1 SW Theorem – Converse

**Converse:** By providing $X_n^2$ to encoder 1 (and decoder), we have $R_1 \geq H(X_1|X_2)$. Similarly, $R_2 \geq H(X_2|X_1)$. Joint encoding converse indicates $R_1 + R_2 \geq H(X_1X_2)$.

2 Generalisation to Many Sources

**Theorem 1.** *(Cover)*: Suppose $X^n_{[J]}$ is iid with generic distribution $p_{X^n_{[J]}}(x_{[J]})$. The achievable rate region is

$$\left\{ R_{[J]} \in \mathbb{R}_+^d : R(S) \geq H(X_S|X_{S^c}), \forall S \subseteq [J] \right\}$$

- Can extend to jointly ergodic sources. Only AEP is needed with notion of typical set modified to entropy typicality.

**Theorem 2.** *(Han 1979)*: With $\sigma(S) := H(X_S|X_{S^c}), \forall S \subseteq [J]$, $\sigma$ is a supermodular rank function, i.e.,

1. $\sigma(\emptyset) = 0$
2. $\sigma(S) \leq \sigma(T)$ if $S \subseteq T \subseteq [J]$
3. $\sigma(S \cup T) + \sigma(S \cap T) \geq \sigma(S) + \sigma(T)$

**Proof.** Exercise.

**Remark 1.**
- Let $\mathcal{C}(\sigma) := \left\{ R \in \mathbb{R}_+^d : R(S) \geq \sigma(S), \forall S \subseteq [J] \right\}$, a polyhedron.
- $([J],\sigma)$ : contrapolymatroid. $\mathcal{C}(\sigma)$ associated polyhedron.
- $R \in \mathcal{C}(\sigma)$ is a minimal extreme point iff $R_{\pi_1} = \sigma(\{\pi_1\})$, $R_{\pi_k} = \sigma(\{\pi_1,\cdots,\pi_k\}) - \sigma(\{\pi_1,\cdots,\pi_{k-1}\})$.
- Any $R$ dominates some convex combination of these $J!$ extreme points.