Interference-Constrained Optimal Power-Adaptive Amplify-and-Forward Relaying and Selection for Underlay Cognitive Radios

B. Sainath, *Student Member, IEEE*, and Neelesh B. Mehta, *Senior Member, IEEE*

***Abstract*—In an underlay cognitive radio (CR) system, a secondary user can transmit when the primary is transmitting but is subject to tight constraints on the interference it causes to the primary receiver. Amplify-and-forward (AF) relaying is an effective technique that significantly improves the performance of a CR by providing an alternate path for the secondary transmitter's signal to reach the secondary receiver. We present and analyze a novel optimal relay gain adaptation policy (ORGAP) in which the relay is interference aware and optimally adapts both its gain and transmit power as a function of its local channel gains. ORGAP minimizes the symbol error probability at the secondary receiver subject to constraints on the average relay transmit power and on the average interference caused to the primary. It is different from *ad hoc* AF relaying policies and serves as a new and fundamental theoretical benchmark for relaying in an underlay CR. We also develop a near-optimal and simpler relay gain adaptation policy that is easy to implement. An extension to a multirelay scenario with selection is also developed. Our extensive numerical results for single and multiple relay systems quantify the power savings achieved over several *ad hoc* policies for both MPSK and MQAM constellations.**

***Index Terms*—Cooperative communications, underlay cognitive radio, relays, amplify-and-forward, fading channels, power constraint, interference constraint, symbol error probability.**

I. Introduction

COGNITIVE radio (CR) promises to address the urgent need for more bandwidth and higher data rates [1], [2]. In a common paradigm of CR, two classes of users are defined, namely, primary users (PUs) and secondary users (SUs). Different CR access modes have been proposed such as overlay, interweave, and underlay [1]. In underlay, which is the focus of this paper, an SU can transmit simultaneously even when the PU is transmitting. However, the SU's transmissions are subject to tight constraints on the interference they can cause to the primary receiver ($\mathrm{P_{Rx}}$). These constraints can severely limit the data rate and coverage of an SU.

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The authors are with the Department of Electrical Communication Engineering, Indian Institute of Science, Bangalore 560 012, India (e-mail: lsnb290580@gmail.com; nbmehta@ece.iisc.ernet.in).

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The use of cooperative relaying in an underlay CR system enhances its reliability and data rates [3]–[12]. A secondary underlay relay node helps forward data from a secondary transmitter ($\mathrm{S_{Tx}}$) to a secondary receiver ($\mathrm{S_{Rx}}$) and exploits spatial diversity to alleviate the impact of the interference constraint. Amplify-and-forward (AF) relaying is a popular technique that has been extensively used for underlay CR [5]–[13]. In it, the relay (R) simply amplifies the signal it receives from the $\mathrm{S_{Tx}}$ and forwards it to the $\mathrm{S_{Rx}}$. In this paper, we focus on AF relaying in underlay CR given its simplicity and effectiveness. We first summarize below the literature on AF relaying for underlay CR. This leads to some key observations about the shortcomings of the approaches that have been pursued thus far.

A. Literature on AF Relaying in Underlay CR

In [8], [11], the outage probability of an underlay AF relay network, in which the relay transmit power is a function of the R–$\mathrm{P_{Rx}}$ link is investigated. In [7], a quotient relay selection rule is proposed that selects the relay with the largest ratio of an upper bound of the signal-to-noise ratio (SNR) at the $\mathrm{S_{Rx}}$ and the interference it causes to the $\mathrm{P_{Rx}}$, and the selected relay transmits with fixed power. Unlike [7], in [5], [6], the selected relay's gain is kept constant. In [9], the asymptotic outage performance of different AF relay selection rules is analyzed with perfect and imperfect channel state information (CSI). In [12], a different AF protocol in which the $\mathrm{S_{Tx}}$ transmits data to one relay and the $\mathrm{S_{Rx}}$ over orthogonal channels is considered. The $\mathrm{S_{Tx}}$ and R are each subject to peak interference constraints. A multi-relay selection scheme in which the total interference power from all the transmitting relays is constrained is considered in [10].

In the above papers, the following three AF relaying policies are inevitably used. To describe them, we adopt the following notation: γ_{rp}, γ_{sr}, and γ_{rd} denote the R–$\mathrm{P_{Rx}}$, $\mathrm{S_{Tx}}$–R, and R–$\mathrm{S_{Rx}}$ channel power gains, respectively, and P_s and P_R denote the $\mathrm{S_{Tx}}$ and relay transmit powers, respectively.

1) *Interference-power Relaying*: In [8], [10], [11], the relay amplifies the signal it receives by a factor $\sqrt{\Theta}$, where

$$\Theta = \frac{I_{\mathrm{th}}}{\gamma_{rp}(P_s\gamma_{sr} + \delta_1)}, \quad (1)$$

where δ_1 is noise plus interference power at the relay. This ensures that an interference of I_{th} is caused to the

P_{Rx} at any time. This is a simple modification of the classical fixed-power AF relaying [14]. Thus, P_R is inversely proportional to γ_{rp} and γ_{sr}, and is not a function of γ_{rd}.

2) *Fixed-power Relaying*: In [7], Θ is set as

$$\Theta = \frac{Q}{P_s \gamma_{sr} + \delta_1}, \quad (2)$$

where Q depends on the interference constraint. This is classical fixed-power AF relaying. Thus, the relay gain only depends on γ_{sr}.

3) *Fixed-gain Relaying*: In [5], [6], [13], Θ is fixed, which is similar to the classical fixed-gain relaying policy [15]. Thus, the gain is not a function of γ_{sr}, γ_{rd}, and γ_{rp}. However, we note that whether a relay is considered for selection does depend on γ_{rp} in [5]–[7].

Thus, the AF relaying policies that have been considered for underlay CR are simple adaptations of the classical fixed-gain and fixed-power relaying policies. However, the average interference constraint imposed by underlay CR can fundamentally alter the way a relay amplifies its received signal. How to optimally adapt the relay gain and transmit power for underlay CR as a function of the local CSI available at the relay has been an open problem, which we solve in this paper. We show that the above *ad hoc* policies are sub-optimal and can be much improved upon. The functional form of the optimal solution and its insightful bounds that we derive, mathematically explain why this is so.

B. Contributions

We make the following contributions in this paper. First, we determine how an AF underlay relay should optimally adapt its gain as a function of the channel power gains of its local links to the S_{Tx}, P_{Rx}, and S_{Rx} in order to minimize the symbol error probability (SEP) at the S_{Rx} while satisfying constraints on the average relay transmit power and the average interference caused by the relay to the P_{Rx}. In particular, the proposed optimal relay gain adaptation policy (ORGAP) minimizes a tractable Chernoff upper bound on the SEP of MPSK. Our approach also easily generalizes to other constellations such as MQAM, results for which are also shown.

Second, we analyze the SEP of ORGAP, which turns out to be considerably more involved than that of conventional AF relaying. We gain significant insights by developing novel bounds and an asymptotic analysis, which cannot be done automatically using a symbolic computation software such as Mathematica. For example, when $\gamma_{rd}/\gamma_{rp} \gg 1$, the optimal gain Θ_{opt} turns out to decay at least as fast as $1/(\gamma_{sr}\sqrt{\gamma_{rd}\gamma_{rp}})$. This is markedly different from fixed-power relaying in which Θ is proportional to $1/\gamma_{sr}$ and interference-power relaying in which Θ is proportional to $1/(\gamma_{rp}\gamma_{sr})$. Diversity order under two insightful scaling regimes is also analyzed.

Third, we develop a novel, near-optimal, and simpler relay gain adaptation policy (SRGAP) in which the relay gain is specified in a closed-form that involves only elementary functions of the local channel power gains. Fourth, we also extend the results to a system with multiple relays that employs relay selection. Finally, extensive benchmarking with the aforementioned underlay AF relaying policies is also presented. These quantify the significant gains that ORGAP can deliver for various scenarios, and identify—for the first time—regimes in which some *ad hoc* policies are near-optimal.

C. Comparisons and Comments

We note that the models in [12], [16], [17] also adapt the relay gain. However, there are several fundamental differences in our model, objectives, and results. They are as follows.

In [16], [17], the interference constraint is not considered. As a result, the relaying policies presented in them are not interference-aware, i.e., they do not take into consideration γ_{rp}, and may not even be feasible for underlay CR. The policy in [17] is a special case of ORGAP when the interference constraint is inactive. Note that inclusion of an additional constraint can fundamentally alter the optimization problem. Thus, ORGAP is not a straightforward extension of the policy in [17], and is a new, fundamental theoretical benchmark for relaying in underlay CR. In [16], the end-to-end average SNR at a destination node is maximized; the optimal relay gain and power are computed numerically by exploiting quasi-concavity. On the other hand, we minimize the SEP at the S_{Rx} that receives signals from an interference-constrained relay. Since the relationship between SNR and SEP is non-linear, maximizing the *average* SNR is different from minimizing the *average* SEP, and leads to different solutions.

Our approach also facilitates the development of an insightful bound for the optimal relay gain, an SEP analysis of ORGAP, and a practically amenable near-optimal policy called SRGAP, all of which are unlike [16]. Our asymptotic analysis clearly brings out how ORGAP functionally differs from the aforementioned *ad hoc* policies. Further, the extensive benchmarking that we present is not available in [12], [16]. The AF relaying policy considered in [12] is similar to fixed-power relaying (cf. (2)), except that the relay transmit power is optimized. Further, in it, the S_{Tx} transmits data to the relay and S_{Rx} over separate orthogonal channels, which is fundamentally different from our model.

Compared to [17], several differences arise due to the underlay CR model that we focus on. First, the R–P_{Rx} interference link state pervades all the derivations and expressions in our work, unlike [17]. It makes the relay interference-aware. Second, the new interference constraint also makes the proof and structure of the optimal policy more involved. For example, in ORGAP, the relay operates in one of three regimes, namely, power-constrained regime, interference-constrained regime, and power- cum interference-constrained regime. On the other hand, in [17], the relay operates only in a power-constrained regime. Consequently, the SEP analysis is more involved in our paper. Third, while bounds on the optimal relay gain are also used in [17], the efficacy of such an approach given the new interference constraint is not obvious *a priori*.

D. Outline and Notation

The paper is organized as follows. Section II describes the system model. Section III derives the ORGAP policy. Its SEP

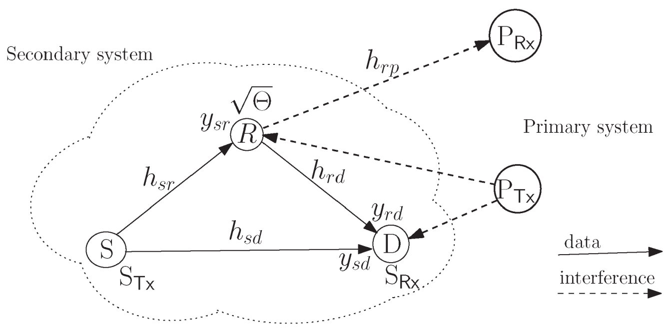

Fig. 1. An $\mathrm{S_{Tx}}$ transmits data to an $\mathrm{S_{Rx}}$ with the help of an AF relay R. The transmissions by R cause interference at a $\mathrm{P_{Rx}}$. Also shown is a $\mathrm{P_{Tx}}$ that causes interference to R and $\mathrm{S_{Rx}}$.

is analyzed in Section IV. Numerical results and our conclusions follow in Sections V and VI, respectively. Mathematical derivations are relegated to the Appendix.

