} or not, we get

$$I^{1-\mathrm{bit}} = L\mathbb{E}_{h_{SD},\mathbf{h}}\left[P_r |h_{1P}|^2 \Pr(\beta=1|h_{SD},\mathbf{h},\gamma_{SD}>\gamma_{\mathrm{th}}) 1_{\{\gamma_{SD}>\gamma_{\mathrm{th}}\}}\right] + L\mathbb{E}_{h_{SD},\mathbf{h}}\left[P_r |h_{1P}|^2 \Pr(\beta=1|h_{SD},\mathbf{h},\gamma_{SD}\le\gamma_{\mathrm{th}}) 1_{\{\gamma_{SD}\le\gamma_{\mathrm{th}}\}}\right]. \tag{25}$$

Substituting the values of $\Pr(\beta = 1|h_{SD},\mathbf{h},\gamma_{SD} > \gamma_{\mathrm{th}})$ and $\Pr(\beta = 1|h_{SD},\mathbf{h},\gamma_{SD} \le \gamma_{\mathrm{th}})$ from Appendix B into (25), averaging over the channel gains, and simplifying further yields

$$I^{1\text{-bit}} \approx \frac{L e^{\frac{1}{q\overline{\gamma}'}}}{2\lambda q\overline{\gamma}'} \sum_{k=0}^{\infty} \frac{1}{k!}\left(\frac{1}{\lambda P_r \mu_{RP}}\right)^{k+1} \sum_{i=0}^{1} c_i^2 \sum_{l=1}^{W} w_l y_{i,l}^k \times e^{-\frac{y_{i,l}}{\lambda P_r \mu_{RP}}} \left[\mathrm{E}_k\left(\frac{c_i}{q\overline{\gamma}' y_{i,l}}\right) - \mathrm{E}_{k+1}\left(\frac{c_i}{q\overline{\gamma}' y_{i,l}}\right)\right] \times \left[1 - e^{-\frac{1}{q\overline{\gamma}'}\left(\frac{c_i}{y_{i,l}}-1\right)} + \frac{c_i e^{\left(\frac{1}{q\overline{\gamma}'} - \frac{y_{i,l}}{\lambda P_r \mu_{RP}}\right)}}{q\overline{\gamma}'} \times \sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{1}{\lambda P_r \mu_{RP}}\right)^k y_{i,l}^{k-1} \mathrm{E}_k\left(\frac{c_i}{q\overline{\gamma}' y_{i,l}}\right)\right]^{L-1}, \tag{26}$$

where $y_{i,l} \triangleq c_i(1+x_l)/2$, for $0 \le i \le 1$, c_0 and c_1 are given in (17), and x_l and w_l, $1 \le l \le W$, are the Gauss-Legendre abscissas and weights, respectively. We have found that $W=5$ terms are sufficient for the parameters of interest to accurately compute $I^{1\text{-bit}}$. Now, λ is the solution of the equation $I^{1\text{-bit}} = I_{\mathrm{avg}}$; it can be easily computed using routines such as `fsolve` in Matlab.

A. Optimizing γ_{th}

Let γ_{th}^* denote the optimal value of γ_{th} that minimizes the SEP. We see from (20) that $\mathrm{SEP}^{1-\mathrm{bit}}$ is an involved function of γ_{th}, and does not yield much insight about γ_{th}^*. More insights about γ_{th}^* can be obtained by studying the asymptotic regime in which $\mu \to \infty$ when $\mu_{SD} = \mu_{SR} = \mu_{RD} = \mu_{RP} = \mu$.

Result 3: Given $\lambda > 0$, P_t, and P_r, the SEP of the 1-bit rule when $\mu \to \infty$ simplifies to

$$\mathrm{SEP}_{\mathrm{asym}}^{1\text{-bit}} = \frac{\left[1 - \frac{1}{M} + \frac{1}{2\pi}\sin\left(\frac{2\pi}{M}\right)\right](\sigma_0^2 + \sigma_2^2)}{2qP_t\mu} \times \left[e^{-q\gamma_{\mathrm{th}} - \frac{Lc_1}{\lambda P_r \mu}} + (1 - e^{-q\gamma_{\mathrm{th}}}) e^{-\frac{Lc_0}{\lambda P_r \mu}}\right], \tag{27}$$

where c_0 and c_1 are given in (17).

Proof: The proof is relegated to Appendix C. □

Since the SEP in this regime falls as $1/\mu$, the diversity order is unity. The expression clearly brings out how the SEP decreases as λ decreases or L increases or M decreases. Now, γ_{th}^* can be found using a simple one-dimensional numerical search that minimizes the expression in (27). As we shall see from Fig. 7 in Section V-B, the SEP obtained from this approach is within 0.8 dB of the SEP when γ_{th}^* is determined by minimizing the more involved expression in (20).

