1. Recall the revenue optimal mechanism with iid valuations. The function $m_i(x_i)$ is the expected payment from buyer $i$, conditioned on buyer $i$ valuation of $x_i$, for the revenue optimal mechanism. Suppose now that the payment rule is modified to $P_i(x_i)1\{i \text{ wins}\}$, where the function $P_i$ is chosen so that $E[P_i(X)1\{i \text{ wins}\}|X_i = x_i] = m_i(x_i)$. Thus buyer $i$ knows exactly what he will pay if he wins, at the time of bidding. Is truthful reporting a weakly dominant strategy? Is truthful reporting by all buyers a Bayes equilibrium?

2. For the revenue optimal mechanism, prove that the expected value of the virtual valuation is 0.

3. Suppose that a VCG mechanism is used to sell two objects $O = \{a, b\}$ to three buyers. Each buyer can buy none, one, or both of the objects. Each buyer is asked to report his valuation function, i.e., $u_i = (u_i(\emptyset), u_i(\{a\}), u_i(\{b\}), u_i(\{a, b\}))$. Clarke’s pivotal mechanism is used to allocate the objects and determine the payments. Suppose that

\[
\begin{align*}
    u_1 &= (0, 10, 3, 13) \\
    u_2 &= (0, 2, 8, 10) \\
    u_3 &= (0, 3, 2, 14).
\end{align*}
\]

Determine the assignment of objects and payments. Why might buyer 3 object to the outcome?

4. Consider two buyers for a good and the English auction for its sale. The signal $X_i$ to buyer $i$ is binary valued. The joint distribution is given by

\[
p(0, 0) = 1/10, \quad p(0, 1) = p(1, 0) = 1/5, \quad p(1, 1) = 1/2.
\]

Verify that the joint distribution is affiliated.

The valuation function for each buyer is $v_i(x) = x - i$, i.e., each buyer’s valuation is the other buyer’s signal. Compute the equilibrium strategy. (a) When buyer 1 sees a signal 0, at what price will he drop out of the auction? When he sees a signal 1, at what price will he now drop out?