We shall use the following notation henceforth. The probability of an event B is denoted by $\Pr(B)$. For a random variable (RV) Y, its probability density function (PDF) and expectation are denoted by $p_Y(y)$ and $\mathbf{E}[Y]$, respectively. The notation $X \sim \mathcal{CN}(0,\sigma^2)$ means that X is a zero-mean, circularly symmetric complex Gaussian RV with variance σ^2.

II. System Model and Transmission Protocol

Fig. 1 shows an underlay CR system consisting of a source $\mathrm{S_{Tx}}$ that transmits data to a destination $\mathrm{S_{Rx}}$ with the help of a half-duplex AF relay R, which causes interference to a primary receiver $\mathrm{P_{Rx}}$. A primary transmitter $(\mathrm{P_{Tx}})$ causes interference to R and $\mathrm{S_{Rx}}$. Each node is equipped with a single transmit or receive antenna. The use of single antenna nodes is one of the principal motivations for cooperative communications compared to multiple antenna communications, it is practically well motivated, and has been considered widely in the CR literature [5]–[7], [13] and in the cooperative communications literature [14], [16], [18]. The $\mathrm{S_{Tx}}$–$\mathrm{S_{Rx}}$, R–$\mathrm{P_{Rx}}$, R–$\mathrm{S_{Rx}}$, and $\mathrm{S_{Tx}}$–$\mathrm{S_{Rx}}$ channels undergo frequency-flat Rayleigh fading. They are mutually independent, but need not be statistically identical. All transmissions occur over the same bandwidth.

A. Cooperative Non-Regenerative Relaying: Preliminaries

The AF relaying protocol occurs over two time slots, along the lines of conventional AF relaying. The difference lies in how the relay gain and power are set, as described below.

S_{Tx} Transmission: In the first time slot, the $\mathrm{S_{Tx}}$ broadcasts a data symbol α, which is drawn with equal probability from a constellation of size M, with a fixed power P_s. The received signals include transmissions from the $\mathrm{P_{Tx}}$, which causes interference to the relay R and the destination $\mathrm{S_{Rx}}$. The received signals y_{sd} and y_{sr} at the $\mathrm{S_{Rx}}$ and relay, respectively, are given by

$$y_{sd} = \sqrt{P_s} h_{sd}\alpha + i_{Pd} + n_{sd}, \quad (3)$$

$$y_{sr} = \sqrt{P_s} h_{sr}\alpha + i_{Pr} + n_{sr}, \quad (4)$$

where $h_{sd} \sim \mathcal{CN}(0,\sigma_{sd}^2)$ is the $\mathrm{S_{Tx}}$–$\mathrm{S_{Rx}}$ complex baseband channel gain and $h_{sr} \sim \mathcal{CN}(0,\sigma_{sr}^2)$ is the $\mathrm{S_{Tx}}$–R complex baseband channel gain. Further, the additive noise terms n_{sr} and n_{sd} are modeled as $\mathcal{CN}(0,\sigma^2)$ RVs. The interferences caused by $\mathrm{P_{Tx}}$ at the destination $\mathrm{S_{Rx}}$ and relay R are $i_{Pd} \sim \mathcal{CN}(0,\sigma_d^2)$ and $i_{Pr} \sim \mathcal{CN}(0,\sigma_r^2)$, respectively. This interference model is reasonable when the $\mathrm{P_{Tx}}$–R and $\mathrm{P_{Tx}}$–$\mathrm{S_{Rx}}$ links undergo Rayleigh fading and the $\mathrm{P_{Tx}}$ transmits at a constant amplitude. It is also reasonable when there are many $\mathrm{P_{Tx}}$s due to the central limit theorem. This can happen, for example, in the uplink. In general, it corresponds to a worst-case model for the interference, and has also been assumed in [11], [19] to make the problem tractable.[1] In [21], [22], the interference from the $\mathrm{P_{Tx}}$ is assumed to be negligible. Our model is valid in this simpler scenario as well. The noise and interference terms are independent of each other and the channel gains. For MPSK, $|\alpha|^2 = 1$, while $\mathbf{E}[|\alpha|^2] = 1$ for MQAM. As mentioned, $\gamma_{sr} = |h_{sr}|^2$, $\gamma_{rd} = |h_{rd}|^2$, and $\gamma_{sd} = |h_{sd}|^2$. Let $\overline{\gamma}_{sr} = \mathbf{E}[\gamma_{sr}]$, $\overline{\gamma}_{rd} = \mathbf{E}[\gamma_{rd}]$, and $\overline{\gamma}_{sd} = \mathbf{E}[\gamma_{sd}]$.

Relay Transmission: In the second time slot, the relay amplifies the signal it receives, y_{sr}, by a factor $\sqrt{\Theta}$. Therefore, the $\mathrm{S_{Rx}}$ receives the signal y_{rd}, which is given by

$$y_{rd} = \sqrt{\Theta P_s} h_{sr} h_{rd}\alpha + \sqrt{\Theta} h_{rd} i_{Pr} + \sqrt{\Theta} h_{rd} n_{sr} + i_{Pd} + n_{rd}. \quad (5)$$

The average relay transmit power $\overline{P}_r$ is equal to

$$\overline{P}_r = \mathbf{E}\left[\Theta\left(P_s\gamma_{sr} + \sigma^2 + \sigma_r^2\right)\right]. \quad (6)$$

In our model, the relay gain Θ is a function of its local channel power gains γ_{sr}, γ_{rd}, and γ_{rp}. The relay is assumed to know the channel power gains of its local links $\mathrm{S_{Tx}}$–R, R–$\mathrm{S_{Rx}}$, and R–$\mathrm{P_{Rx}}$, as has also been assumed in [14], [16], [23]. These can be estimated by exploiting reciprocity and overhearing the transmissions from $\mathrm{S_{Tx}}$, $\mathrm{S_{Rx}}$, and $\mathrm{P_{Rx}}$. Note that the relay need not know the phases of the complex baseband channel gains of any these links. Thus, simple receive energy-based estimation techniques can be employed. *The relay does not know the state of the direct S_{Tx}–S_{Rx} link and the link between the P_{Tx} and P_{Rx}*. For AF policies that are designed assuming less CSI, e.g., [5], [6], this model serves as a useful benchmark.

Reception: The receiver $\mathrm{S_{Rx}}$ employs coherent detection to detect α using its two observables y_{sd} and y_{rd}. The output SNR Γ_E at the destination $\mathrm{S_{Rx}}$, when it employs maximal ratio combining, can be shown to be equal to

$$\Gamma_E = \frac{P_s\gamma_{sd}}{\sigma^2 + \sigma_d^2} + \frac{P_s\gamma_{sr}\gamma_{rd}}{\gamma_{rd}\left(\sigma^2 + \sigma_r^2\right) + \Theta^{-1}\left(\sigma^2 + \sigma_d^2\right)}. \quad (7)$$

B. Constraints at P_{Rx}

The baseband interference signal i_p seen by the $\mathrm{P_{Rx}}$ due to the relay is given by

$$i_p = \sqrt{\Theta} y_{sr} h_{rp}, \quad (8)$$

[1] We do not use the interference model of [20] because it assumes that the interference and transmitted signal power itself are both Gaussian.

where $h_{rp} \sim \mathcal{CN}(0, \sigma_{rp}^2)$ is the R–$\mathrm{P_{Rx}}$ channel gain. Let $\gamma_{rp} = |h_{rp}|^2$ and $\overline{\gamma}_{rp} = \mathbf{E}[\gamma_{rp}]$.

The relay is subject to the following two constraints:

- *Average Interference Power Constraint:* It requires that the average interference power at the $\mathrm{P_{Rx}}$ from the relay should be less than or equal to an average interference threshold I_{th}. Note that I_{th} is a system parameter and is chosen based on how much interference the $\mathrm{P_{Rx}}$ can tolerate on average. The average interference power $\overline{I}_r$ due to the relay's transmissions is equal to

$$\overline{I}_r = \mathbf{E}\left[|i_p|^2\right] = \mathbf{E}\left[\Theta\gamma_{rp}\left(P_s\gamma_{sr} + \sigma^2 + \sigma_r^2\right)\right]. \quad (9)$$

  Therefore, $\mathbf{E}[\Theta\gamma_{rp}(P_s\gamma_{sr} + \sigma^2 + \sigma_r^2)] \leq I_{\mathrm{th}}$.
- *Average Transmit Power Constraint:* It requires that the average transmit power of the relay should be less than or equal to P_{th}. Therefore, $\mathbf{E}[\Theta(P_s\gamma_{sr} + \sigma^2 + \sigma_r^2)] \leq P_{\mathrm{th}}$.

Generalizations: While the model that we investigate is novel and insightful, more general formulations are possible. One possible extension would also allow the source $\mathrm{S_{Tx}}$, whose transmissions are time-orthogonal to the relay's transmissions, to adapt its transmit power as a function of its local channel gains subject to constraints on its average transmit power and average interference. However, determining the optimal joint $\mathrm{S_{Tx}}$ and relay power adaptation policy is an open problem because the local CSI available at the $\mathrm{S_{Tx}}$ and the relay, which are geographically separated, is different. This joint optimization is beyond the scope of this paper. Our approach is consistent with [5]–[7], in which the relaying policy is not a function of the interference generated by the $\mathrm{S_{Tx}}$.

III. Interference-Aware Relay Gain Adaptation

We first derive the optimum relay gain $\Theta_{\mathrm{opt}}(\gamma_{sr}, \gamma_{rp}, \gamma_{rd})$, which is a mapping from $(\mathbb{R}^+)^3$ to $\mathbb{R}^+$. For notational simplicity, we henceforth do not explicitly show its dependence on γ_{sr}, γ_{rd}, and γ_{rp}. We consider MPSK first. The corresponding analysis for MQAM is presented in Section III-C.

The fading-averaged SEP for MPSK at the destination $\mathrm{S_{Rx}}$ is given by [24, (8.23)]

$$\mathrm{SEP} = \frac{1}{\pi}\int_0^{\left(\frac{M-1}{M}\right)\pi} \mathbf{E}\left[\exp\left(-\Gamma_E \frac{m}{\sin^2\psi}\right)\right] d\psi, \quad (10)$$

where $m = \sin^2(\pi/M)$. Averaging over γ_{sd}, which is independent of γ_{sr}, γ_{rp}, and γ_{rd}, we get

$$\mathrm{SEP} = \frac{1}{\pi}\int_0^{\left(\frac{M-1}{M}\right)\pi} \frac{\mathbf{E}\left[\exp\left(-\frac{P_s\gamma_{sr}\gamma_{rd}}{\gamma_{rd}\delta_1+\Theta^{-1}\delta_2}\frac{m}{\sin^2\psi}\right)\right]}{1+\frac{P_s\overline{\gamma}_{sd}}{\delta_2}\frac{m}{\sin^2\psi}} d\psi, \quad (11)$$

where $\delta_1 = \sigma^2 + \sigma_r^2$ and $\delta_2 = \sigma^2 + \sigma_d^2$.