Further Insights: Since $0 \le c_1 \le c_0 \le 1$, as $\mu \to \infty$, we have $\frac{Lc_1}{\lambda P_r \mu} \ll 1$ and $\frac{Lc_0}{\lambda P_r \mu} < 1$. Therefore, $e^{-\frac{Lc_1}{\lambda P_r \mu}} \approx 1 - \frac{Lc_1}{\lambda P_r \mu}$ and $e^{-\frac{Lc_0}{\lambda P_r \mu}} \approx 1 - \frac{Lc_0}{\lambda P_r \mu}$. Further, when $\frac{\gamma_{\mathrm{th}}}{\overline{\gamma}_{SD}} \ll 1$, it can be shown that $c_0 = e^{-q\gamma_{\mathrm{th}}}$. Substituting the above values of $e^{-\frac{Lc_1}{\lambda P_r \mu}}$, $e^{-\frac{Lc_0}{\lambda P_r \mu}}$, $c_0 = e^{-q\gamma_{\mathrm{th}}}$, and $c_1 = e^{-q(\gamma_{\mathrm{th}} + \overline{\gamma}_{SD})}$ in (27), we get

$$\mathrm{SEP}_{\mathrm{asym}}^{1\text{-bit}} \approx \frac{\left[1 - \frac{1}{M} + \frac{1}{2\pi}\sin\left(\frac{2\pi}{M}\right)\right](\sigma_0^2 + \sigma_2^2)}{2qP_t\mu} \times \left[1 - \frac{L}{\lambda P_r \mu}\left(e^{-q\gamma_{\mathrm{th}}} - e^{-2q\gamma_{\mathrm{th}}}(1 - e^{-q\overline{\gamma}_{SD}})\right)\right]. \tag{28}$$

Differentiating (28) with respect to γ_{th} and equating it to zero, we get the following closed-form expression for γ_{th}^*:

$$\gamma_{\mathrm{th}}^* = \arg\min_{\gamma_{\mathrm{th}}>0}\left\{\mathrm{SEP}_{\mathrm{asym}}^{1\text{-bit}}\right\} = \frac{1}{q}\ln\left(2 - 2e^{-q\overline{\gamma}_{SD}}\right), \approx \frac{1}{q}(\ln 2 - e^{-q\overline{\gamma}_{SD}}). \tag{29}$$

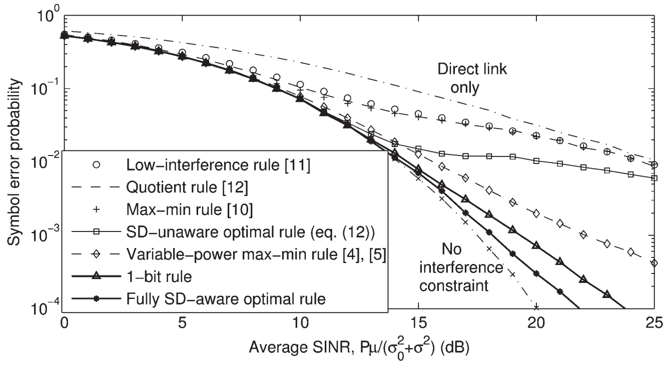

Fig. 3. Comparison of the SEPs of the optimal and 1-bit RS rules with several other RS rules proposed in the literature (8PSK, $L = 4$, and $I_{\text{avg}} = 15$ dB).

V. Numerical Results and Benchmarking

We now present extensive Monte Carlo simulation results to verify our analysis and quantify the benefits of SD-aware RS. For the sake of illustration, we use $P_t = P_r = P = 10$ dB, $\sigma_0^2 = 0$ dB, and $\sigma_1^2 = \sigma_2^2 = \sigma^2 = 3.36$ dB. Therefore, $\sigma_0^2 + \sigma^2 = 5$ dB. Unless mentioned otherwise, we set $\mu_{SD} = \mu_{SR} = \mu_{RD} = \mu_{RP} = \mu$, and vary μ from -5 dB to 20 dB. Thus, the average SINR of the various links, $P\mu \big/ \left(\sigma_0^2 + \sigma^2\right)$, varies from 0 to 25 dB.

A. Comparison and Benchmarking of Proposed Rules

Fig. 3 compares the SEPs of the fully SD-aware optimal rule and 1-bit rule with optimal threshold γ_{th}^* as a function of the average SINR $P\mu \big/ \left(\sigma_0^2 + \sigma^2\right)$. The optimization of γ_{th} is discussed in Section V-B. As a reference, the SEPs of a conventional relay network ($I_{\text{avg}} = \infty$) and a non-cooperative network that uses only the direct SD link ($I_{\text{avg}} = 0$) are also shown. All the SEPs lie between these two curves. When $P\mu \big/ \left(\sigma_0^2 + \sigma^2\right) \le 10$ dB, the network is not interference-constrained. Hence, the SEPs of the 1-bit and optimal rules are the same as that of the unconstrained rule. When $P\mu \big/ \left(\sigma_0^2 + \sigma^2\right) > 10$ dB, the network is interference-constrained and the SEPs of the two rules differ. The optimal rule has the lowest SEP. The SEP of the 1-bit rule is marginally worse for high average SINRs.

Also plotted in the figure are the SEPs of the fixed-power SD-unaware optimal rule in (12), low-interference rule [11], max-min rule [10], and quotient rule [12]. These have been adapted to our model with the interference threshold set as I_{avg}. Also shown is the SEP of the variable-power max-min rule [4], [5], which has been adapted to our model with the peak transmit powers of the source and relays set as P. We see that the two proposed rules outperform all the benchmark rules over the entire range of average SINRs. For example, at an average SINR of 17 dB, the fully SD-aware optimal rule lowers the SEP by a factor of 16.5, 6.0, and 3.0 as compared to the quotient rule, SD-unaware optimal rule, and variable-power max-min rule, respectively. Equivalently, this translates into significant power savings at the source and the relays for the same SEP. Also, the performance gap increases as the average SINR increases. This is because the SEPs of the low-interference, quotient, max-min, and SD-unaware optimal rules approach that of a non-cooperative network in order to satisfy the interference constraint.

