Remarks:

- Collaboration, discussion, and working in teams to solve problems is strongly encouraged.
- To test your understanding, write the solution to each problem in your own words without referring to a friend, text, or class notes.

Problems:

1. Recall the notation that \( \leq_n \) stands for the relation "is less than or equals for all sufficiently large \( n \)". Suppose that for each \( \varepsilon > 0 \), we have \( a_n \leq_n a + \varepsilon \). Show that \( \lim \sup_{n \to \infty} a_n \leq a \).

2. Let \( a_n \leq b_n \). Show that \( \lim \sup_{n \to \infty} a_n \leq \lim \sup_{n \to \infty} b_n \).

3. Let \( a = \lim \sup_{n \to \infty} a_n \in \mathbb{R} \). Show that for every \( \varepsilon > 0 \), the inequality \( a_n > a - \varepsilon \) occurs infinitely often.

4. What are the analogous statements for \( \lim \inf \)?

5. Show that \( \lim \inf_{n \to \infty} a_n \leq \lim \sup_{n \to \infty} a_n \).

6. Show that \( \lim \inf_{n \to \infty} a_n = \lim \sup_{n \to \infty} a_n = a \in \mathbb{R} \) if and only if the following holds: for every \( \varepsilon > 0 \), there exists an \( N \) such that \( n \geq N \) implies \( |a_n - a| \leq \varepsilon \). This establishes that the usual notion of a limit and the one via \( \lim \sup \) and \( \lim \inf \) are equivalent.

7. Problem 3.3 of Cover and Thomas (2nd edition).


For this problem, generate \( \binom{n}{k} p^k (1 - p)^{n-k} \) and \(-n^{-1} \log p(x^n)\) values by yourselves (via matlab or otherwise) and check if they match with the text.