The above expression involves four integrals over γ_{sr}, γ_{rd}, γ_{rp}, and ψ, and the relay gain is an implicit function of γ_{sr}, γ_{rp}, and γ_{rd}. To gain insights, we derive below an analytically tractable, integral-free upper bound for the SEP that we then minimize. Using the inequality $\sin^2\psi \leq 1$ only for the term inside the expectation in the integrand in (11), we get

$$\begin{aligned}\mathrm{SEP} &\leq \left[\frac{1}{\pi}\int_0^{\left(\frac{M-1}{M}\right)\pi} \frac{1}{1+\frac{P_s\overline{\gamma}_{sd}}{\delta_2}\frac{m}{\sin^2\psi}} d\psi\right] \\ &\quad \times \mathbf{E}\left[\exp\left(-\frac{mP_s\gamma_{sr}\gamma_{rd}}{\gamma_{rd}\delta_1+\Theta^{-1}\delta_2}\right)\right], \\ &= \Xi_0 \mathbf{E}\left[\exp\left(-\frac{mP_s\gamma_{sr}\gamma_{rd}}{\gamma_{rd}\delta_1+\Theta^{-1}\delta_2}\right)\right], \end{aligned} \quad (12)$$

where Ξ_0 captures the contribution of the direct $\mathrm{S_{Tx}}$–$\mathrm{S_{Rx}}$ link, and is given in closed-form as [24]

$$\Xi_0 = \frac{M-1}{M} - \frac{\left(\frac{1}{2} + \frac{1}{\pi}\tan^{-1}\sqrt{\frac{1-m}{m+\frac{\delta_2}{P_s\overline{\gamma}_{sd}}}}\right)}{\sqrt{1+\frac{\delta_2}{mP_s\overline{\gamma}_{sd}}}}. \quad (13)$$

This form is similar to the integral-free SEP approximation used for optimal link adaptation in [25, Chap. 9].

A. Optimization Problem and Solution

Since Ξ_0 is a constant, minimizing SEP reduces to the following constrained, stochastic optimization problem:

$$\min_{\Theta} \quad \mathbf{E}\left[\exp\left(-\frac{mP_s\gamma_{sr}\gamma_{rd}}{\gamma_{rd}\delta_1+\Theta^{-1}\delta_2}\right)\right] \quad (14)$$

$$\text{s.t.} \quad \mathbf{E}\left[\Theta(P_s\gamma_{sr}+\delta_1)\right] \leq P_{\mathrm{th}}, \quad (15)$$

$$\mathbf{E}\left[\Theta\gamma_{rp}(P_s\gamma_{sr}+\delta_1)\right] \leq I_{\mathrm{th}}, \text{ and} \quad (16)$$

$$\Theta \geq 0, \quad \forall\, \gamma_{sr} \geq 0,\ \gamma_{rp} \geq 0,\ \gamma_{rd} \geq 0. \quad (17)$$

A policy that satisfies (15), (16), and (17) is said to be feasible. Our goal is to find a feasible policy that minimizes the average SEP. We shall refer to this policy as an optimal policy Θ_{opt}. It is easy to see that for the optimal policy at least one of the two constraints in (15) and (16) must be active. The following result characterizes an optimal policy.

Result 1: Let

$$\mathcal{B}(\gamma_{sr}, \gamma_{rp}) \triangleq \delta_2 \frac{\lambda_R + \gamma_{rp}\lambda_I}{m}\left(1 + \frac{\delta_1}{P_s\gamma_{sr}}\right). \quad (18)$$

The optimal relay gain Θ_{opt} is as follows:

$$\Theta_{\mathrm{opt}} = 0, \quad \text{if} \quad \gamma_{rd} < \mathcal{B}(\gamma_{sr}, \gamma_{rp}). \quad (19)$$

Else, for $\gamma_{rd} \geq \beta(\gamma_{sr}, \gamma_{rp})$, Θ_{opt} is the unique positive root of the following transcendental equation in θ:

$$\begin{aligned}\Delta(\theta) \triangleq{}& \exp\left(\frac{mP_s\gamma_{sr}\gamma_{rd}}{\gamma_{rd}\delta_1+\theta^{-1}\delta_2}\right)(\delta_1\theta\gamma_{rd}+\delta_2)^2 \\ &- \frac{mP_s\delta_2\gamma_{sr}\gamma_{rd}}{(\lambda_R+\gamma_{rp}\lambda_I)(P_s\gamma_{sr}+\delta_1)} = 0. \end{aligned} \quad (20)$$

Here, $\lambda_R \geq 0$ and $\lambda_I \geq 0$ are constants, which are chosen as follows. Let Θ_{opt}^R correspond to the above policy in which

$\lambda_I = 0$, i.e., the interference constraint is inactive. In this scenario, $\lambda_R > 0$ is chosen such that the average transmit power is equal to $P_{\rm th}$. If the interference generated by $\Theta_{\rm opt}^R$ is less than or equal to $I_{\rm th}$, then the policy $\Theta_{\rm opt}^R$ is optimal. Else, let $\Theta_{\rm opt}^I$ correspond to the above policy in which $\lambda_R = 0$, i.e., the transmit power constraint is inactive. In this scenario, $\lambda_I > 0$ is chosen such that the average interference power is equal to $I_{\rm th}$. If the average power consumed by $\Theta_{\rm opt}^I$ is less than or equal to $P_{\rm th}$, then $\Theta_{\rm opt}^I$ is optimal.[2] Else, $\lambda_R > 0$ and $\lambda_I > 0$ are chosen to satisfy both (15) and (16) with equality, and such a choice always exists.

Proof: The proof is given in Appendix A. ■

We shall henceforth refer to the regime in which $\lambda_R > 0$ and $\lambda_I = 0$ as the *power-constrained regime*, the regime in which $\lambda_I > 0$ and $\lambda_R = 0$ as the *interference-constrained regime*, and the regime in which $\lambda_R > 0$ and $\lambda_I > 0$ as the *power- cum interference-constrained regime*.

We make the following comments about the optimal policy. (i) We note that the Chernoff upper bound on the SEP is minimized, and not the exact SEP in (11), which is similar to the approach used in [4], [25, Chap. 9]. However, for ease of exposition, we shall continue to refer to the resulting policy as ORGAP. This step ensures tractability and leads to significant analytical insights. We note that it is very challenging to develop tractable error bounds or approximation ratios to quantify the sub-optimality incurred in doing so. (ii) From (19), we see that $\mathcal{B}(\gamma_{sr}, \gamma_{rp})$ defines the boundary of a region in which the relay shuts down if γ_{rd} is weak. It is an affine and monotonically increasing function of γ_{rp}. Thus, the relay is shut down over a wider range of values of γ_{rd} when the R–$\mathrm{P_{Rx}}$ link is strong so as avoid excessive interference at the $\mathrm{P_{Rx}}$. This brings out the interference-aware nature of ORGAP, which sets it apart from the policies presented in [16], [17]. (iii) The constants λ_R and λ_I do not depend on the instantaneous channel gains, and need to be computed only once. Such a parametric specification of the optimal policy is typical of constrained optimization problems in wireless communications, e.g., water-filling in space, time, or frequency [26] and optimal rate adaptation [25, Chap. 9]. (iv) Since $\Theta_{\rm opt}$ is specified for each γ_{sr}, γ_{rp}, and γ_{rd} realization, the fading distribution only affects the values of the constants λ_R and λ_I.

In order to compute the optimal relay gain, (20) needs to be solved numerically for each realization of γ_{sr}, γ_{rp}, and γ_{rd}. While it is optimal with respect to minimizing the SEP upper bound, it is impractical to solve in real-time. To tackle this issue, we now present a functionally simpler policy that is practically implementable.

B. Relay Gain Bound and SRGAP

We first present below an upper bound for $\Theta_{\rm opt}$. The benefits of this bound are two-fold. Firstly, its explicit form provides analytical insights about the dependence of $\Theta_{\rm opt}$ on the channel gains. Secondly, it leads to an implementation-friendly approximation for $\Theta_{\rm opt}$.

[2]The policy derived in [17] corresponds to the special case in which the interference constraint is inactive.

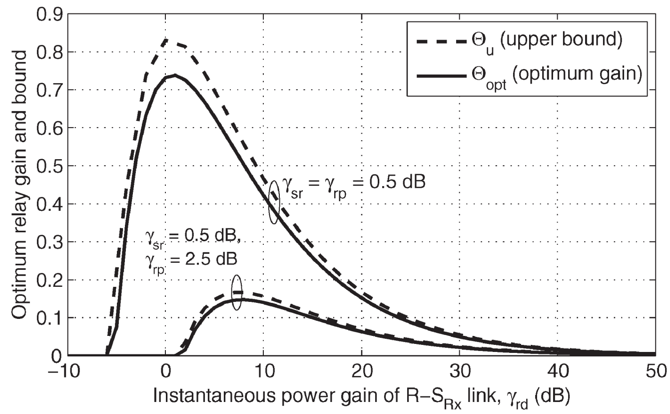

Fig. 2. Behavior of optimum relay gain $\Theta_{\rm opt}$ and its upper bound Θ_u as a function of the instantaneous power gain of R–$\mathrm{S_{Rx}}$ link γ_{rd} ($\lambda_I = 0.1$, $\lambda_R = 0$, $P_s = 0$ dB, $\delta_1 = 1$, $\delta_2 = 1$, and QPSK).

Result 2: The optimal AF relay gain $\Theta_{\rm opt}$ is less than or equal to Θ_u, where

$$\Theta_u = \begin{cases} \dfrac{\frac{-\delta_1\delta_2}{\delta_1^2+m^2P_s^2\gamma_{sr}^2}+\sqrt{\frac{\gamma_{rd}\delta_2^2}{\mathcal{B}(\gamma_{sr},\gamma_{rp})\left(\delta_1^2+m^2P_s^2\gamma_{sr}^2\right)}}}{\gamma_{rd}}, & \gamma_{rd}\geq\mathcal{B}(\gamma_{sr},\gamma_{rp}),\\ 0, & \text{otherwise.}\end{cases} \tag{21}$$

Proof: The proof is relegated to Appendix B. ■

Notice the dependence on γ_{rp} that is brought out by Θ_u. Fig. 2 plots $\Theta_{\rm opt}$ and Θ_u as a function of γ_{rd} for two pairs of values of γ_{sr} and γ_{rp}. The figure verifies that Θ_u is an upper bound. While the bound need not always be tight, we see that its behavior is qualitatively similar to $\Theta_{\rm opt}$. For example, the relay shuts down when the R–$\mathrm{S_{Rx}}$ link is in a deep fade. Similarly, the relay power decreases when the R–$\mathrm{S_{Rx}}$ link is strong, so as to conserve transmit power. The relay shuts down over a wider range of values of γ_{rd} when γ_{rp} increases from 0.5 dB to 2.5 dB so as to avoid causing excessive interference at the $\mathrm{P_{Rx}}$. Further, the relay gain decreases as γ_{rp} increases.