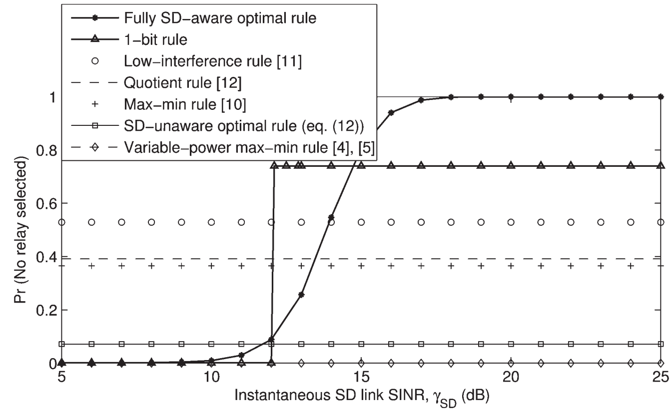

Fig. 4. *Effect of SD link on RS*: Probability of no relay selection as a function of instantaneous SINR of the SD link using different RS rules (8PSK, $L = 2$, $I_{\text{avg}} = 15$ dB, $\mu = 3$ dB, and $\gamma_{\text{th}}^* = 12$ dB).

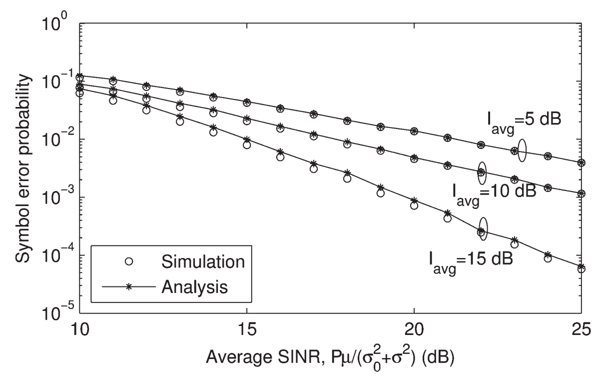

Fig. 5. *Impact of average interference threshold*: SEP of the 1-bit RS rule as a function of average SINR for different values of I_{avg} (8PSK and $L = 4$).

In order to understand the behavior of the proposed rules better, Fig. 4 plots the probability $\Pr(\beta = 0)$ that no relay gets selected as a function of instantaneous SINR γ_{SD} of the SD link for the proposed and benchmark rules. As expected, the probability that no relay gets selected is constant over the entire range of γ_{SD} considered for the low-interference, quotient, max-min, and SD-unaware optimal rules since they do not consider γ_{SD} for RS. Unlike the other rules, in which a relay transmits at a fixed power, in the variable-power max-min rule, a relay is always selected and its transmit power is adjusted to meet the interference constraint. Hence, $\Pr(\beta = 0)$ is zero for it. On the other hand, $\Pr(\beta = 0)$ increases monotonically as γ_{SD} increases for the fully SD-aware optimal rule. Thus, the relays transmit less often when the SD link is stronger. As can be seen from (17), $\Pr(\beta = 0)$ for the 1-bit rule takes only two values, which explains the staircase shape for it. Furthermore, in (17), since $c_1 < c_0$, the probability that no relay gets selected is greater when $\gamma_{SD} > \gamma_{\text{th}}^*$.

Fig. 5 studies the impact of the average interference threshold I_{avg} on the 1-bit rule. We see that the SEP monotonically decreases as the average SINR increases. Further, as I_{avg} increases, SEP decreases since the interference constraint becomes more relaxed. We also see that the SEP approximation in (20) is within 0.6 dB of the SEP from simulations.

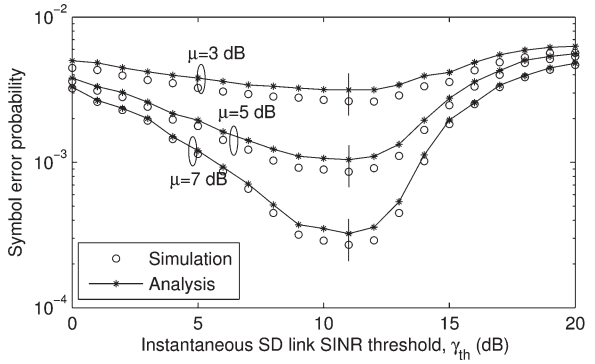

Fig. 6. *Optimizing* γ_{th}: SEP as a function of γ_{th} using 1-bit RS rule for different average channel power gain μ (QPSK, $L = 2$, and $I_{\text{avg}} = 15$ dB).

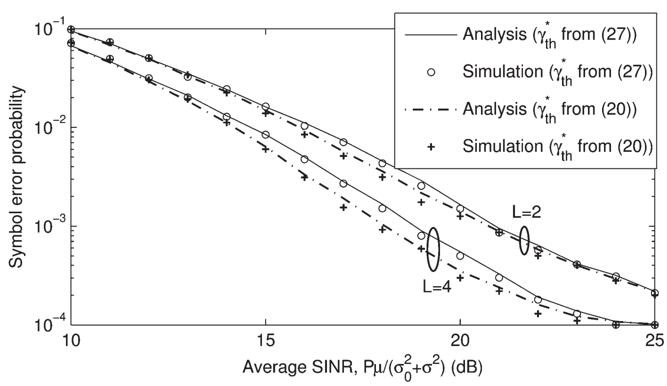

Fig. 7. *Efficacy of the 1-bit rule*: SEPs of the fully SD-aware optimal and 1-bit rules as a function of average SINR for different L (8PSK and $\lambda = 0.0001$).