1) Asymptotic Insights: Consider the asymptotic regime in which $\gamma_{rd}/\gamma_{rp} \gg 1$. In this case, it can be shown that Θ_u is proportional to $(\propto)$ $1/(\gamma_{sr}\sqrt{\gamma_{rd}\gamma_{rp}})$. Thus, Θ decays at least as fast as $1/(\gamma_{sr}\sqrt{\gamma_{rd}\gamma_{rp}})$. This behavior is different from fixed-power relaying [7] in which $\Theta \propto 1/\gamma_{sr}$, fixed-gain relaying [5], [6] in which $\Theta \propto 1$, and interference-power relaying [8], [11] in which $\Theta \propto 1/(\gamma_{rp}\gamma_{sr})$.

2) SRGAP: Using Θ_u Instead of $\Theta_{\rm opt}$ as Relay Gain: We observed that Θ_u is qualitatively similar to $\Theta_{\rm opt}$. Furthermore, it is available in an explicit closed-form. Therefore, we propose using it instead of $\Theta_{\rm opt}$ as the AF relay gain. We shall refer to this policy as SRGAP. When SRGAP is used, the values of λ_R and λ_I are chosen so that the relay satisfies the average relay transmit power and average interference constraints, which are given by $\mathbf{E}[\Theta_u\gamma_{rp}(P_s\gamma_{sr}+\delta_1)] \leq I_{\rm th}$ and $\mathbf{E}[\Theta_u(P_s\gamma_{sr}+\delta_1)] \leq P_{\rm th}$, with equality holding under conditions similar to those for ORGAP in Result 1.

C. Analysis for MQAM

The SEP for MQAM is [24]

$$\mathrm{SEP} = \frac{4m_1}{\pi}\int_0^{\frac{\pi}{2}} \mathbf{E}\left[e^{-\frac{m'\Gamma_E}{\sin^2\psi}}\right] d\psi - \frac{4m_1^2}{\pi}\int_0^{\frac{\pi}{4}} \mathbf{E}\left[e^{-\frac{m'\Gamma_E}{\sin^2\psi}}\right] d\psi,$$

where $m' = 1.5/(M-1)$ and $m_1 = 1-(1/\sqrt{M})$. As before, upper bounding the SEP, we get

$$\mathrm{SEP} \le \frac{M-1}{M\left(1+\frac{m'P_s\overline{\gamma}_{sd}}{\delta_2}\right)}\mathbf{E}\left[e^{-\frac{m'P_s\gamma_{sr}\gamma_{rd}}{\gamma_{rd}\delta_1+\Theta^{-1}\delta_2}}\right]. \quad (22)$$

Thus, (22) has the same form as (12) except for a different scaling constant in front of the expectation and also inside the exponential, which does not affect the optimization problem in (14). Therefore, for MQAM, the expressions for Θ_{opt} and Θ_u are similar to those in (20) and (21), respectively, with m replaced by m'.

IV. SEP Analysis of ORGAP

We now analyze the SEP of ORGAP. Expanding the expectation term in the integrand in (11) results in the following quadruple-integral, which averages over ψ, γ_{sr}, γ_{rd}, and γ_{rp}:

$$\mathrm{SEP} = \frac{1}{\pi\overline{\gamma}_{sr}\overline{\gamma}_{rp}\overline{\gamma}_{rd}}\int_0^\infty\int_0^\infty\int_0^\infty\int_0^{\left(\frac{M-1}{M}\right)\pi} \frac{\exp\left(-\frac{mP_s\gamma_{sr}\gamma_{rd}}{\gamma_{rd}\delta_1+\Theta^{-1}\delta_2}\right)}{1+\frac{P_s\overline{\gamma}_{sd}}{\delta_2}\frac{m}{\sin^2\psi}} \times e^{-\frac{\gamma_{sr}}{\overline{\gamma}_{sr}}} e^{-\frac{\gamma_{rp}}{\overline{\gamma}_{rp}}} e^{-\frac{\gamma_{rd}}{\overline{\gamma}_{rd}}} d\psi\, d\gamma_{rd}\, d\gamma_{rp}\, d\gamma_{sr}. \quad (23)$$

Note that Θ_{opt} in the integrand itself is an implicit function of γ_{sr}, γ_{rp}, and γ_{rd} (cf. Result 1). Hence, this quadruple integral cannot be simplified further. To circumvent this challenge, we present below a more tractable SEP upper bound SEP_u. The expression is much simpler than (23) as it has two integrals and has no implicit dependence on the gains in the integrand.

Result 3: The SEP of ORGAP is upper bounded as

$$\mathrm{SEP} \le \mathrm{SEP}_u \triangleq T_1 + T_2, \quad (24)$$

where

$$T_1 = \Xi_0 - \frac{\Xi_0 e^{-\frac{\lambda_R\delta_2}{m\overline{\gamma}_{rd}}}}{(\delta_2\lambda_I\overline{\gamma}_{rp}+m\overline{\gamma}_{rd})} \times \left[\Delta K_1(\Delta) - \frac{\delta_1\delta_2\lambda_I}{P_s\overline{\gamma}_{sr}} \times \int_0^\infty \frac{\exp\left(-\left(\frac{\gamma_{sr}}{\overline{\gamma}_{sr}}+\frac{\delta_1\delta_2\lambda_R}{mP_s\overline{\gamma}_{rd}\gamma_{sr}}\right)\right)}{\left(\frac{1}{\overline{\gamma}_{rp}}+\frac{\delta_2\lambda_I}{m\overline{\gamma}_{rd}}\right)\gamma_{sr}+\frac{\delta_1\delta_2\lambda_I}{mP_s\overline{\gamma}_{rd}}} d\gamma_{sr}\right], \quad (25)$$

$$T_2 = \frac{\Xi_0\delta_2}{\overline{\gamma}_{sr}\overline{\gamma}_{rp}\overline{\gamma}_{rd}} \times \int_0^\infty\int_0^\infty \left[\sqrt{\frac{4\pi\overline{\gamma}_{rd}m^4P_s^4\gamma_{sr}^4}{\omega(\gamma_{sr})^3\mathcal{B}(\gamma_{sr},\gamma_{rp})}}\operatorname{erfc}\left(\sqrt{\frac{\mathcal{B}(\gamma_{sr},\gamma_{rp})}{\overline{\gamma}_{rd}}}\right) + \frac{m^4P_s^4\gamma_{sr}^4}{\omega(\gamma_{sr})^2}E_1\left(\frac{\mathcal{B}(\gamma_{sr},\gamma_{rp})}{\overline{\gamma}_{rd}}\right) + \frac{\overline{\gamma}_{rd}e^{-\frac{\mathcal{B}(\gamma_{sr},\gamma_{rp})}{\overline{\gamma}_{rd}}}}{\omega(\gamma_{sr})\mathcal{B}(\gamma_{sr},\gamma_{rp})}\right] \times \mathcal{B}(\gamma_{sr},\gamma_{rp})e^{-\frac{\gamma_{rp}}{\overline{\gamma}_{rp}}}e^{-\frac{\gamma_{sr}}{\overline{\gamma}_{sr}}}d\gamma_{rp}\,d\gamma_{sr}, \quad (26)$$

Ξ_0 is given by (13), $\Delta = \sqrt{4\delta_1\delta_2\lambda_R/(mP_s\overline{\gamma}_{sr}\overline{\gamma}_{rd})}$, $\omega(\gamma_{sr}) = \delta_1^2 + m^2P_s^2\gamma_{sr}^2$, $K_1(\cdot)$ denotes the modified Bessel function of second kind and first order [27, (9.6)], $\operatorname{erfc}(\cdot)$ denotes the complementary error function [28, (8.250.4)], and $E_i(x)$ is the Euler exponential integral [27, (5.1.1)].

Proof: The derivation is presented in Appendix C. ■

While SEP_u is in a double integral form, it is simpler and easier to evaluate than (23), and is a contribution of this paper. It can be simplified further using Gauss-Laguerre quadrature [27] and expressed as a summation of a few terms. The final form is not shown here to conserve space.

To gain more insights, we further simplify (24) in the interference-constrained regime below.

Result 4: An upper bound on the SEP of ORGAP in the interference-constrained regime is

$$\mathrm{SEP}_u = \Xi_0\left(\frac{\omega_2P_s\overline{\gamma}_{sr}}{\delta_1\delta_2}+\frac{\omega_2e^{\omega_2}E_1(\omega_2)}{\omega_1}\right) + \frac{\Xi_0\delta_2}{\overline{\gamma}_{rd}\overline{\gamma}_{rp}\overline{\gamma}_{sr}} \times \int_0^\infty \left[\frac{\overline{\gamma}_{rp}\overline{\gamma}_{rd}}{\omega(\gamma_{sr})\left(\omega_3(\gamma_{sr})+1\right)} + \frac{2m^2P_s^2\gamma_{sr}^2\overline{\gamma}_{rd}^2}{\mathcal{B}_I(\gamma_{sr})\omega(\gamma_{sr})^{\frac{3}{2}}} \times \left(\frac{\tan^{-1}\left(\sqrt{\omega_3(\gamma_{sr})}\right)}{\omega_3(\gamma_{sr})^{\frac{3}{2}}} - \frac{1}{\omega_3(\gamma_{sr})\left(\omega_3(\gamma_{sr})+1\right)}\right) + \frac{\overline{\gamma}_{rd}^2m^4P_s^4\gamma_{sr}^4}{\mathcal{B}_I(\gamma_{sr})\omega(\gamma_{sr})^2} \times \left[-\frac{1}{\omega_3(\gamma_{sr})\left(\omega_3(\gamma_{sr})+1\right)} + \frac{\ln\left(1+\omega_3(\gamma_{sr})\right)}{\omega_3(\gamma_{sr})^2}\right]\right] e^{-\frac{\gamma_{sr}}{\overline{\gamma}_{sr}}}d\gamma_{sr}, \quad (27)$$

where $\omega_1 = (\delta_2\lambda_I\overline{\gamma}_{rp}+m\overline{\gamma}_{rd})/(m\overline{\gamma}_{rd})$, $\omega_2 = \delta_1\delta_2\lambda_I\overline{\gamma}_{rp}/(P_s\overline{\gamma}_{sr}(\lambda_I\overline{\gamma}_{rp}+m\overline{\gamma}_{rd}))$, $\omega_3(\gamma_{sr}) = \overline{\gamma}_{rd}/(\mathcal{B}_I(\gamma_{sr})\overline{\gamma}_{rp})$, and $\mathcal{B}_I(\gamma_{sr}) = \delta_2\lambda_I(1+\delta_1/(P_s\gamma_{sr}))/m$.

TABLE I
GAINS OF VARIOUS AF RELAYING POLICIES

Relaying policy	Gain
ORGAP	$\Theta_{\text{opt}}(\gamma_{sr}, \gamma_{rp}, \gamma_{rd})$ (cf. Result 1)
Interference-power	$\Theta = \frac{I_{\text{th}}}{\gamma_{rp}(P_s\gamma_{sr}+\delta_1)}$ $(P_{\text{th}} > \frac{I_{\text{th}}}{\overline{\gamma}_{rp}})$ & $\Theta = \frac{P_{\text{th}}}{P_s\gamma_{sr}+\delta_1}$ $(P_{\text{th}} \leq \frac{I_{\text{th}}}{\overline{\gamma}_{rp}})$
Fixed-power	$\Theta = \frac{I_{\text{th}}}{\overline{\gamma}_{rp}(P_s\gamma_{sr}+\delta_1)}$ $(P_{\text{th}} > \frac{I_{\text{th}}}{\overline{\gamma}_{rp}})$ & $\Theta = \frac{P_{\text{th}}}{P_s\gamma_{sr}+\delta_1}$ $(P_{\text{th}} \leq \frac{I_{\text{th}}}{\overline{\gamma}_{rp}})$
Fixed-gain	$\Theta = \frac{I_{\text{th}}}{\overline{\gamma}_{rp}(P_s\overline{\gamma}_{sr}+\delta_1)}$ $(P_{\text{th}} > \frac{I_{\text{th}}}{\overline{\gamma}_{rp}})$ & $\Theta = \frac{P_{\text{th}}}{P_s\overline{\gamma}_{sr}+\delta_1}$ $(P_{\text{th}} \leq \frac{I_{\text{th}}}{\overline{\gamma}_{rp}})$

Proof: The derivation is given in Appendix D. ■

Further simplification can again be done using Gauss-Laguerre quadrature. In the power-constrained regime, the corresponding SEP upper bound is given by [17, (22)].

A. Diversity Order Analysis

We now analyze the diversity order of ORGAP for the following two scaling regimes:

- *Conventional scaling regime* [18]*:* In this, $P_s \to \infty$ and $P_{\text{th}} \to \infty$, but the average interference threshold and the mean channel power gains are kept fixed. In such a case, it can be shown that an error floor occurs. Therefore, the diversity order is zero.
- *Alternate scaling regime:* Along the lines of [20],[3] we consider the following alternate scaling regime in which $P_s \to \infty$, I_{th}/P_s is constant, $\overline{\gamma}_{rp} \to 0$, and P_{th}/P_s is either constant or decreases to zero. The other mean channel power gains are kept fixed. Thus, the average power constraint is inactive, i.e., $\lambda_R = 0$ and λ_I is a strictly positive constant. As shown in Appendix E, in this regime, a full diversity order of two is achieved.

V. NUMERICAL RESULTS AND PERFORMANCE BENCHMARKING

We now characterize the SEP performance of ORGAP using Monte Carlo simulations that were performed using MATLAB and used up to 10^6 channel fading and noise realizations. For each realization of fading and noise, a symbol from the chosen constellation is drawn and transmitted as per ORGAP or the benchmarking policies simulated, and is then decoded at the S_{Rx}. We present results for two scenarios: (i) where the direct S_{Tx}–S_{Rx} link is comparable in strength to the S_{Tx}–R, R–P_{Rx}, and R–S_{Rx} links ($\overline{\gamma}_{sd} = 1$), and (ii) where the direct S_{Tx}–S_{Rx} link is absent ($\overline{\gamma}_{sd} = 0$). We also benchmark the SEP of ORGAP with interference-power, fixed-power, and fixed-gain AF relaying. In order to ensure a meaningful comparison, these policies now also adhere to the same average power and average interference constraints. Table I summarizes and compares the various relaying policies.

[3] In [20], the diversity order is evaluated for an alternate scaling regime in which primary and secondary transmit powers increase to infinity, the mean power gain of the channel between S_{Tx} and P_{Rx} tends to zero, and the other mean channel power gains are fixed. Furthermore, it is assumed that S_{Rx} has the ability to completely cancel the interference caused by P_{Tx}.

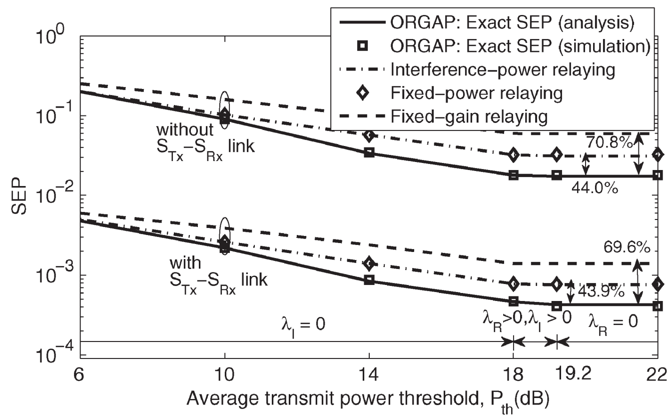

Fig. 3. Effect of direct link: Zoomed-in view of SEPs of ORGAP and benchmark policies as a function of P_{th} ($\overline{\gamma}_{sr} = \overline{\gamma}_{rp} = \overline{\gamma}_{rd} = 1$, $P_s = 18$ dB, $I_{\text{th}} = 18$ dB, $\delta_1 = 1$, $\delta_2 = 1$, and QPSK. With direct link $\overline{\gamma}_{sd} = 1$, and without direct link $\overline{\gamma}_{sd} = 0$).

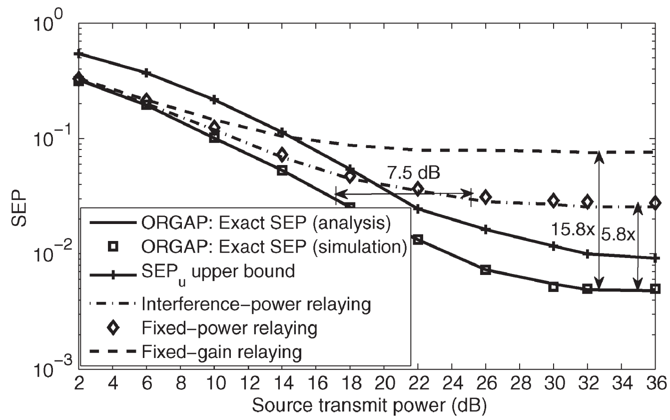

Fig. 4. Effect of P_s: Comparison of SEPs of ORGAP, fixed-gain, fixed-power relaying, and interference-power relaying. Also shown is the SEP upper bound ($\overline{\gamma}_{sr} = \overline{\gamma}_{rd} = \overline{\gamma}_{rp} = 1$, $\overline{\gamma}_{sd} = 0$, $I_{\text{th}} = 15$ dB, $\delta_1 = 1$, $\delta_2 = 1$, $\lambda_R = 0$, and QPSK).

Fig. 3 plots the SEP of ORGAP and the three benchmark policies for QPSK as a function of the average relay transmit power threshold P_{th} with and without the direct link. We observe that the SEP analysis and simulation results of ORGAP match well. The three regimes of operation of the system for ORGAP are also demarcated. An error floor occurs in the interference-constrained regime. In the presence of the direct link, the error floor of ORGAP is 69.6%, 44.7%, and 43.9% lower than that of fixed-gain, fixed-power, and interference-power relaying, respectively. Without the direct link, the error floor of ORGAP is 70.8%, 46.8%, and 44.0% lower than that of fixed-gain, fixed-power, and interference-power relaying, respectively. As expected, the SEP increases when $\overline{\gamma}_{sd} = 0$.

Fig. 4 plots the SEPs of ORGAP and the benchmark policies as a function of the source transmit power P_s in the interference-constrained regime for QPSK. The simulations match well with the analysis results for ORGAP. Compared to ORGAP, the error floors of fixed-gain, interference-power, and fixed-power relaying are greater by a factor of 15.8, 5.3, and 5.8, respectively. At an SEP of 0.03, ORGAP requires 7.5 dB less SNR than interference-power relaying. This corresponds to a significant power saving of 82.2%. These show the efficacy

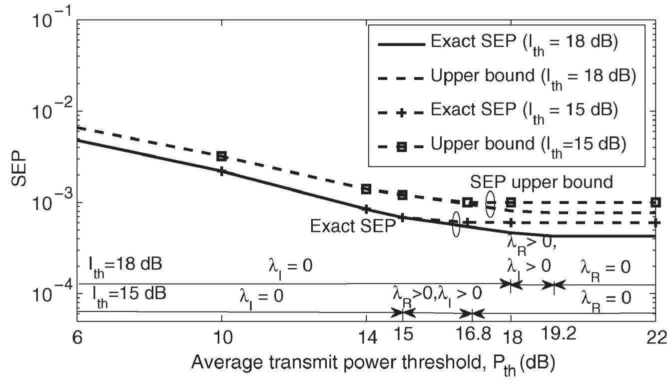

Fig. 5. Effect of $P_{\rm th}$ and $I_{\rm th}$: SEP and SEP_u of ORGAP as a function of $P_{\rm th}$ for different values of $I_{\rm th}$ ($\overline{\gamma}_{sr} = \overline{\gamma}_{rp} = \overline{\gamma}_{rd} = \overline{\gamma}_{sd} = 1$, $P_s = 18$ dB, $\delta_1 = 1$, $\delta_2 = 1$, and QPSK).

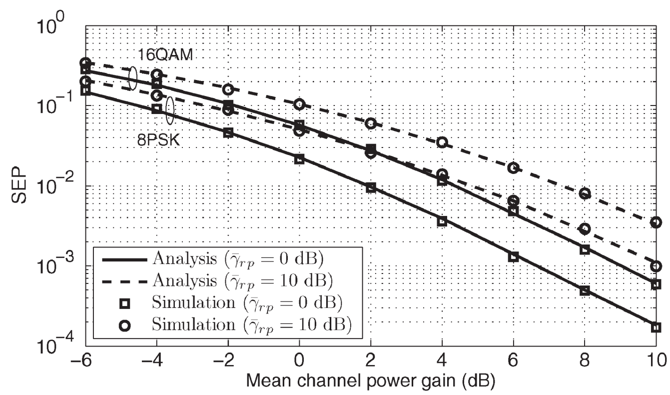

Fig. 6. Effect of R–P_{Rx} channel statistics ($\overline{\gamma}_{rp}$) in interference-constrained regime: SEP of ORGAP as a function of $\gamma_{\rm av}$ ($\overline{\gamma}_{sr} = \overline{\gamma}_{rd} = \overline{\gamma}_{sd} \stackrel{\Delta}{=} \gamma_{\rm av}$, $\delta_1 = 1$, $\delta_2 = 1$, $\lambda_R = 0$, $P_s = 15$ dB, $I_{\rm th} = 15$ dB, and $P_{\rm th} = 20$ dB).