B. Optimization of Threshold γ_{th} of 1-Bit Rule

We now delve deeper into the problem of optimizing γ_{th} for the 1-bit rule. Fig. 6 plots the SEP from simulations and the formula in (20) as a function of γ_{th} for different values of the average channel power gain μ. As γ_{th} increases, the SEP initially decreases, reaches a minimum at γ_{th}^* (which is indicated by a vertical line), and then starts increasing. The simulation and analytical results both yield the same optimal threshold. The marginal mismatch between the two is due to the use of the exponential approximation for the end-to-end SINR. Furthermore, as μ increases, the SEP decreases.

Fig. 7 plots the SEPs of the 1-bit rule for different numbers of relays when γ_{th}^* is obtained by minimizing the involved SEP expression in (20) and its simpler asymptotic version in (27). We see that the two SEPs are within 0.8 dB and 0.5 dB of each other for $L = 4$ and $L = 2$, respectively. Thus, the simpler asymptotic approach is relevant for a wide range of average SINR. As L increases, the SEP decreases. Also shown are Monte Carlo simulation results, which validate the analysis.

VI. Conclusion

We proposed two novel SD-aware AF relay selection rules for an average interference-constrained underlay CR system. We first derived a general form of the fully SD-aware optimal rule, in which the choice of the relay depended on the SINR of the SD link. We saw that a relay got selected less often when the SD link was stronger. This was in marked contrast to the SD-unaware RS rules proposed in the literature. The 1-bit rule was a simpler and low feedback variant of the fully SD-aware optimal rule, in which the choice of the relay depended on whether or not the SINR of the SD link exceeded a threshold. It incurred only a marginal performance loss compared to the optimal rule, despite its considerably simpler and practically amenable form. Furthermore, the SD-aware RS rules outperformed several SD-unaware RS rules proposed in the literature over a wide range of SINRs.

Appendix

A. Proof of Result 1

When $I_{\text{un}} \le I_{\text{avg}}$, the unconstrained rule in (10) is feasible. This is also the optimal rule. This follows because the SEP in (6) is a monotonically decreasing function of γ_β, and the optimal selected relay β^* is given by

$$\beta^* = \underset{i \in \{1,2,\dots,L\}}{\arg\min} \left\{ e^{-q(\gamma_{SD}+\gamma_i)} \right\} = \underset{i \in \{1,2,\dots,L\}}{\arg\max} \{\gamma_i\}. \tag{30}$$

Now consider the case when $I_{\text{un}} > I_{\text{avg}}$. In this case, the unconstrained rule is not feasible and, thus, cannot be optimal. An RS rule in which no relay transmits causes zero relay interference to P_{Rx} and is, thus, feasible for any I_{avg}. Therefore, the set of all feasible RS rules, $\mathcal{Z}$, is a non-empty set. Let $\phi \in \mathcal{Z}$ be a feasible rule. For a constant $\lambda > 0$, define an auxiliary function $L_\phi(\lambda)$ associated with ϕ as

$$L_\phi(\lambda) \triangleq \mathbb{E}_{h_{SD},\mathbf{h}} \left[\text{SEP}\left(|h_{SD}|^2, |h_{S\beta}|^2, |h_{\beta D}|^2 \right) + \lambda P_\beta |h_{\beta P}|^2 \right]. \tag{31}$$

Note that $L_\phi(\lambda)$ is a function of both ϕ and λ. Further, define a new rule ϕ^* in terms of the relay β^* it selects as follows:

$$\beta^* = \underset{i \in \{0,1,\dots,L\}}{\arg\min} \left\{ \text{SEP}\left(|h_{SD}|^2, |h_{Si}|^2, |h_{iD}|^2 \right) + \lambda P_i |h_{iP}|^2 \right\}, \tag{32}$$

where λ is chosen such that $\mathbb{E}_{h_{SD},\mathbf{h}}\left[P_{\beta^*} |h_{\beta^* P}|^2 \right] = I_{\text{avg}}$.[4] Thus, ϕ^* is a feasible rule.

We now prove that ϕ^* is the desired optimal RS rule. From (32), it follows that $L_{\phi^*}(\lambda) \le L_\phi(\lambda)$. Therefore,

$$\begin{aligned} &\mathbb{E}_{h_{SD},\mathbf{h}} \left[\text{SEP}\left(|h_{SD}|^2, |h_{S\beta^*}|^2, |h_{\beta^* D}|^2 \right) \right] \\ &\quad \le \mathbb{E}_{h_{SD},\mathbf{h}} \left[\text{SEP}\left(|h_{SD}|^2, |h_{S\beta}|^2, |h_{\beta D}|^2 \right) \right] \\ &\qquad + \lambda \left(\mathbb{E}_{h_{SD},\mathbf{h}} \left[P_\beta |h_{\beta P}|^2 \right] - I_{\text{avg}} \right). \end{aligned} \tag{33}$$

Since ϕ is a feasible rule, $\mathbb{E}_{h_{SD},\mathbf{h}}\left[P_\beta |h_{\beta P}|^2 \right] \le I_{\text{avg}}$. Thus,

$$\begin{aligned} &\mathbb{E}_{h_{SD},\mathbf{h}} \left[\text{SEP}\left(|h_{SD}|^2, |h_{S\beta^*}|^2, |h_{\beta^* D}|^2 \right) \right] \\ &\qquad \le \mathbb{E}_{h_{SD},\mathbf{h}} \left[\text{SEP}\left(|h_{SD}|^2, |h_{S\beta}|^2, |h_{\beta D}|^2 \right) \right]. \end{aligned} \tag{34}$$

[4] Such a unique choice of λ exists can be proved using the intermediate value theorem by observing that $0 \le I_{\text{avg}} < I_{\text{un}}$, and proving that the average interference is a continuous and monotonically decreasing function of $\lambda \ge 0$.

Hence, ϕ^* yields the lowest average SEP among all feasible rules. It is, therefore, optimal.

B. SEP Analysis of 1-Bit Rule

As mentioned, $\gamma_i \approx \gamma_i'$, where the latter is an exponential RV. The SEP conditioned on h_{SD} and $\mathbf{h}$, which we denote by $\Pr(\text{Err}|h_{SD}, \mathbf{h})$, can be written as $\Pr(\text{Err}|h_{SD}, \mathbf{h}) = \Pr(\beta = 0, \text{Err}|h_{SD}, \mathbf{h}) + \sum_{i=1}^{L} \Pr(\beta = i, \text{Err}|h_{SD}, \mathbf{h})$. Averaging over h_{SD} and $\mathbf{h}$, and by the chain rule, we get

$$\text{SEP} = \mathbb{E}_{h_{SD},\mathbf{h}}\left[\Pr(\text{Err}|h_{SD}, \mathbf{h})\right] = T_1 + LT_2, \quad (35)$$

where

$$T_1 = \mathbb{E}_{h_{SD},\mathbf{h}}\left[\Pr(\beta = 0|h_{SD}, \mathbf{h}) \Pr(\text{Err}|\beta = 0, h_{SD}, \mathbf{h})\right],$$
$$T_2 = \mathbb{E}_{h_{SD},\mathbf{h}}\left[\Pr(\beta = 1|h_{SD}, \mathbf{h}) \Pr(\text{Err}|\beta = 1, h_{SD}, \mathbf{h})\right].$$

1) Evaluating T_1: Using the law of total probability and conditioning on whether γ_{SD} exceeds γ_{th} or not, we get

$$\begin{aligned} T_1 = \mathbb{E}_{h_{SD},\mathbf{h}}\Big[\big\{&\Pr(\beta = 0|h_{SD}, \mathbf{h}, \gamma_{SD} > \gamma_{\text{th}}) 1_{\{\gamma_{SD}>\gamma_{\text{th}}\}} \\ &+ \Pr(\beta = 0|h_{SD}, \mathbf{h}, \gamma_{SD} \le \gamma_{\text{th}}) 1_{\{\gamma_{SD}\le\gamma_{\text{th}}\}}\big\} \\ &\times \big\{\Pr(\text{Err}|\beta = 0, h_{SD}, \mathbf{h}, \gamma_{SD} > \gamma_{\text{th}}) 1_{\{\gamma_{SD}>\gamma_{\text{th}}\}} \\ &+ \Pr(\text{Err}|\beta = 0, h_{SD}, \mathbf{h}, \gamma_{SD} \le \gamma_{\text{th}}) 1_{\{\gamma_{SD}\le\gamma_{\text{th}}\}}\big\}\Big]. \end{aligned}$$

Since $1_{\{\gamma_{SD}>\gamma_{\text{th}}\}} 1_{\{\gamma_{SD}\le\gamma_{\text{th}}\}} = 0$, we get $T_1 = T_{11} + T_{12}$, where

$$\begin{aligned} T_{11} = \mathbb{E}_{h_{SD},\mathbf{h}}\Big[&\Pr(\beta = 0|h_{SD}, \mathbf{h}, \gamma_{SD} > \gamma_{\text{th}}) \\ &\times \Pr(\text{Err}|\beta = 0, h_{SD}, \mathbf{h}, \gamma_{SD} > \gamma_{\text{th}}) 1_{\{\gamma_{SD}>\gamma_{\text{th}}\}}\Big], \\ T_{12} = \mathbb{E}_{h_{SD},\mathbf{h}}\Big[&\Pr(\beta = 0|h_{SD}, \mathbf{h}, \gamma_{SD} \le \gamma_{\text{th}}) \\ &\times \Pr(\text{Err}|\beta = 0, h_{SD}, \mathbf{h}, \gamma_{SD} \le \gamma_{\text{th}}) 1_{\{\gamma_{SD}\le\gamma_{\text{th}}\}}\Big]. \end{aligned}$$