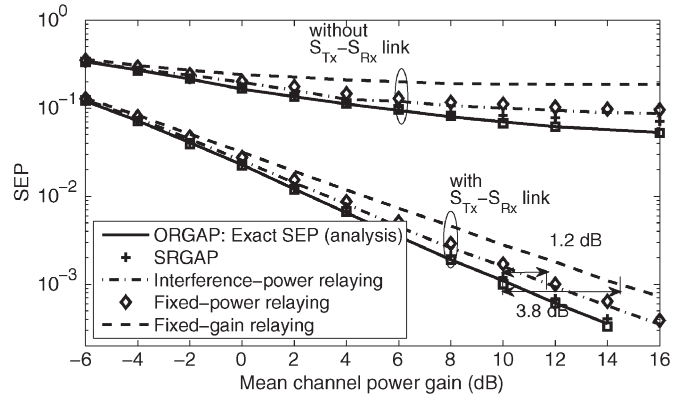

Fig. 7. Comparison of SEPs of ORGAP, SRGAP, and benchmark policies as a function of mean channel power gain $\gamma_{\rm av}$ ($\overline{\gamma}_{sr} = \overline{\gamma}_{rp} = \overline{\gamma}_{rd} \stackrel{\Delta}{=} \gamma_{\rm av}$, $\delta_1 = 1$, $\delta_2 = 1$, $P_s = 15$ dB, $I_{\rm th} = 15$ dB, $P_{\rm th} = 24$ dB, $\lambda_R = 0$, and 8PSK. With the direct link, $\overline{\gamma}_{sd} = \gamma_{\rm av}$, and without the direct link, $\overline{\gamma}_{sd} = 0$).

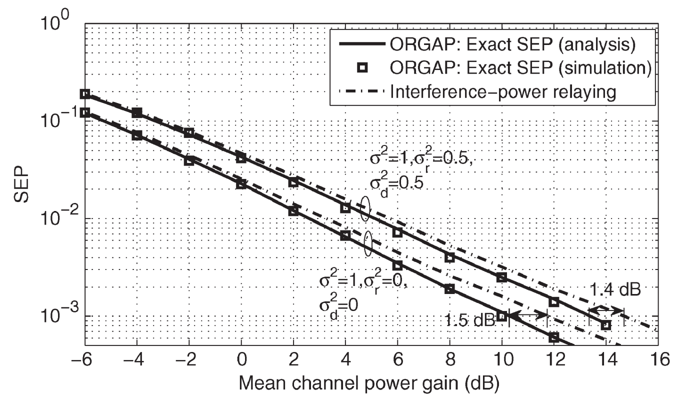

Fig. 8. Impact of the interference from P_{Tx}: Comparison of SEPs of ORGAP, SRGAP, and interference-power relaying as a function of mean channel power gain $\gamma_{\rm av}$ ($\overline{\gamma}_{sr} = \overline{\gamma}_{rp} = \overline{\gamma}_{rd} = \overline{\gamma}_{sd} \stackrel{\Delta}{=} \gamma_{\rm av}$, $P_s = 15$ dB, $P_{\rm th} = 26$ dB, $I_{\rm th} = 15$ dB, $\lambda_R = 0$, and 8PSK).

of the proposed approach. Also shown is the SEP upper bound (cf. (27)). We see that it tracks the exact SEP well, albeit with an offset. The results for 16QAM are similar and are not shown due to space constraints.

Fig. 5 compares the exact SEP of ORGAP and its upper bound (cf. (24)) for QPSK as a function of $P_{\rm th}$ for different $I_{\rm th}$. The three regimes of operation of ORGAP are also demarcated. We observe that the larger the value of $I_{\rm th}$, the smaller the range of $P_{\rm th}$ in which both constraints are simultaneously active. Further, the higher the value of $I_{\rm th}$, the lower the error floor, which is intuitive. We observe that upper bound tracks the SEP well, albeit with a 2 dB shift.

Fig. 6 evaluates the impact of the mean channel power gain of the R–P_{Rx} interference link on the SEP in the interference-constrained regime. As the interference link becomes stronger, i.e., as $\overline{\gamma}_{rp}$ increases, the SEP increases for both constellations. The analysis and simulation results are shown for both MQAM and MPSK, and they match each other well.

SRGAP vs. ORGAP: Fig. 7 compares the SEPs of ORGAP and SRGAP in the interference-constrained regime for 8PSK. The simulation results of ORGAP are also shown for reference. Also plotted are the SEPs of fixed-power, fixed-gain, and interference-power relaying. We see that the SEP of SRGAP is within 0.3 dB of that of ORGAP when the direct link is present. In this case, SRGAP outperforms interference-power relaying by 1.2 dB (24.1% power savings), fixed-power relaying by 1.4 dB (27.6%), and fixed-gain relaying by 3.8 dB (58.3%) at an SEP of 0.001. Without the direct link, the error floor of SRGAP is marginally higher than that of ORGAP. However, SRGAP again outperforms benchmark policies.

Impact of Interference From Primary: Fig. 8 studies the impact of the interference from P_{Tx} on the secondary system. It plots the SEPs of ORGAP and interference-power relaying as a function of $\gamma_{\rm av}$ in the interference-constrained regime for 8PSK. The results for other benchmark policies are not shown to avoid clutter. Results with and without interference from P_{Tx} at R and S_{Rx} are shown. When there is no interference from the P_{Tx}, ORGAP requires 1.5 dB (29.2%) less SNR than interference-power relaying, at an SEP of 0.001. In the presence of interference from P_{Tx}, ORGAP requires 1.4 dB (27.6%) less SNR than interference-power relaying.

A. Extension to Multiple Relays and Selection

We now extend our model to include multiple AF relays and focus on selection because it avoids synchronization problems and is practically appealing [5]–[7]. The relays use the proposed

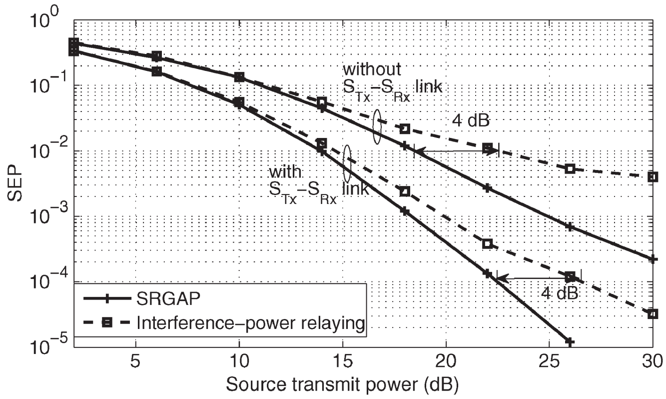

Fig. 9. Multiple relays with selection: Comparison of SEPs of SRGAP and interference-power relaying ($\overline{\gamma}_{sr} = \overline{\gamma}_{rd} = \overline{\gamma}_{rp} = 1$, $I_{\rm th} = 15$ dB, $P_{\rm th} = 24$ dB, $\sigma^2 = 1$, $\delta_1 = 1.5$, $\delta_2 = 1.5$, 4 relays, $\lambda_R = 0$, and 8PSK. With direct link $\overline{\gamma}_{sd} = 1$, and without direct link $\overline{\gamma}_{sd} = 0$).

variable gain adaptation model. The relay that maximizes the end-to-end SNR is selected. In it, the SNR using relay i is given by $\Gamma_i = (P_s\gamma_{sd}/\delta_2) + (P_s\gamma_{si}\gamma_{id}/(\gamma_{id}\delta_1 + \Theta_i^{-1}\delta_2))$, where Θ_i is the gain of relay i (if selected), and is given in Result 1 for ORGAP and in Result 2 for SRGAP. As in the single relay case, the constants λ_R and λ_I (cf. Result 1) is determined numerically once to satisfy the average interference constraint.

Fig. 9 plots the SEP of SRGAP with relay selection as a function of P_s for $L = 4$ relays and compares it with interference-power relaying in the interference constrained regime ($\lambda_R = 0$). The channel gains for different relays are independent and identically distributed. Results with and without the $\mathrm{S_{Tx}}$–$\mathrm{S_{Rx}}$ link are shown. In both cases, we see that SRGAP outperforms interference-power relaying by 4.0 dB, which amounts to a power saving of 60.2%. Thus, SRGAP delivers significant gains with relay selection as well. The gains with respect to the other policies are even greater, and are not shown in order to avoid clutter.

VI. Conclusion

We proposed a novel and optimal AF relay gain adaptation policy for cooperative underlay CR systems in which the relay adapts its gain as a function of its local channel gains. It is unlike the *ad hoc* fixed-power, fixed-gain, and interference-power policies that have been studied in the underlay CR literature. We also analyzed the SEP of ORGAP. More insights were obtained by investigating the power-constrained and interference-constrained regimes, developing bounds, and asymptotics. We saw that ORGAP outperforms the *ad hoc* relaying policies. Depending on the system parameters, the power savings achieved by ORGAP were as large as to 82.2%.

While ORGAP, given its optimality, serves as a fundamental benchmark, it is difficult to implement in practice. SRGAP circumvents this problem by specifying the relay gain in a closed form, and, yet, incurs only a marginal performance penalty compared to ORGAP. Interesting avenues for future work involve developing the optimal relaying policy with multiple antennas, and modeling imperfect or partial CSI.

Appendix

A. Proof of Result 1

Let $\Theta_{\rm opt}^R$ be the optimal relay gain when the relay is subject only to the average transmit power constraint in (15) but not (16). If it is feasible, then it clearly is optimal since it is the solution to a less constrained optimization problem. Else, let $\Theta_{\rm opt}^I$ be the optimal relay gain when the relay is subject only to the interference constraint in (16) but not (15). If it is feasible, then it clearly must be optimal.[4] Else, when both $\Theta_{\rm opt}^R$ and $\Theta_{\rm opt}^I$ are not feasible, let $f(\theta) \triangleq \exp(-mP_s\gamma_{sr}\gamma_{rd}/(\gamma_{rd}\delta_1 + \theta^{-1}\delta_2))$. For any policy, whose relay gain is the function Θ, define the following auxiliary function $L_\Theta(\tilde{\lambda}_I, \tilde{\lambda}_R)$, in terms of two constants $\tilde{\lambda}_R \geq 0$ and $\tilde{\lambda}_I \geq 0$, as follows:

$$L_\Theta(\tilde{\lambda}_I, \tilde{\lambda}_R) = \mathbf{E}\left[f(\Theta)\right] + \tilde{\lambda}_R \mathbf{E}\left[\Theta(P_s\gamma_{sr} + \delta_1)\right] + \tilde{\lambda}_I \mathbf{E}\left[\Theta\gamma_{rp}(P_s\gamma_{sr} + \delta_1)\right]. \quad (28)$$

Consider the policy $\tilde{\Theta}$ that sets its relay gain as follows:

$$\tilde{\Theta} = \arg\min_{\theta\geq 0}\left\{f(\theta) + \tilde{\lambda}_R\theta(P_s\gamma_{sr}+\delta_1) + \tilde{\lambda}_I\theta\gamma_{rp}(P_s\gamma_{sr}+\delta_1)\right\}. \quad (29)$$

Note that $\tilde{\Theta}$ is a function of three variables γ_{sr}, γ_{rd}, and γ_{rp}. For this policy, let $\lambda_R > 0$ and $\lambda_I > 0$ denote the values of $\tilde{\lambda}_R$ and $\tilde{\lambda}_I$, respectively, such that both the constraints in (15) and (16) are met with equality.[5] For these specific values of λ_I and λ_R, let $\Theta_{\rm opt}$ denote the relay gain policy.