Evaluating T_{11}: Conditioning on $\gamma_{SD} > \gamma_{\text{th}}$ and h_{SD} is the same as conditioning on $\gamma_{SD} > \gamma_{\text{th}}$ for the 1-bit rule. And, conditioning on $\beta = 0$, the decoding error event is independent of $\gamma_{SD} > \gamma_{\text{th}}$ and $\mathbf{h}$. Therefore, T_{11} reduces to

$$\begin{aligned} T_{11} = \mathbb{E}_{\mathbf{h}}&\left[\Pr(\beta = 0|\mathbf{h}, \gamma_{SD} > \gamma_{\text{th}})\right] \\ &\times \mathbb{E}_{h_{SD}}\left[\Pr(\text{Err}|\beta = 0, h_{SD}) 1_{\{\gamma_{SD}>\gamma_{\text{th}}\}}\right]. \quad (36) \end{aligned}$$

Now, $\mathbb{E}_{\mathbf{h}}\left[\Pr(\beta = 0|\mathbf{h}, \gamma_{SD} > \gamma_{\text{th}})\right] = \Pr(\beta = 0|\gamma_{SD} > \gamma_{\text{th}})$.

Let $g_i = P_r|h_{iP}|^2$ and $y_i = c_1/(1 + q\gamma_i')$, for $1 \le i \le L$. Since $\{\gamma_i'\}_{i=1}^{L}$ are i.i.d. exponential RVs with mean $\overline{\gamma}'$, $\{y_i\}_{i=1}^{L}$ are i.i.d. RVs, whose PDF can be shown to be $f_{y_i}(y) = \frac{c_1}{q\overline{\gamma}' y^2} e^{-\frac{1}{q\overline{\gamma}'}\left(\frac{c_1}{y} - 1\right)}$, for $0 \le y \le c_1$. Then, from (17), we get $\Pr(\beta = 0|\gamma_{SD} > \gamma_{\text{th}}) = \Pr(y_1 + \lambda g_1 > c_1, y_2 + \lambda g_2 > c_1, \ldots, y_L + \lambda g_L > c_1) = \left[\Pr(y_1 + \lambda g_1 > c_1)\right]^L$. Substituting the PDFs of g_1 and y_1, integrating over g_1, and using $t = 1/y_1$, we get

$$\Pr(y_1 + \lambda g_1 > c_1) = \frac{c_1}{q\overline{\gamma}'} e^{\frac{1}{q\overline{\gamma}'} - \frac{c_1}{\lambda P_r \mu_{RP}}} \int_{1/c_1}^{\infty} e^{\frac{1}{\lambda P_r \mu_{RP} t} - \frac{c_1 t}{q\overline{\gamma}'}} \, dt. \quad (37)$$

Expanding $e^{\frac{1}{\lambda P_r \mu_{RP} t}}$ as $\sum_{k=0}^{\infty} \frac{1}{k!}\left(\frac{1}{\lambda P_r \mu_{RP} t}\right)^k$ and interchanging the order of integration and summation, which is justified by the dominated convergence theorem (DCT) [33], the first term of T_{11} is given by $\phi(c_1, \lambda)$, which is defined in the result statement.

Now, from (6), the second product term of T_{11} in (36) is

$$\begin{aligned} &\mathbb{E}_{h_{SD}}\left[\Pr(\text{Err}|\beta = 0, h_{SD}) 1_{\{\gamma_{SD}>\gamma_{\text{th}}\}}\right] \\ &= \frac{1}{\pi \overline{\gamma}_{SD}} \int_0^{m\pi} \int_{\gamma_{\text{th}}}^{\infty} e^{-\left(\frac{q\gamma_{SD}}{\sin^2\theta} + \frac{\gamma_{SD}}{\overline{\gamma}_{SD}}\right)} d\gamma_{SD} d\theta, \\ &= \frac{1}{\pi} \int_0^{m\pi} \left(1 + \frac{q\overline{\gamma}_{SD}}{\sin^2\theta}\right)^{-1} e^{-\frac{\gamma_{\text{th}}}{\overline{\gamma}_{SD}}\left(1 + \frac{q\overline{\gamma}_{SD}}{\sin^2\theta}\right)} d\theta. \quad (38) \end{aligned}$$

Substituting $\cot\theta = x$, using [34, (3.466)], and by the definition of Owen's T function, we get $\mathbb{E}_{h_{SD}}\left[\Pr(\text{Err}|\beta = 0, h_{SD}) 1_{\{\gamma_{SD}>\gamma_{\text{th}}\}}\right] = \psi(q\gamma_{\text{th}}, q\overline{\gamma}_{SD})$, which is defined in the result statement. Thus, $T_{11} = \phi(c_1, \lambda)\psi(q\gamma_{\text{th}}, q\overline{\gamma}_{SD})$.

Similarly, it can be shown that $T_{12} = \phi(c_0, \lambda)\left[\text{SEP}_0 - \psi(q\gamma_{\text{th}}, q\overline{\gamma}_{SD})\right]$. Here, $\text{SEP}_0 = \frac{1}{\pi}\int_0^{m\pi}\left(1 + \frac{q\overline{\gamma}_{SD}}{\sin^2\theta}\right)^{-1} d\theta$, and is given in closed-form in the result statement.

2) Evaluating T_2: We can write $T_2 = T_{21} + T_{22}$, where

$$\begin{aligned} T_{21} = \mathbb{E}_{h_{SD},\mathbf{h}}\Big[&\Pr(\beta = 1|h_{SD}, \mathbf{h}, \gamma_{SD} > \gamma_{\text{th}}) \\ &\times \Pr(\text{Err}|\beta = 1, h_{SD}, \mathbf{h}, \gamma_{SD} > \gamma_{\text{th}}) 1_{\{\gamma_{SD}>\gamma_{\text{th}}\}}\Big], \\ T_{22} = \mathbb{E}_{h_{SD},\mathbf{h}}\Big[&\Pr(\beta = 1|h_{SD}, \mathbf{h}, \gamma_{SD} \le \gamma_{\text{th}}) \\ &\times \Pr(\text{Err}|\beta = 1, h_{SD}, \mathbf{h}, \gamma_{SD} \le \gamma_{\text{th}}) 1_{\{\gamma_{SD}\le\gamma_{\text{th}}\}}\Big]. \end{aligned}$$

Evaluating T_{21}: Conditioning on $\gamma_{SD} > \gamma_{\text{th}}$, the event that relay 1 gets selected ($\beta = 1$) is only a function of $\mathbf{h}$. This combined with the SEP expression in (6) yields

$$T_{21} = \mathbb{E}_{h_{SD},\mathbf{h}}\left[\Pr(\beta = 1|\mathbf{h}, \gamma_{SD} > \gamma_{\text{th}}) \times \frac{1}{\pi}\int_0^{m\pi} e^{-\frac{q(\gamma_{SD} + \gamma_1')}{\sin^2\theta}} d\theta \, 1_{\{\gamma_{SD}>\gamma_{\text{th}}\}}\right]. \quad (39)$$

Interchanging the order of the finite integral and expectation, averaging over h_{SD}, and from the law of total expectation,

$$T_{21} = \frac{1}{\pi}\int_0^{m\pi} \frac{e^{-\frac{\gamma_{\text{th}}}{\overline{\gamma}_{SD}}\left(1 + \frac{q\overline{\gamma}_{SD}}{\sin^2\theta}\right)}}{1 + \frac{q\overline{\gamma}_{SD}}{\sin^2\theta}} \mathbb{E}_{y_1,g_1}\left[e^{-\left(\frac{c_1}{y_1} - 1\right)\csc^2\theta} \times \Pr(\beta = 1|y_1, g_1, \gamma_{SD} > \gamma_{\text{th}})\right] d\theta. \quad (40)$$

Now, from (17), we get $\Pr(\beta = 1|y_1, g_1, \gamma_{SD} > \gamma_{\text{th}}) = [1 - F_{y_2+\lambda g_2}(y_1 + \lambda g_1)]^{L-1} 1_{\{y_1+\lambda g_1 < c_1\}}$. Substituting the PDFs of g_2 and y_2, and simplifying further, we get

$$F_{y_2+\lambda g_2}(y_1 + \lambda g_1) = e^{-\frac{c_1-(y_1+\lambda g_1)}{q\overline{\gamma}'(y_1+\lambda g_1)}} - \frac{c_1 e^{\frac{1}{q\overline{\gamma}'} - \frac{y_1+\lambda g_1}{\lambda P_r \mu_{RP}}}}{q\overline{\gamma}'(y_1 + \lambda g_1)} \sum_{k=0}^{\infty} \frac{1}{k!} \times \left(\frac{y_1 + \lambda g_1}{\lambda P_r \mu_{RP}}\right)^k \mathrm{E}_k\left(\frac{c_1}{q\overline{\gamma}'(y_1 + \lambda g_1)}\right). \quad (41)$$

Substituting (41) back into $\Pr(\beta = 1|y_1, g_1, \gamma_{SD} > \gamma_{\text{th}})$, unconditioning over y_1 and g_1, and using $y_1 + \lambda g_1 = x$ and $x - \lambda g_1 = 1/t$ yields

$$T_{21} = \frac{c_1 e^{\frac{1}{q\overline{\gamma}'}}}{\pi \lambda P_r \mu_{RP} q\overline{\gamma}'} \int_0^{m\pi} \int_0^{c_1} \int_{\frac{1}{x}}^{\infty} (\kappa(x))^{L-1} e^{-\frac{\gamma_{\text{th}}}{\overline{\gamma}_{SD}}\left(1+\frac{q\overline{\gamma}_{SD}}{\sin^2\theta}\right)} \times \left(1 + \frac{q\overline{\gamma}_{SD}}{\sin^2\theta}\right)^{-1} e^{-\left(\frac{x}{\lambda P_r \mu_{RP}} - \csc^2\theta\right)} \times e^{-\left[c_1 t\left(\csc^2\theta + \frac{1}{q\overline{\gamma}'}\right) - \frac{1}{\lambda P_r \mu_{RP} t}\right]} dt dx d\theta, \quad (42)$$

where $\kappa(x) = 1 - e^{-\frac{1}{q\overline{\gamma}'}\left(\frac{c_1}{x} - 1\right)} + \frac{c_1 e^{\frac{1}{q\overline{\gamma}'} - \frac{x}{\lambda P_r \mu_{RP}}}}{q\overline{\gamma}'} \sum_{k=0}^{\infty} \frac{x^{k-1}}{k!} \left(\frac{1}{\lambda P_r \mu_{RP}}\right)^k \times \mathrm{E}_k\left(\frac{c_1}{q\overline{\gamma}' x}\right)$. Simplifying further, we get

$$T_{21} = \frac{c_1 e^{\frac{1}{q\overline{\gamma}'}}}{\pi q\overline{\gamma}'} \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{1}{\lambda P_r \mu_{RP}}\right)^{k+1} \int_0^{m\pi} \int_0^{c_1} (\kappa(x))^{L-1} \times \mathrm{E}_k\left(\frac{c_1}{x}\left(\csc^2\theta + \frac{1}{q\overline{\gamma}'}\right)\right) e^{-\frac{\gamma_{\text{th}}}{\overline{\gamma}_{SD}}\left(1+\frac{\overline{\gamma}_{SD}(q\gamma_{\text{th}}-1)}{\gamma_{\text{th}}\sin^2\theta}\right)} \times \left(1 + \frac{q\overline{\gamma}_{SD}}{\sin^2\theta}\right)^{-1} x^{k-1} e^{-\frac{x}{\lambda P_r \mu_{RP}}} dx d\theta. \quad (43)$$