From (29), it directly follows that $L_{\Theta_{\rm opt}}(\lambda_I, \lambda_R) \leq L_\Theta(\lambda_I, \lambda_R)$. Substituting this in (28) yields

$$\mathbf{E}\left[f(\Theta_{\rm opt})\right] \leq \mathbf{E}\left[f(\Theta)\right] + \lambda_R\left(\mathbf{E}\left[(P_s\gamma_{sr} + \delta_1)\Theta\right] - P_{\rm th}\right) + \lambda_I\left(\mathbf{E}\left[\gamma_{rp}(P_s\gamma_{sr} + \delta_1)\Theta\right] - I_{\rm th}\right). \quad (30)$$

Since Θ is a feasible policy, we know that $\mathbf{E}[\gamma_{rp}(P_s\gamma_{sr} + \delta_1)\Theta] - I_{\rm th} \leq 0$ and $\mathbf{E}[(P_s\gamma_{sr} + \delta_1)\Theta] - P_{\rm th} \leq 0$. Therefore, from (30), we get $\mathbf{E}[f(\Theta_{\rm opt})] < \mathbf{E}[f(\Theta)]$. Since $\Theta_{\rm opt}$ is feasible and has the lowest SEP among all feasible policies, it is optimal.

It is easy to verify that $\Lambda(\theta) \triangleq f(\theta) + \lambda_R\theta(P_s\gamma_{sr} + \delta_1) + \lambda_I\theta\gamma_{rp}(P_s\gamma_{sr} + \delta_1)$ is strictly convex in θ. Therefore, the

[4]The derivations of the policies $\Theta_{\rm opt}^R$ and $\Theta_{\rm opt}^I$ are simpler versions of the derivation presented here. The former policy corresponds to setting $\lambda_I = 0$ and the latter corresponds to setting $\lambda_R = 0$. Therefore, we do not show these proofs separately.

[5]Their existence follows from the following properties, which enable the intermediate value theorem to be applied. Firstly, the optimal relay gain $\tilde{\Theta}$ is monotonic and continuous in $\tilde{\lambda}_R$ and $\tilde{\lambda}_I$ given the channel gains. Specifically, given any $\tilde{\lambda}_R$, as $\tilde{\lambda}_I$ increases, $\tilde{\Theta}$ decreases. Hence, the average interference and the average relay transmit power decrease (cf. (20), (6), and (9)). Similarly, given any $\tilde{\lambda}_I$, as λ_R increases, the average interference and average transmit power decrease. Secondly, the set of feasible policies is non-empty because transmitting with zero power always is a feasible policy. This policy is optimal when $\tilde{\lambda}_R \to \infty$ or $\tilde{\lambda}_I \to \infty$. Thirdly, since Θ_R is infeasible, there exist a $\tilde{\lambda}_R > 0$ and $\tilde{\lambda}_I = 0$ such that the average interference is strictly greater than $I_{\rm th}$ and the average power is equal to $P_{\rm th}$. Similarly, since Θ_I is infeasible, there exist a $\tilde{\lambda}_R = 0$ and $\tilde{\lambda}_I > 0$ such that the average power is strictly greater than $P_{\rm th}$ and the average interference is equal to $I_{\rm th}$.

optimal solution is unique. It is the non-negative solution of

$$\frac{\partial \Lambda(\theta)}{\partial \theta} = -\exp\left(-\frac{mP_s\gamma_{sr}\gamma_{rd}}{\gamma_{rd}\delta_1+\theta^{-1}\delta_2}\right)\frac{mP_s\delta_2\gamma_{sr}\gamma_{rd}}{(\gamma_{rd}\delta_1\theta+\delta_2)^2} + (\lambda_R+\lambda_I\gamma_{rp})(P_s\gamma_{sr}+\delta_1)=0, \quad (31)$$

if it exists, and is 0, otherwise. Simplifying (31) results in (20). The boundary of the region in which Θ_{opt} is 0 is obtained by substituting $\theta = 0$ in (20).

B. Brief Proof of Result 2

We know that $\exp(mP_s\gamma_{sr}\gamma_{rd}/(\gamma_{rd}\delta_1+\theta^{-1}\delta_2)) \geq 1 + (mP_s\gamma_{sr}\gamma_{rd}/(\gamma_{rd}\delta_1+\theta^{-1}\delta_2))^2$, for $\theta \geq 0$. Substituting this in (20) and rearranging terms, we can show that $\Delta(\theta) \geq \Psi(\theta)$, where $\Psi(\theta)$ is defined as

$$\Psi(\theta) \triangleq \gamma_{rd}^2\left(\delta_1^2+m^2P_s^2\gamma_{sr}^2\right)\theta^2 + 2\delta_1\delta_2\gamma_{rd}\theta + \delta_2\left(\delta_2 - \frac{mP_s\gamma_{sr}\gamma_{rd}}{(\lambda_R+\lambda_I\gamma_{rp})(P_s\gamma_{sr}+\delta_1)}\right). \quad (32)$$

From the properties of quadratic equations, it can be verified that when $\gamma_{rd} \geq \mathcal{B}(\gamma_{sr},\gamma_{rp})$, $\Psi(\theta)$ has exactly one positive root η, which is equal to

$$\eta = \frac{\delta_2\left(-\delta_1+\sqrt{\frac{mP_s\gamma_{sr}\gamma_{rd}\left(\delta_1^2+m^2P_s^2\gamma_{sr}^2\right)}{\delta_2(\lambda_R+\lambda_I\gamma_{rp})(P_s\gamma_{sr}+\delta_1)} - m^2P_s^2\gamma_{sr}^2}\right)}{\gamma_{rd}\left(\delta_1^2+m^2P_s^2\gamma_{sr}^2\right)}.$$

Since $\Delta(\theta) \geq \Psi(\theta)$ and since both are convex for $\theta \geq 0$, it can be easily shown that $\Theta_{\text{opt}} \leq \eta$. Finally, by dropping the negative $m^2P_s^2\gamma_{sr}^2$ term inside the square root, we get $\eta \leq \Theta_u$, where Θ_u is given in (21).[6] Therefore, $\Theta_{\text{opt}} \leq \Theta_u$.

C. Derivation of Result 3

To simplify the SEP expression in (23), we make the following observations. First, replacing $\sin^2\psi$ with unity yields a Chernoff upper bound for the SEP that involves three integrals. Second, for $\gamma_{rd} < \mathcal{B}(\gamma_{sr},\gamma_{rp})$, we have $\Theta_{\text{opt}} = 0$. Thus, we can replace $\exp(mP_s\gamma_{sr}\gamma_{rd}/(\gamma_{rd}\delta_1+\Theta_{\text{opt}}^{-1}\delta_2))$ with unity in this region. Third, for $\gamma_{rd} \geq \mathcal{B}(\gamma_{sr},\gamma_{rp})$, we know from (20) that

$$e^{-\frac{mP_s\gamma_{sr}\gamma_{rd}}{\gamma_{rd}\delta_1+\Theta_{\text{opt}}^{-1}\delta_2}} = \frac{(\lambda_R+\gamma_{rp}\lambda_I)(P_s\gamma_{sr}+\delta_1)}{mP_s\delta_2\gamma_{sr}\gamma_{rd}}(\delta_1\Theta_{\text{opt}}\gamma_{rd}+\delta_2)^2. \quad (33)$$

From (21), we have $\delta_1\Theta_{\text{opt}}\gamma_{rd}/\delta_2+1 \leq \delta_1\Theta_u\gamma_{rd}/\delta_2+1 = b_1+\sqrt{b_2\gamma_{rd}}$, where $b_1 = m^2P_s^2\gamma_{sr}^2/(\delta_1^2+m^2P_s^2\gamma_{sr}^2)$ and $b_2 = \mathcal{B}(\gamma_{sr},\gamma_{rp})^{-1}(\delta_1^2+m^2P_s^2\gamma_{sr}^2)^{-1}$.

[6] There are two differences between the above derivation and that in [17]. Firstly, the R–P_{Rx} gain γ_{rp} occurs throughout the derivation due to the interference constraint and needs to be kept track of. Secondly, Θ_u has a simpler form than in [17, (17)].

Combining all the above observations, we can show that SEP $\leq T_1 + T_2$, where

$$T_1 = \frac{\Xi_0}{\overline{\gamma}_{sr}\overline{\gamma}_{rp}\overline{\gamma}_{rd}}\int_0^\infty\int_0^\infty\int_0^{\mathcal{B}(\gamma_{sr},\gamma_{rp})} e^{-\frac{\gamma_{sr}}{\overline{\gamma}_{sr}}}e^{-\frac{\gamma_{rp}}{\overline{\gamma}_{rp}}} \times e^{-\frac{\gamma_{rd}}{\overline{\gamma}_{rd}}}d\gamma_{sr}\,d\gamma_{rp}\,d\gamma_{rd}, \quad (34)$$

$$T_2 = \frac{\delta_2\Xi_0}{\overline{\gamma}_{sr}\overline{\gamma}_{rp}\overline{\gamma}_{rd}}\int_0^\infty\int_0^\infty\int_{\mathcal{B}(\gamma_{sr},\gamma_{rp})}^\infty e^{-\frac{\gamma_{sr}}{\overline{\gamma}_{sr}}}e^{-\frac{\gamma_{rp}}{\overline{\gamma}_{rp}}}e^{-\frac{\gamma_{rd}}{\overline{\gamma}_{rd}}} \times \frac{\mathcal{B}(\gamma_{sr},\gamma_{rp})(b_1+\sqrt{b_2\gamma_{rd}})^2}{\gamma_{rd}}d\gamma_{sr}\,d\gamma_{rp}\,d\gamma_{rd}. \quad (35)$$

Expression for T_1: Simplifying (34) further using [28, (3.310), (3.324.1)] and partial fractions yields (25).