The double integral above cannot be simplified further because of its involved form. Hence, we replace it with its upper bound, which is obtained as follows. We split the region of integration over θ into sub-intervals $[\theta_0, \theta_1], \ldots, [\theta_3, \theta_4]$, where $\theta_0 = 0, \theta_1 = \pi/4, \theta_2 = \pi/2, \theta_3 = 3\pi/4$, and $\theta_4 = m\pi$ [35]. In each sub-interval $[\theta_{i-1}, \theta_i]$, we replace θ by θ_i for $1 \le i \le 2$, and by θ_{i-1} for $3 \le i \le 4$, in the term $\mathrm{E}_k\left(\frac{c_1}{x}\left(\csc^2\theta + \frac{1}{q\overline{\gamma}'}\right)\right)$ in (43), and finally, integrate over θ. This yields the expression for T_{21} in (23).

T_{22} can be evaluated similarly, and is given by (24).

C. Asymptotic SEP Analysis of 1-Bit Rule

As mentioned, we assume $\mu_{SD} = \mu_{SR} = \mu_{RD} = \mu_{RP} = \mu$. As $\mu \to \infty$, it can be shown from (23) and (24) that the contribution from the relay links $L(T_{21} + T_{22})$ is $o(1/\mu^3)$. Thus, it has a negligible effect on the asymptotic SEP. Therefore, to obtain the asymptotic equivalent of the SEP in (20), we need to derive the asymptotic expressions for T_{11} and T_{12} in (21) and (22), respectively.

As $\mu \to \infty$, it can be shown from (37) that $\phi(c_1, \lambda) \to e^{-\frac{Lc_1}{\lambda P_r \mu}}$ and $\phi(c_0, \lambda) \to e^{-\frac{Lc_0}{\lambda P_r \mu}}$. Further, as $\mu \to \infty$ (i.e., $\overline{\gamma}_{SD} \to \infty$), we get from (38)

$$\psi(q\gamma_{\text{th}}, q\overline{\gamma}_{SD}) \to \frac{1}{\pi q\overline{\gamma}_{SD}} \int_0^{m\pi} \sin^2\theta e^{-\frac{q\gamma_{\text{th}}}{\sin^2\theta}} d\theta. \quad (44)$$

To get insights, we simplify the integral by replacing $e^{-\frac{q\gamma_{\text{th}}}{\sin^2\theta}}$ with its upper bound $e^{-q\gamma_{\text{th}}}$. This yields $\psi(q\gamma_{\text{th}}, q\overline{\gamma}_{SD}) \to \frac{e^{-q\gamma_{\text{th}}}(\sigma_0^2+\sigma_2^2)}{2qP_t\mu}\left[m + \frac{1}{2\pi}\sin\left(\frac{2\pi}{M}\right)\right]$. Similarly, as $\mu \to \infty$, we can show that $\mathrm{SEP}_0 - \psi(q\gamma_{\text{th}}, q\overline{\gamma}_{SD}) \to \frac{(1-e^{-q\gamma_{\text{th}}})(\sigma_0^2+\sigma_2^2)}{2qP_t\mu}\left[m + \frac{1}{2\pi}\sin\left(\frac{2\pi}{M}\right)\right]$.

Substituting these expressions in (20) yields (27).

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Priyanka Das (S'15) received the B.Tech. degree in radio physics and electronics from the University of Calcutta, West Bengal, India, in 2009, and the M.Tech. degree in digital signal processing from the Indian Institute of Technology (IIT), Guwahati, in 2011. She worked in Dell R&D, India from August 2011 to July 2012. She is now pursuing the Ph.D. degree in the Department of Electrical Communication Engineering, Indian Institute of Science (IISc), Bangalore. Her research interests include the design and performance analysis of cooperative relaying systems and cognitive radio networks.

Neelesh B. Mehta (S'98–M'01–SM'06) received the B.Tech. degree in electronics and communications engineering from the Indian Institute of Technology (IIT), Madras in 1996, and the M.S. and Ph.D. degrees in electrical engineering from the California Institute of Technology, Pasadena, CA, USA in 1997 and 2001, respectively. He is now an Associate Professor in the Department of Electrical Communication Engineering, Indian Institute of Science (IISc), Bangalore, India. He serves as an Executive Editor of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATION, and as an Editor of the IEEE TRANSACTIONS ON COMMUNICATIONS and IEEE WIRELESS COMMUNICATION LETTERS. He served as the Director of Conference Publications on the Board of Governors of the IEEE ComSoc in 2012–2013, and currently serves as a Member-at-Large on the Board.