Expression for T_2: It can be recast as

$$T_2 = \frac{\delta_2\Xi_0}{\overline{\gamma}_{sr}\overline{\gamma}_{rp}\overline{\gamma}_{rd}}\int_0^\infty\int_0^\infty \mathcal{B}(\gamma_{sr},\gamma_{rp})e^{-\frac{\gamma_{rp}}{\overline{\gamma}_{rp}}} \times e^{-\frac{\gamma_{sr}}{\overline{\gamma}_{sr}}}T_2^{\text{in}}(\gamma_{sr},\gamma_{rp})d\gamma_{rp}\,d\gamma_{sr}, \quad (36)$$

where $T_2^{\text{in}} = b_1^2\varphi_1 + b_2\varphi_2 + 2b_1\sqrt{b_2}\varphi_3$ and

$$\varphi_1 \triangleq \int_{\mathcal{B}(\gamma_{sr},\gamma_{rp})}^\infty \frac{e^{-\frac{\gamma_{rd}}{\overline{\gamma}_{rd}}}}{\gamma_{rd}}d\gamma_{rd} = E_1\left(\frac{\mathcal{B}(\gamma_{sr},\gamma_{rp})}{\overline{\gamma}_{rd}}\right), \quad (37)$$

$$\varphi_2 \triangleq \int_{\mathcal{B}(\gamma_{sr},\gamma_{rp})}^\infty e^{-\frac{\gamma_{rd}}{\overline{\gamma}_{rd}}}d\gamma_{rd} = \overline{\gamma}_{rd}e^{-\frac{\mathcal{B}(\gamma_{sr},\gamma_{rp})}{\overline{\gamma}_{rd}}}, \quad (38)$$

$$\varphi_3 \triangleq \int_{\mathcal{B}(\gamma_{sr},\gamma_{rp})}^\infty \frac{e^{-\frac{\gamma_{rd}}{\overline{\gamma}_{rd}}}}{\sqrt{\gamma_{rd}}}d\gamma_{rd} = \sqrt{\pi\overline{\gamma}_{rd}}\,\text{erfc}\left(\sqrt{\frac{\mathcal{B}(\gamma_{sr},\gamma_{rp})}{\overline{\gamma}_{rd}}}\right). \quad (39)$$

Substituting (37)–(39) in (36) results in (26).

D. Brief Derivation of Result 4

To derive this, we follow the approach employed in the previous proof. Since $\lambda_R = 0$ in the interference-constrained regime, the boundary region $\mathcal{B}(\gamma_{sr},\gamma_{rp})$ simplifies to $\gamma_{rp}\mathcal{B}_I(\gamma_{sr})$, where $\mathcal{B}_I(\gamma_{sr}) = (\delta_2\lambda_I/m)(1+(\delta_1/(P_s\gamma_{sr})))$. As in Appendix C, we can show that SEP $\leq J_1 + J_2$, where

$$J_1 = \frac{\Xi_0}{\overline{\gamma}_{sr}\overline{\gamma}_{rp}\overline{\gamma}_{rd}}\int_0^\infty\int_0^\infty\int_0^{\mathcal{B}(\gamma_{sr},\gamma_{rp})} e^{-\frac{\gamma_{sr}}{\overline{\gamma}_{sr}}} \times e^{-\frac{\gamma_{rp}}{\overline{\gamma}_{rp}}}e^{-\frac{\gamma_{rd}}{\overline{\gamma}_{rd}}}d\gamma_{sr}\,d\gamma_{rp}\,d\gamma_{rd}, \quad (40)$$

$$J_2 = \frac{\Xi_0 \delta_2}{\overline{\gamma}_{sr}\overline{\gamma}_{rp}\overline{\gamma}_{rd}} \times \int_0^\infty \int_0^\infty \left[\left(\frac{2m^2 P_s^2 \gamma_{sr}^2}{\omega(\gamma_{sr})} \right) \sqrt{\frac{\pi \overline{\gamma}_{rd}}{\gamma_{rp}\mathcal{B}_I(\gamma_{sr})\omega(\gamma_{sr})}} \times \operatorname{erfc}\left(\sqrt{\frac{\gamma_{rp}\mathcal{B}_I(\gamma_{sr})}{\overline{\gamma}_{rd}}} \right) + \frac{\overline{\gamma}_{rd} e^{-\frac{\gamma_{rp}\mathcal{B}_I(\gamma_{sr})}{\overline{\gamma}_{rd}}}}{\gamma_{rp}\mathcal{B}_I(\gamma_{sr})\omega(\gamma_{sr})} + \left(\frac{m^2 P_s^2 \gamma_{sr}^2}{\omega(\gamma_{sr})} \right)^2 E_1\left(\frac{\gamma_{rp}\mathcal{B}_I(\gamma_{sr})}{\overline{\gamma}_{rd}} \right) \right] \times \gamma_{rp}\mathcal{B}_I(\gamma_{sr}) e^{-\frac{\gamma_{rp}}{\overline{\gamma}_{rp}}} e^{-\frac{\gamma_{sr}}{\overline{\gamma}_{sr}}} d\gamma_{rp}\, d\gamma_{sr}, \quad (41)$$

where $\omega(\gamma_{sr})$ is defined in Result 4. Averaging over γ_{rd} and γ_{rp}, it can be shown using [27, (5.1.1)] that $J_1 = \Xi_0(\omega_2(P_s\overline{\gamma}_{sr}/(\delta_1\delta_2)) + (\omega_2 e^{\omega_2} E_1(\omega_2)/\omega_1))$, where ω_1 and ω_2 are defined in Result 4. Similarly, using [28, (6.227.1), (6.292.1)], J_2 can be simplified. Summing J_1 and J_2 yields (27).

E. Diversity Order Analysis

We show that the diversity order is two for the alternate scaling regime in which $P_s \to \infty$, I_{th}/P_s is constant, P_{th}/P_s is a constant or tends to zero, $\overline{\gamma}_{rp} \to 0$, and the other mean channel power gains are fixed, as follows. First, we know that the diversity order cannot exceed two because it is two even for interference-unconstrained conventional AF relaying [17], [18]. Next, we derive below a lower bound on the SEP and show from it that the diversity order is at least two. The above two facts together imply that the diversity order is two.

The expression for the exact SEP of ORGAP is

$$\frac{1}{\pi} \int_0^{\left(\frac{M-1}{M}\right)\pi} \frac{\mathbf{E}\left[\exp\left(-\frac{P_s \gamma_{sr}\gamma_{rd}}{\gamma_{rd}\delta_1 + \delta_2 \Theta_{\text{opt}}^{-1}} \frac{m}{\sin^2\psi} \right) \right]}{1 + \frac{P_s \overline{\gamma}_{sd}}{\delta_2} \frac{m}{\sin^2\psi}} d\psi.$$

We obtain a lower bound by replacing $\sin^2\psi$ with its lower bound of 0, for $0 \le \psi < \pi/4$ and $3\pi/4 < \psi < \pi$, and with its lower bound of 1/2, for $\pi/4 \le \psi \le 3\pi/4$. This yields

$$\text{SEP} \ge \frac{\mathbf{E}\left[\exp\left(-\frac{2mP_s\gamma_{sr}\gamma_{rd}}{\gamma_{rd}\delta_1 + \Theta_{\text{opt}}^{-1}\delta_2} \right) \right]}{2\left(1 + \frac{2mP_s\overline{\gamma}_{sd}}{\delta_2} \right)}. \quad (42)$$

As $P_s \to \infty$, $\mathcal{B}(\gamma_{sr}, \gamma_{rp}) \to \delta_2\lambda_I\gamma_{rp}/m$. When $\gamma_{rd} < \lambda_I\gamma_{rp}/m$, $\exp(-2mP_s\gamma_{sr}\gamma_{rd}/(\gamma_{rd}\delta_1 + \Theta_{\text{opt}}^{-1}\delta_2))$ is unity because $\Theta_{\text{opt}} = 0$. Using this to expand the expectation in (42), we get

$$\mathbf{E}\left[\exp\left(-\frac{2mP_s\gamma_{sr}\gamma_{rd}}{\gamma_{rd}\delta_1 + \Theta_{\text{opt}}^{-1}\delta_2} \right) \right] = \frac{1}{\overline{\gamma}_{sr}\overline{\gamma}_{rp}\overline{\gamma}_{rd}} \times \int_0^\infty \int_0^\infty \int_0^{\frac{\delta_2\lambda_I\gamma_{rp}}{m}} e^{-\frac{\gamma_{sr}}{\overline{\gamma}_{sr}}} e^{-\frac{\gamma_{rp}}{\overline{\gamma}_{rp}}} e^{-\frac{\gamma_{rd}}{\overline{\gamma}_{rd}}} d\gamma_{sr}\, d\gamma_{rp}\, d\gamma_{rd} + \frac{1}{\overline{\gamma}_{sr}\overline{\gamma}_{rp}\overline{\gamma}_{rd}} \int_0^\infty \int_0^\infty \int_{\frac{\delta_2\lambda_I\gamma_{rp}}{m}}^\infty \exp\left(-\frac{2mP_s\gamma_{sr}\gamma_{rd}}{\gamma_{rd}\delta_1 + \Theta_{\text{opt}}^{-1}\delta_2} \right) \times e^{-\frac{\gamma_{sr}}{\overline{\gamma}_{sr}}} e^{-\frac{\gamma_{rp}}{\overline{\gamma}_{rp}}} e^{-\frac{\gamma_{rd}}{\overline{\gamma}_{rd}}} d\gamma_{sr}\, d\gamma_{rp}\, d\gamma_{rd}. \quad (43)$$

Using [28, (3.310))] and $e^{-2mP_s\gamma_{sr}\gamma_{rd}/(\gamma_{rd}\delta_1 + \Theta_{\text{opt}}^{-1}\delta_2)} \ge e^{-2mP_s\gamma_{sr}/\delta_1}$, we can show that

$$\text{SEP} \ge \frac{1}{2} \frac{1}{\left(1 + \frac{2mP_s\overline{\gamma}_{sd}}{\delta_2} \right)} \left[\frac{\delta_2\lambda_I\overline{\gamma}_{rp}}{\delta_2\lambda_I\overline{\gamma}_{rp} + m\overline{\gamma}_{rd}} + \frac{m\delta_1\overline{\gamma}_{rd}}{(2mP_s\overline{\gamma}_{sr} + \delta_1)(\delta_2\lambda_I\overline{\gamma}_{rp} + m\overline{\gamma}_{rd})} \right]. \quad (44)$$

Taking limits as $\overline{\gamma}_{rp} \to 0$ and $P_s \to \infty$, we can show that $\text{SEP} \ge \delta_1\delta_2/(8m^2\overline{\gamma}_{sr}\overline{\gamma}_{rd}P_s^2)$. Thus, the diversity order is at least two.

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B. Sainath received the B.Tech. degree in electronics and communications engineering from Sri Venkateswara University College of Engineering (SVUCE), Tirupati, India, and the M.Tech. degree from the National Institute of Technology, Calicut, India, in 2001 and 2007, respectively. He is currently working toward the Ph.D. degree with the Department of Electrical Communication Engineering, Indian Institute of Science, Bangalore, India. After a brief stint with Zuari Cements, he joined and served as a Faculty Member with various academic institutions from October 2001 to November 2004. From June 2007 to December 2008, he worked with Reliance Communications, Navi Mumbai, India.

Neelesh B. Mehta (S'98–M'01–SM'06) received the Bachelor of Technology degree in electronics and communications engineering from the Indian Institute of Technology Madras, Chennai, India, and the M.S. and Ph.D. degrees in electrical engineering from the California Institute of Technology, Pasadena, CA, USA, in 1996, 1997, and 2001, respectively. He is currently an Associate Professor with the Department of Electrical Communication Engineering, Indian Institute of Science, Bangalore, India. Dr. Mehta currently serves as a Member-at-Large on the Board of Governors of the IEEE COMMUNICATIONS SOCIETY. He is an editor of the IEEE TRANSACTIONS ON COMMUNICATIONS AND IEEE WIRELESS COMMUNICATION LETTERS, and an Executive Editor of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS.